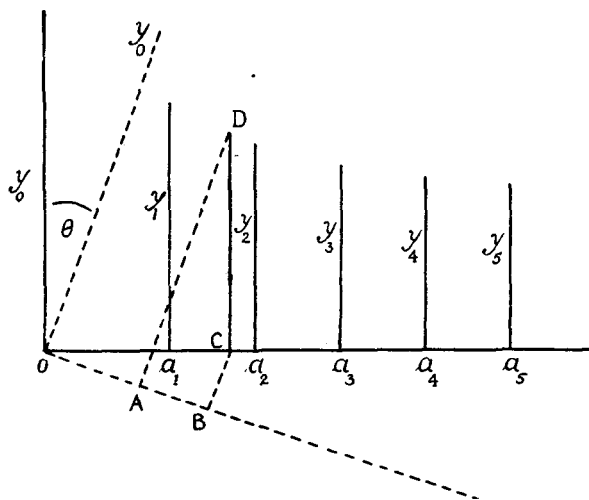## An Apparatus for Determining Coefficients in Power Series

By the late R. W. M. THOMSON, B.Sc.   (Communicated by
Professor Peddie.)*

The diagrammatic figure illustrates the principle involved.   The
vertical full lines represent ordinates of a curve of which the

abscissae are Oa_1, Oa_2, etc.   Let now all these vertical lines be rotated
round the points O, a_1, a_2, etc., while retaining their original lengths,

---

through the same angle θ. The curve passing through the new positions of the end points is now to be referred to the tilted axis Oy_0 and the line OAB, drawn at right angles thereto. If we put $OC = x$, $CD = y$, and let x', y', be the corresponding coordinates with reference to the tilted axis we get

$$y' = y + x \sin \theta,$$
$$x' = x \cos \theta.$$

Hence, if the original curve, were approximately representable by the equation

$$y = y_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots,$$

the new curve would represent the relation

$$y' = y_0 + (a_1 + \sin \theta) \sec \theta . x' + a_2 \sec^2 \theta . x'^2 + a_3 \sec^3 \theta . x'^3 + \ldots.$$

If we write this as

$$y' = y_0 + a_1' x' + a_2' x'^2 + a_3' x'^3 + \ldots,$$

we obtain, for the new constants, the simple relations

$$y_0' = y_0,$$
$$a_1' = (a_1 + \sin \theta) \sec \theta,$$
$$a_2' = a_2 \sec^2 \theta,$$
$$a_3' = a_3 \sec^3 \theta,$$
$$\ldots \ldots \ldots \ldots$$
$$\ldots \ldots \ldots \ldots$$
$$a_n' = a_n \sec^n \theta.$$

The photographs show the actual apparatus, as used, in the normal and tilted positions. The constants y_0, a_1, a_2, etc., are got from data of measurement once for all. The lower ends of the tiltable rods are pivoted on a subsidary beam which is parallel to the main beam, and the tilt of the rods is produced by sliding the subsidiary beam parallel to itself. The original curve (no tilt) was selected as being suitable for the special set of experimental curves which were dealt with. If the lengths of the rods were made adjustable, other original curves could be used.

Two fixed pins, at a slight distance apart, pass through the upper end of each rod; and two light, flat, steel springs, one at each side of the set of rods, pass through the space between each pair of pins. These springs are clamped together by a pair of clamps which can be seen in the illustrations. The two springs thus act as a single spring, which can slide in the spaces between each pair of pins, and is fixed by clamping screws at the end nearest the

main beam. This point of fixture can be taken as an additional point on the curve corresponding to $y = 0$ on the untilted curve. Half a dozen V-shaped notch pieces, two fixed on the longest rod, and four (visible as such in the illustrations) fixed on the main beam or its
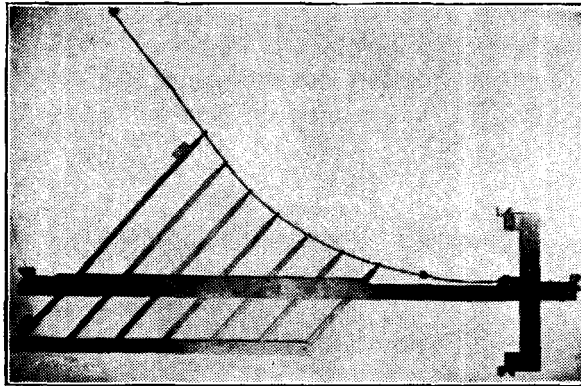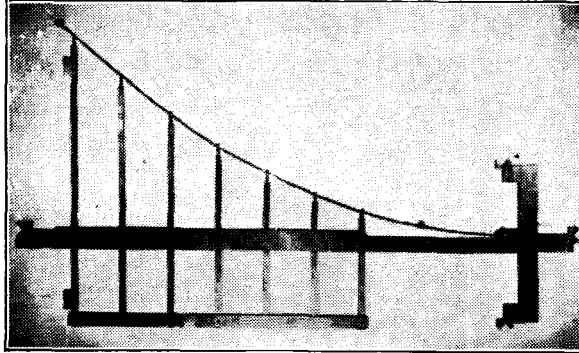
cross piece, serve for adjustment of the apparatus in a vertical frame-work if desired. Otherwise, the apparatus can lie flat on the paper on which the experimental curve is traced, and adjustment be made until the curve of the spring coincides as closely as may be desired with the experimental curve, to which an empirical formula is thus readily found.

The spring of course merely gives a smooth curve passing through the points fixed by the rods. The diagrams show how very different may be the shape of the curve relatively to the untilted and the tilted axes. The origin for y' may clearly be shifted to any desired position along the y' axis.