

Introduction

We have discussed in the previous part several of the most popular QCD non-perturbative methods other than the QCD spectral sum rules (QSSR). Now, we shall dedicate this part of the book to the discussion of this non-perturbative approach, which has been used successfully for understanding the hadron properties and hadronic matrix elements, using those parameters (QCD coupling, quark masses and QCD condensates), derived from QCD first principles. This method was introduced by SVZ in 1979 [1] and reviewed in a book [3], numerous reviews and lecture notes [356–365]. Its basic concepts, based on the operator product expansion and dispersion relations, are well understood in quantum field theory, and is a fully relativistic approach in contrast to potential models, for example. Its applications are quite simple and transparent. However, on the one hand it has a limited accuracy (usually about 10–20% depending on the process), and some uses of the method in some QCD-like models show that its accuracy cannot be improved iteratively. On the other hand, confinement is not a result of the method but is put into it via the introduction of different QCD condensates. In practice, one has to introduce some assumptions, and the results are obtained from self-consistency.

However, in some cases, results obtained from the sum rules disagree with each other and have led to some polemics, which some people use to discredit the approach. We shall see how it is important to check that the results satisfy *some stability criteria* and that the matching between the low-energy hadronic region and the perturbative region described by QCD, which is called *global duality tests* in the literature, are obtained.

A further limitation of the method comes from the fact that the Green's function is computed in the Euclidian region from which observable quantities can be extracted by duality. Due to the approximate nature of the method, one can only extract the properties of the ground state of given quantum numbers but not those of its radial excitations, which are smeared by the perturbative QCD continuum used to parametrize these states of higher masses.

In this part, we shall follow closely the discussions in [3], not with the aim of reproducing this book, but to update the different discussions therein. However, the present book cannot replace the former as we shall not repeat the detailed derivations already included therein. We shall also not be able to give a complete presentation of the different existing QSSR results due to the large number of its applications. Instead, we will try to limit ourselves to

some specific applications, which in my opinion, are representative of the QSSR results. This part of the book is organized as follows: in the first two chapters, we give an introduction to the method of QSSR and in the remaining chapters, we shall review the main developments and results from the method.

In the first part of this book, we have already discussed some current algebra sum rules prior to QCD, such as the Adler–Weisberger sum rules, the Weinberg and DMO sum rules, the electromagnetic $\pi^+ - \pi^0$ mass difference. These current algebra sum rules are prototype QSSR. We shall discuss some of them in the context of QCD.