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Computational approaches to combinatorial group theory

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Computers have been successfully applied to research in pure mathematics mainly by following algorithms or heuristics where the calculation is beyond the scope of hand computation.

In view of the great computational power of current computers, programs have been written to follow known algorithms and have achieved significant new results. New algorithms have been constructed whose application is beyond human capabilities but which, in machine implementation, have achieved new results.

The object of this thesis is to consider the application of computers to group theoretic problems. Computer implementations of certain algorithms are made to investigate a number of difficult mathematical problems and some advances are achieved. The actual algorithms are studied and consideration is given to the problems arising in their machine implementation.

Chapters 1, 2 and 3 deal with finitely presented groups and, in particular, with the enumeration of the cosets of subgroups of finite index. A study of a coset enumeration algorithm is made and a number of new applications are described. Substantial parts of these chapters have been published in Cannon, Dimino, Havas and Watson [1] and in Havas [2].

Chapter 4 contains a brief description of some of the most recent algorithms and implementations in the field of group theory. Particular reference is made to presentation finding algorithms and nilpotent group calculation methods.

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Chapter 5 deals with an algebraic approach to a topological problem. Machine calculation of group theoretic invariants is used to assist in topological classification of knots and links.

Chapter 6 describes a computational approach to calculation within finitely generated Lie algebras. The relationship between Lie algebras and groups is discussed and certain problems in group theory are investigated.

Chapter 7 discusses the programming languages and data structures required for efficient transferral of the mathematical problems to the machine.

In the Appendix are listed four major programs which implement algorithms described in the body of the thesis. Each of the implementations is made in ANSI standard FORTRAN.

References

[1] John J. Cannon, Lucien A. Dimino, George Havas and Jane M. Watson, "Implementation and analysis of the Todd-Coxeter algorithm", *Math. Comp.* 27 (1973), 463-490.

[2] George Havas, "A Reidemeister-Schreier program", *Proc. Second Internat. Conf. Theory of Groups*, Australian National University, Canberra, 1973, 347-356 (Lecture Notes in Mathematics, 372. Springer-Verlag, Berlin, Heidelberg, New York, 1974).