THE INFLUENCE OF A MODIFIED MIXING-LENGTH THEORY AND OF AN ADOPTED DESCRIPTION OF THE ATMOSPHERE ON THE SOLAR FIVE-MINUTE OSCILLATIONS

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ABSTRACT. The influence of the modified treatment of subphotospheric convection, as recommended by Deupree (1979) and by Deupree and Varner (1980), on the frequencies of solar five-minute oscillations of degree l = 1-100 is studied. As compared with the results for a standard solar model, the convection theory modification has practically no effect on the frequencies near the low frequency edge of the observational interval (at $\nu \simeq$ 2000 μHz), but causes a frequency decrease for high overtones; the effect is larger at larger frequencies. At $\nu \simeq$ 4000 μHz the frequency decrease is about 4-6 μHz for all $l \leqslant 40$ and about 8-10 μHz for $l \simeq$ 60-100. If, additionally, we use the dependence $T(\tau)$ according to Holweger and Müller (1974) instead of the HSRA model, the joint effect is 1.5-2 times larger. In this case the slope of the theoretical curves in "echelle diagrams" turns out to be in agreement with the observational one for $l \leqslant 20$, but the frequencies themselves are approximately 10 μHz lower than those from observations.

Deupree (1979) and Deupree and Varner (1980) proposed to modify the mixing-length theory which is very often used to describe non-adiabatic convection in stellar envelopes. The most significant of these modifications concerns the necessity of taking into account temperature differences between rising and sinking convective elements when computing the opacity of the matter at a given depth in the stellar envelope. Another modification, based on the results of two-dimensional hydrodynamic computations of convection, proposes to use a variable ratio of mixing-length to pressure scale height. This ratio is given in Deupree and Varner (1980) as a simple analytical function of the temperature. Lester, Lane and Kurucz (1982) investigated the influence of these modifications on the structure of stellar atmospheres. A comparison of the modified theoretical models with semi-empirical solar atmosphere models shows good agreement for $\tau \leqslant 2.5$. For deeper layers, due to the lack of accurate experimental data on the temperature distribution with depth, it is not possible to distinguish among the different theoretical models of convection.

We have studied the influence of the suggested modifications in the mixing-length theory on the theoretical spectrum of five-minute oscillations

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and compared the computed frequencies with published data from observations. In preliminary computations, performed in collaboration with V. A. Baturin (to be published), we used outer boundary conditions applied to the photosphere, therefore the results are not enough precise.

To construct models of the present Sun we used Schwarzschild's fitting method and computed the evolution of one solar mass from zero age main sequence. The heavy element abundance Z was taken to be equal to 0.02. The initial hydrogen abundance X and the mixing length parameter α were adjusted to give solar luminosity and radius at the age of 4.75 billion years. The structure of the solar atmosphere was computed using either the $T(\tau)$ dependence of the HSRA model (Gingerich et al., 1971) or the one from Holweger and Müller (1974). Outer boundary conditions in pulsation calculations were applied at the level $\tau \simeq 10^{-4}$.

The opacity coefficient was taken from tables by Alexander (1975) and by Cox and Stewart (1969). The nuclear reaction rates for pp and CNO cycles were taken from Fowler et al. (1975). We used the equation of state, taking partial degeneracy into account according to Paczynski's stellar evolution code (Paczynski, 1970).

The basic characteristics of the models are given in Table I. Each model consists of ≃490 layers, with roughly 40 layers in the atmosphere (from $\tau \simeq 10^{-4}$ to $\tau \simeq 2/3$) and about 230 layers in the convection zone.

