The Predictive Power of Computational Astrophysics as a Discovery Tool
Proceedings IAU Symposium No. 362, 2023
D. Bisikalo, D. Wiebe & C. Boily, eds.
doi:10.1017/S1743921322001247

Simulations of dark matter with frequent self-interactions

Moritz S. Fischer

Hamburger Sternwarte, Universität Hamburg, Gojenbergsweg 112, D-21029 Hamburg, Germany

Abstract. Self-interacting dark matter (SIDM) is promising to solve or at least mitigate small-scale problems of cold collisionless dark matter. N-body simulations have proven to be a powerful tool to study SIDM within the astrophysical context. However, it turned out to be difficult to simulate dark matter (DM) models that typically scatter about a small angle, for example, light mediator models. We developed a novel numerical scheme for this regime of frequent self-interactions that allows for N-body simulations of systems like galaxy cluster mergers or even cosmological simulations. We have studied equal and unequal mass mergers of galaxies and galaxy clusters and found significant differences between the phenomenology of frequent self-interactions and the commonly studied large-angle scattering (rare self-interactions). For example, frequent self-interactions tend to produce larger offsets between galaxies and DM than rare self-interactions.

Keywords. astroparticle physics, methods: numerical, galaxies: haloes, dark matter

1. Introduction to SIDM

The cosmological standard model ΛCDM has been quite successful in explaining the observed large scale structure. Large cosmological N-body simulations have been used, e.g. the Millenium simulations (Springel et al. 2005), to make predictions from ΛCDM. The first such simulations were DM-only and did not take baryonic physics into account. For about two decades, it is known that these simulations deviate on small, i.e. galactic, scales from the observed matter distribution. But at the same time, they are remarkably successful in explaining the distribution of matter on large scales.

There exist several problems or maybe better curiosities on small scales, together they form the small-scale crisis of ΛCDM. One of these problems is the core-cusp problem. DM-only simulations predict cuspy haloes that are fairly well described by an NFW profile. However, cored haloes with a lower central density are observed. Besides, there exists the too-big-to-fail problem, the diversity problem and the plane of satellite problem, and maybe more. For a review of the small-scale challenges see Bullock & Boylan-Kolchin (2017).

Many potential solutions have been proposed to the small-scale problems. Several of them rely on an alternative DM model, e.g. warm DM (Dodelson & Widrow 1994), self-interacting DM (Spergel & Steinhardt 2000) or fuzzy DM (Hu et al. 2000). Another branch of solutions relies on the inclusion of baryonic physics in cosmological simulations. Unfortunately, modelling the baryonic processes comes with high uncertainty, but it has been shown that a proper inclusion of the processes, like star formation, supernovae and AGNs can at least contribute to a solution of the small-scale crisis. Besides, there are also other attempts to solve the problems on small scale by introducing an alternative theory of gravity. In addition, improvements in modelling the internal structure of observed galaxies may contribute to a solution too (Oman et al. 2019).

From now on, we will focus on one particular potential solution, which is DM with self-interactions. SIDM refers to a class of particle physics models that assume that DM particles interact with each other through an additional force beyond gravity. But this affects only DM and no interaction with standard model particles is assumed. SIDM has been studied for about two decades and it has been shown that self-interactions can resolve or at least mitigate several small-scale problems. For instance, SIDM can create density cores in DM haloes, by transferring heat inward. For a review of SIDM, see Tulin & Yu (2018).

There exist several methods to model SIDM. The gravothermal fluid model and the Jeans approach make simplifying assumptions such that they can only be applied to relaxed haloes. In contrast, N-body simulations do not simplify the problem but are computationally much more expensive. In the following, we focus on N-body simulation, i.e. describe how to faithfully model self-interactions even in complicated systems.

To model SIDM, one needs to solve the Vlasov-Poisson equation with an additional collision term, see Eq. (1). Thus, SIDM is neither collisionless like CDM nor fully collisional like a fluid. Hence, the 6d phase-space information is required.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f - \nabla_x \Phi \cdot \nabla_v f = \left(\frac{\partial f}{\partial t} \right)_{\rm coll} . \tag{1}$$

The self-interactions are described by the collision term. This term follows from the differential cross-section of the particle physics model. Here we distinguish two regimes, the large-angle and the small-angle scattering. If the typical scattering angle is large, a significant amount of momentum is transferred per scattering event. Thus not many scattering events are necessary to alter the DM distribution. Therefore, the self-interactions are called rare. On the other hand, if particles scatter at small angles, only a little amount of momentum is transferred between the particles per interaction. Hence, this type of interaction must be frequent to have a significant effect on the distribution of DM. It has turned out that this regime is more difficult to model within N-body simulations. For about two decades researchers have been performing N-body simulations of SIDM. However, almost all of them fall into the regime of rare self-interactions, mostly an isotropic cross-section has been studied. However, in Fischer et al. (2021a) we introduced a novel scheme that allows, for the first time, to model fSIDM within N-body simulations from first principles. In contrast to the work of Kummer et al. (2019), we can simulate more complicated setups like mergers or do cosmological simulations. So far we have focused on idealized simulations of equal and unequal mass mergers (Fischer et al. 2021a,b).

