ACCELERATION OF RUNAWAY ELECTRONS AND JOULE HEATING IN SOLAR FLARES

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D.C. electric fields provide the simplest and most direct means of accelerating electrons out of a thermal plasma. Most solar flare models result in the production of D.C. electric fields. On the other hand, microwave and hard X-ray observations of flares provide specific requirements for the number and energy of energetic electrons produced during a flare, and the timescales involved in accelerating them. The microwave emission from flares is understood to be gyrosynchrotron radiation from electrons with energies of 100 keV or greater. The hard X-ray emission ($\gtrsim$ 25 keV) can be interpreted as being either thick-target bremsstrahlung from non-thermal electrons (thin-target radiation may also contribute to the X-ray emission, but the process is less efficient), or thermal bremsstrahlung from hot, impulsively heated plasma. Hence, it is of interest to study the electric field acceleration of "runaway" electrons and the simultaneous Joule heating of the thermal plasma in light of these results from flare observations, without recourse to a specific flare model. Some of the results of such a study are summarized here.

When a thermal plasma is subjected to an electric field, a fraction of the current-carrying ($J = \sigma E$) electrons, those with a velocity greater than a critical velocity, v_c, will be freely accelerated out of the thermal distribution (Dreicer 1959). The velocity v_c is determined by the temperature (T_e), density (n) and resistivity ($\eta = 1/\sigma$) of the plasma, and by the electric field strength (E). As this tail of "runaway" electrons is produced, new particles are supplied to the runaway region ($v \geq v_c$) by collisions. (This assumes that $v_c > v_e$, the thermal electron velocity. Otherwise, the runaway region includes the entire distribution of thermal electrons.) The runaway electrons are further accelerated by the electric field as long as they remain in the current channel.

1. It is of interest to estimate the maximum number of electrons that can be supplied by the original distribution of thermal electrons, $N = n(v > v_c)\, V_J$, where V_J is the volume of the acceleration region, without considering the additional particles made available by collisions. The

M. R. Kundu and G. D. Holman (eds.), Unstable Current Systems and Plasma Instabilities in Astrophysics, 191–196.

volume of the current channel and, hence, the acceleration region is not arbitrary, since the total allowable current is limited by the induction magnetic field associated with it. From Ampere's law, the magnetic field associated with a current sheet of total current $I = Jw\delta r$ (w is the width of the sheet, δr its thickness) is $B_J = (2\pi/c)(I/w) = (2\pi/c)(J\delta r)$. Requiring $B_J \lesssim B$, the maximum magnetic field strength in the acceleration region, yields $\delta r \lesssim (c/2\pi)(B/J)$. With $V_J = A\delta r$ and $J = nev_D$, where A is the area of the current sheet and v_D is the mean drift velocity of the current-carrying electrons, one finds that $V_J \lesssim (c/2\pi e)(BA/nv_D)$. The maximum number of electrons that can be accelerated is obtained when $v_D = v_c = v_e$, giving

$$N \lesssim \frac{BA}{2\pi e}\left(\frac{c}{v_e}\right) = 4.3\times10^{29}\left(\frac{B}{100\ \mathrm{G}}\right)\left(\frac{A}{5.3\times10^{17}\ \mathrm{cm}^2}\right)\left(\frac{T}{10^7\ \mathrm{K}}\right)^{-1/2} \text{ electrons.} \quad (1)$$

Note that this result is independent of the density and the resisitivity of the plasma. The sheet area chosen here corresponds to a 10" x 10" sheet on the sun, as seen from the earth. It should also be noted that a much smaller limit is obtained if the current is assumed to be confined to a cylindrical volume, rather than to a sheet geometry.