TABLE I. Basic characteristics of the models

Model	1	2	3
Mixing-length theory	STANDARD	MODIFIED	MODIFIED
Model of atmosphere	HSRA	HSRA	Holweger & Müller
$X_{initial}$	0.7466	0.74655	0.7465
$\alpha = l/H_p$	1.615	3.45	6.50
Age, 10^9 yrs	4.752	4.760	4.750
R, 10^{10} cm	6.9605	6.9600	6.9599
L, 10^{33} erg/s	3.8238	3.8265	3.8254
T_{center}, 10^6 K	14.948	14.953	14.949
ρ_{center}, g/cm^3	153.28	153.47	153.25
X_{center}	0.3965	0.3959	0.3966
D_{conv}, km	183200	183400	184500
m_{conv}, $M_\odot$	0.01670	0.01675	0.01696
T_{bottom}, 10^6 K	1.923	1.923	1.932
ρ_{bottom}, g/cm^3	0.1330	0.1334	0.1349

In computing Models 2 and 3 we used, for T < 10000 K, the variable ratio of the mixing length to the pressure scale height, as suggested by Deupree and Varner (1980). For temperatures between 10000 K and, approximately, 13000 K we assumed that $\lg(\alpha)$ varies linearly with

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temperature, reaching a specified value which was considered to be constant for deeper layers. This value is given in the Table I. It is necessary to mention that the functional dependence of α on T, which we adopted for T > 10000 K, is *ad hoc*. Unfortunately, the choice of behaviour of α near this region (where convection is strongly non-adiabatic) contributes significantly to the effect of the proposed modifications of the mixing-length theory on pulsation frequencies.

In all models, the depth of the convection zone and the physical conditions in the center are almost the same. They are also close to the corresponding values in Model 1 by Christensen-Dalsgaard (1982). The differences between all our models lie in the structure of the atmosphere and the non-adiabatic convection region.

The frequencies of linear adiabatic oscillations were computed using programs by Dziembowski (1977) for modes of the followong spherical harmonic degree: l = 0, 1, 2, 3, 4, 10, 20, 40, 60 and 100. For $l > 20$ the Cowling approximation was used. The computations were performed for the models both with the number of mesh-points in the equilibrium model and with four times finer a grid mesh-points produced by interpolation. The oscillation frequencies in the limit of the infinite number of mesh-points were then calculated by analogy with Shibahashi and Osaki (1981).

To compare the theoretical frequencies with those from observations we used equally weighted averages of data published by Claverie et al. (1981), Grec et al. (1983), Duvall and Harvey (1983), Harvey and Duvall (1984), Woodard and Hudson (1983) and Hill (1985).

In Fig. 1, the differences between observational and theoretical frequencies are presented. This figure is similar to Fig. 11 by Christensen-Dalsgaard (1984), but the curves for different values of l were shifted to one another to see more explicitly the differences between different models. The results for our Model 1 are close to those by Christensen-Dalsgaard (1984), with the same morphology of the curves. The modification of the mixing-length theory (Model 2 in comparison with Model 1) has practically no influence on the frequencies near the low frequency edge of the observational interval (at $\nu \simeq 2000$ μHz), but results in frequency decrease for higher overtones. The effect is larger for larger frequencies and only slightly depends on l at $l \leqslant 40$. It is more pronounced for l = 60 and 100. If we consider the modified convection theory and use the solar atmosphere model with the T(τ) dependence according to Holweger and Müller (1974), instead of the one according to HSRA, the effect is 1.5-2 times larger (compare Model 3 with Model 1). The differences ($\langle\nu_{obs}\rangle - \nu_{theor}$) for modified models are more uniform in frequency than those for Model 1. However, the typical values of the differences of about 10 μHz are too large to be considered in good agreement with the observations.

In Fig. 2, the theoretical oscillation frequencies for Models 1 and 3 are plotted together with one series of observational data in the form of "echelle diagrams". The frequency of each oscillation mode was defined as $\nu = \nu_{\bullet} + \delta\nu$, with $\delta\nu$ between 0 and 135 μHz (with the exception of the extreme left and right parts of the figure), and the values of $\nu_{\bullet}$ and $\delta\nu$ are plotted in the figure. The general slope and non-linearities of the curves for Model 3 are similar to those for the observational data, but the theoretical frequencies themselves are approximately 10 μHz lower.

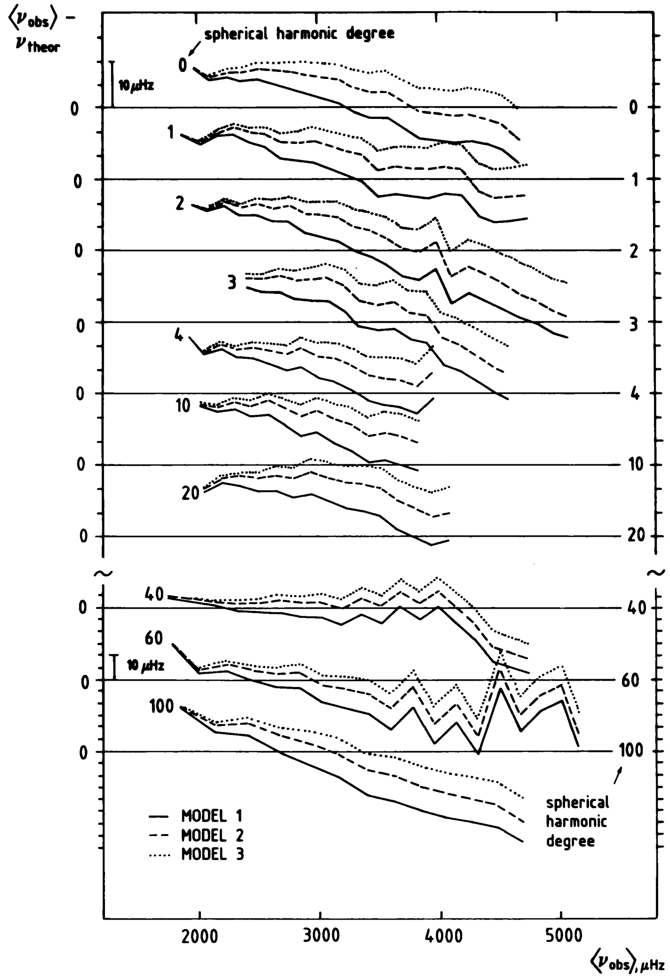

Figure 1. Difference between averaged observational and theoretical frequencies. For clarity, the curves for modes of different degree *l* are shifted one relative to the other. The values of *l* are indicated. Note the different vertical scales for $l \leq 20$ and $l \geq 40$. The positive values of the differences ($\langle\nu_{obs}\rangle - \nu_{theor}$) lie above and the negative ones below each corresponding abscissa.

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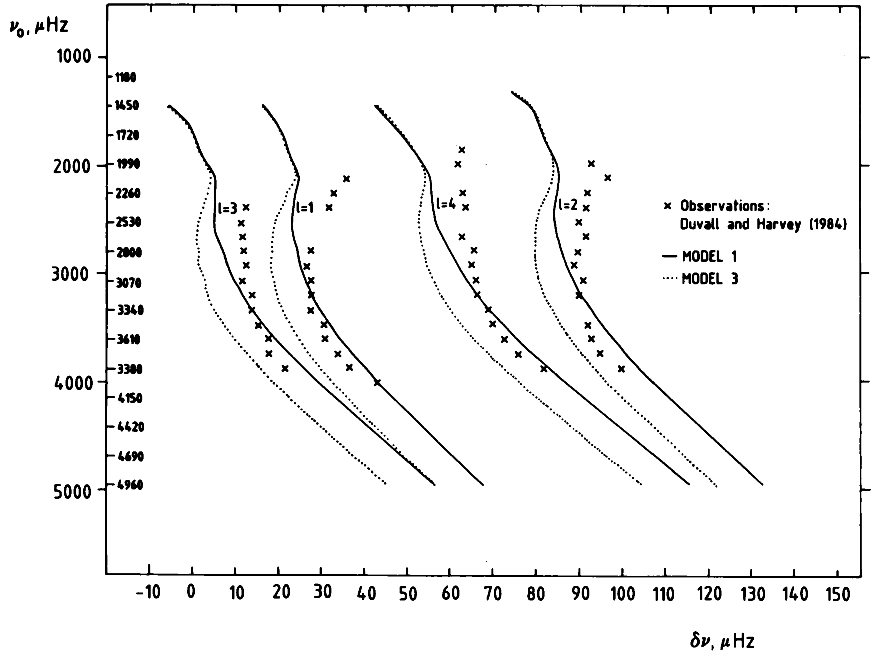

Figure 2. Echelle diagrams for Models 1 and 3 and for a single series of observational data. (Theoretical data for l = 0 are not indicated.)

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