The number of electrons required to produce a typical microwave flare on the sun is $10^{31} - 10^{32}$. Maximum observed magnetic field strengths are on the order of 1000 G. Hence, under extreme conditions it may be possible to generate an observable microwave flare with the original distribution of thermal electrons. In general, however, additional time is required for more electrons to be supplied from the thermal plasma. An important corollary of this result is that *the number of accelerated electrons will, in general, depend upon the thermal collision frequency and, therefore, the resistivity of the plasma.*

2. Interpreting the $\geq$ 25 keV X-ray emission from a flare to be thick-target, non-thermal bremsstrahlung requires a minimum of 10^{35} electrons/sec to be accelerated. The current associated with an electron flux of N electrons/sec, $I = e\dot{N}$, is limited by the induction field it generates, $B_J = (2\pi/c)(I/w)$. Requiring $B_J \leq B$ yields the result

$$\dot{N} \leq \frac{cwB}{2\pi e} = 1.0\times10^{30}\left(\frac{w}{10^9\ \mathrm{cm}}\right)\left(\frac{B}{100\ \mathrm{G}}\right) \text{ electrons/sec.} \quad (2)$$

Hence, since observed magnetic field strengths are limited to $B \lesssim 1000$ G, *the simple acceleration of runaway electrons cannot produce a large enough electron flux to explain the bulk of the observed hard X-ray emission as non-thermal bremsstrahlung.* This result agrees with a similar result obtained by Spicer (1983).

Note that the timescale for the generation of N runaway electrons is $t_N \equiv N/\dot{N}$. Hence, the upper limit on $\dot{N}$ yields a minimum timescale for the generation of the number of electrons required for an observable microwave source:

$$t_N \geq 10 \left(\frac{N}{10^{31}}\right)\left(\frac{w}{10^9 \text{ cm}}\right)^{-1}\left(\frac{B}{100 \text{ G}}\right)^{-1} \text{ sec.} \tag{3}$$

3. The timescale, t_a for the acceleration of an electron in a D.C. electric field from the critical velocity v_c to a velocity v, is limited by the distance over which this acceleration can occur. For the simplest case of a constant electric field, the acceleration distance is $x_a = (1/2)(v_c + v)t_a$. To accelerate an electron to an energy of 100 keV in a distance of 10^9 cm or less requires $t_a \lesssim 0.1$ sec. The electric field strength required to accomplish this is $E = (m/e)(v-v_c)/t_a \gtrsim 3 \times 10^{-7}$ statvolt/cm. If $E < E_D$, the Dreicer field, is also required, so that the entire thermal electron distribution does not run away, the following constraint on the density and temperature in the current channel is obtained:

$$0.18 \left(\frac{n}{10^9 \text{ cm}^{-3}}\right)\left(\frac{T}{10^7 \text{ K}}\right)^{-1}\left(\frac{\ell n\, \Lambda}{23}\right) > 1, \tag{4}$$

where $\ell n\, \Lambda$ is the Coulomb logarithm and the collision frequency is assumed to be classical. Hence, an acceptably high density or a low temperature in the current channel will maintain $E < E_D$. Alternatively, a high (anomalous) resistivity will maintain $E < E_D$.

4. A flare that is well suited for testing the applicability of a simple D.C. electric field acceleration, Joule heating model is the May 14, 1980 (1920 UT) flare, discussed in the preceding paper by Kundu. The 6 cm VLA maps show that interaction among several magnetic loops is involved, with the possible formation of a current sheet between two of the loops. The impulsive 6 cm microwave emission peaks ~ 10 sec before the onset of the hard X-ray flare. The microwave and hard X-ray data can be interpreted as follows:

The microwave emission increases from one-tenth its peak value to its peak value in ~ 30 sec. This timescale can be related to either t_a or to t_N. The results of sections 2 and 3 indicate that t_N is more likely to be relevant, with $t_a \ll t_N$. Since $t_N(N = 0.1\, N_{peak}) = 0.1\, t_N(N = N_{peak})$ and $t_N(N = N_{peak}) - t_N(N = 0.1\, N_{peak}) \approx 30$ sec, $t_N(N = N_{peak}) \simeq 33$ sec.

The hard X-ray emission, in light of the result of Section 2, is taken to be thermal. The electron temperature deduced from the X-ray spectrum is greatest at the onset of the (X-ray) flare. Since the onset of the hard X-ray flare follows the microwave peak by ~ 10 sec, the Joule heating time is $t_J \simeq t_N + 10$ sec $\simeq 43$ sec.

The timescale for the production of runaways is $t_N \equiv N/\dot{N} \simeq N/\gamma n V_J$, where γ is the runaway production rate (Kruskal and Bernstein 1964). This can be written as (a more detailed presentation will be published elsewhere)

$$t_N \simeq 160 \left(\frac{N}{10^{32}}\right)\left(\frac{A}{5.3\times10^{17}\,\mathrm{cm}^2}\right)^{-1}\left(\frac{B}{100\ \mathrm{G}}\right)^{-1}\left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2}\left(\frac{\nu}{20\mathrm{sec}^{-1}}\right)^{-1} / f\left(\frac{v_c}{v_e}\right)\ \mathrm{sec}, \tag{5}$$

where ν is the thermal collision frequency in the current channel and $f(v_c/v_e) \leq 1$ is an exponential function of v_c/v_e:

$$f(v_c/v_e) = 4.66\ (v_c/v_e)^{11/4}\ \exp[-2^{1/2}(v_c/v_e) - (1/4)(v_c/v_e)^2]. \tag{6}$$

This timescale is consistent with the inferred timescale of ~ 33 sec.

The fundamental timescale for Joule heating is $\tau_J \equiv nkT/(J\cdot E)$. The actual timescale ($t_J$) will be longer than this, however, since the volume of gas to be heated will generally be larger than the volume of the current channel. The actual heating time can be estimated to be $t_J \gtrsim \tau_J/\varepsilon$, where $\varepsilon \equiv V_J/V_X$, the volume of the current channel divided by the volume of the X-ray emitting gas. A thermal fit to the X-ray spectrum at the time of the temperature maximum gives an emission measure of $EM \equiv n_X^2 V_X \simeq 3 \times 10^{44}\ \mathrm{cm}^{-3}$. The Joule heating time can be shown to be

$$t_J \gtrsim 3.5\times10^4 \left(\frac{A}{5.3\times10^{17}\,\mathrm{cm}^2}\right)^{-1}\left(\frac{B}{100\ \mathrm{G}}\right)^{-1}\left(\frac{EM}{3\times10^{44}\,\mathrm{cm}^{-3}}\right)\left(\frac{n_X}{1\times10^{9}\,\mathrm{cm}^{-3}}\right)^{-1} \cdot \left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2}\left(\frac{\nu}{20\mathrm{sec}^{-1}}\right)^{-1}\left(\frac{v_c}{v_e}\right)^2\ \mathrm{sec} \tag{7}$$

(n_X is the average electron density in the X-ray emitting volume, n is the density in the current channel).

In order to achieve the inferred heating timescale of ~ 43 sec through Joule heating, *either the resistivity of the plasma in the current channel must be anomalous, or the density in the X-ray emitting region (and, therefore in the current channel as well) must be much greater than 10^9 cm^{-3}*. With the other parameters remaining unchanged from those used in equations (5) and (7), a collision frequency that is 2×10^4 times greater than the classical thermal (Coulomb) collision frequency will give the required timescales. This also requires $v_c/v_e \sim 5$ and $v_D \simeq 1.5\ c_s$, were c_s is the ion sound speed. Hence, the ion acoustic or the ion cyclotron instability is a likely source of the anomalous resistivity. If the resistivity remains classical, a plasma density in the X-ray emitting region and current channel of ~ 10^{11} cm^{-3} will give the required timescales. This also requires $v_c/v_e \sim 4$ and $v_D \simeq 3c_s$, however. Since both the ion acoustic and the electrostatic ion cyclotron instabilities set in near or below this drift velocity, *anomalous resistivity is likely to play a significant role in the development of the flare* •

This work was supported in part by NASA Grant NGR 21-002-199, NASA Contract NSG 5320, NSF Grant ATM 81-03809, and NGL 21-002-033.

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DISCUSSION

Vlahos: The restriction of the maximum number of electrons accelerated by an electric field E to 10^{31} is an artifact of the over-simplified model. For the model you used, for example, what if the region is split into many small volumes with electric fields in opposite directions, or if the field is localized and the return current flows from the boundaries to produce a total current that is small, but copious non-thermal particles?

Or if $E(t) \sim \cos\omega t$, it accelerates electrons in both tails that carry no current!

Holman: Splitting the acceleration region into many small, oppositely directed current channels is indeed a possible way of getting around the induction field limit on the electron flux. This requires at least 10^4 individual current channels to produce a (> 25 keV) hard X-ray burst, however. The plausability of producing this situation, and its stability, requires further study.

I do not see how "return current flow from the boundaries" can result in a small total current, but "copious non-thermal particles". If the current is reduced, so is N, and the nonthermal electron flux is not large enough.

A time oscillating electric field does not necessarily avoid the induction field limitation since, although electrons are accelerated in both directions, they are not cospatial, there is a current, and an induction B field is still generated. Cospatial, oppositely directed beams would avoid the inducation field limitation, but this arrangement will not be produced by simple electric field acceleration.

The model used here is intentionally simple, since its purpose is to test the applicability of the D.C. electric field acceleration of electrons to solar flares. The limitations that I have discussed are within this context. On the other hand, this simple model does appear to adequately explain at least one class of solar flares, such as the May 14 flare. In this sense the model is not at all over simplified.

Vasyliunas: In computing the magnetic field associated with the runaway electron current, you have assumed in effect that the return current is outside the sheet. Could some of the return current (which includes any displacement current effects present) be within the sheet itself, thus reducing the calculated B?

Holman: Since the current is driven by an electric field (which is necessary to accelerate the runaway electrons), the return current cannot be within the sheet itself. Hence, the return current cannot reduce the calculated induction magnetic field.

Bratenahl: There may be another process (other than this Joule theory), namely, acceleration in double layers - does this help the problem? Have you looked into it?

Holman: The formation of double layers is expected to be driven by the interruption of a macroscopic current, with the electrons accelerated by the enhanced electric field within the double layer. This process is limited by the induction B field associated with the current and, therefore, is also subject to this limitation.

Spicer: It makes no difference what flare mechanism one uses in an inductive circuit to produce a flare, because an inductive circuit responds to an enhanced impedance in the form of double layers, anomalous resistivity, or reconnection by producing a backward emf so as to attempt to keep the net magnetic flux constant (it, of course, does not succeed). However, by doing so $\frac{dN}{dt} = I/|e|$ will not increase, but decrease. In addition, the particles accelerated by the emf may be relativistic, but the number of relativistic electrons will not exceed $\frac{dN}{dt} = I/|e|$ (Spicer 1983).

Degaonkar: We find the difference in time of peak emission of hard X-rays and microwaves varies from a few millisecond to a few seconds. On what mechanism or process does this depend? Does it have bearing on the acceleration of particles or the distance between the sources?

Holman: These are certainly important questions, for which I do not claim to have a general answer. For the May 14 flare, the time difference between the hard X-ray and microwave peaks is taken to be the difference between the Joule heating time, t_J, and the runaway production time, t_N (as is required by the acceleration mechanism). The microwave and hard X-ray sources are essentially cospatial. Shorter time delays in other flares may be related to an entirely different process. Fully non-thermal models for the hard X-ray and microwave emission are constrained by the limit on the accelerated electron flux (Equation 2, and Spicer 1983), however.

D. Smith: Have you accounted for the change in the number of runaways which occurs in the presence of ion-acoustic turbulence?

Holman: Yes. When the plasma resistivity is higher, a higher electric field strength is required to accelerate a given number of runaways. This electric field strength is consistent with the other parameters in the acceleration region.