2. *N*-body Simulation with Self-Interactions

In this section, we describe the basic principles of the numerical formulation used to model self-interactions within N-body codes. Before we explain the new scheme for frequent self-interactions, we describe the state-of-the-art scheme for rare self-interactions, and lastly, we show a test problem to validate our numerical scheme and the implementation.

The today widely spread Monte Carlo scheme for rSIDM goes back to Burkert (2000) and has been highly improved by Rocha et al. (2013). The idea is to treat the interaction between N-body particles like physical particles. If two numerical particles are close to each other, a random number is drawn to decide whether they interact with each other or not. In the case of interaction, another random number is needed to decide which angle they scatter. All numerical particle interactions are treated pairwise, this allows conserving momentum and energy explicitly. Note that SIDM physics could be described deterministically at the scales of interest as long as one studies the limit of many physical particles. Thus, it is not affected by the stochastic nature of the interaction of individual particles. However, when modelling self-interactions within the N-body method difficulties arise. To overcome them, random numbers and the formulation analogous to the interaction of physical particles have been introduced.

In principle, the scheme for rSIDM could describe the limit of fSIDM, but time-step limitations make it impractical to use it. The interaction probability of two numerical particles per simulation time step must always be smaller than unity, which gives a time-step constraint. In the fSIDM limit, this time-step constraint implies a time-step of zero. Thus, the simulation can no longer be advanced in time. To overcome these problems, a different formulation of the collision term is required.

For the fSIDM scheme, we no longer describe the interaction of numerical particles like physical particles. As for the rSIDM scheme, we use a stochastic process that converges in the limit of many particles against the deterministic collective behaviour of many physical SIDM particles. At least there exists another stochastical process in the limit of fSIDM, besides the particle physics one, that fulfills this condition. We use this process to overcome the problems with the rSIDM scheme described above. In our fSIDM scheme, physical particles interact with each other if they are close. There is no interaction probability anymore. For close pairs of particles, we use an effective description based on a drag force to describe the self-interactions.

The drag force description we use goes back to Kahlhoefer et al. (2014). The idea is the following: A DM particle is travelling through a constant background density at rest. While doing so, it scatters frequently with the background particles; every interaction alters the direction of motion by a tiny angle. The corresponding velocity changes perpendicular to the direction of motion average out, but the one parallel to the direction of motion sum up and decelerate the DM particle. This is the drag force we use for the numerical scheme. The energy taken from the forward motion goes into the perpendicular component. To understand this, it might be helpful to think of a phase-space patch travelling through the background density. The energy taken from the forward motion does not dissipate but increases the velocity dispersion perpendicular to the direction of motion. The phase-space patch is heated and its particles no longer have the same velocity. But their mean direction of motion does not change.

In order to apply this to the numerical particles, we treat close pairs in two steps. First, we compute the momentum change due to the drag force and decelerate the particles. Here, the numerical particles represent phase-space patches that overlap in configuration space and thus can interact and decelerate each other. Secondly, we re-add the energy lost in the first step into a random direction but within a plane perpendicular to the direction of motion.

We have implemented the novel scheme for fSIDM into the cosmological N-body code Gadget-3, which is a successor of Gadget-2 (Springel 2005). Our implementation conserves momentum and energy explicitly. This makes the parallelisation more complicated than, for example, in SPH. But reasonable large simulations with fSIDM can be run. For DM-only simulations, the self-interactions slow down the simulation by a factor of 4 or more. The exact number depends largely on the specific simulation setup.

To demonstrate that the implementation in Gadget-3 works as expected, we use a test problem similar to Rutherford's experiment. A beam of DM particles scatters on a target consisting of DM particles and the distribution of the deflection angles is measured. In contrast to Rutherford, we do not use a thin but a thick target. An analytical solution for this problem is given by Moliere (1948). In Fig. 1, we compare our simulation results (black) to the exact solution (blue). The distributions of the deflection angle agree well

https://doi.org/10.1017/S1743921322001247 Published online by Cambridge University Press

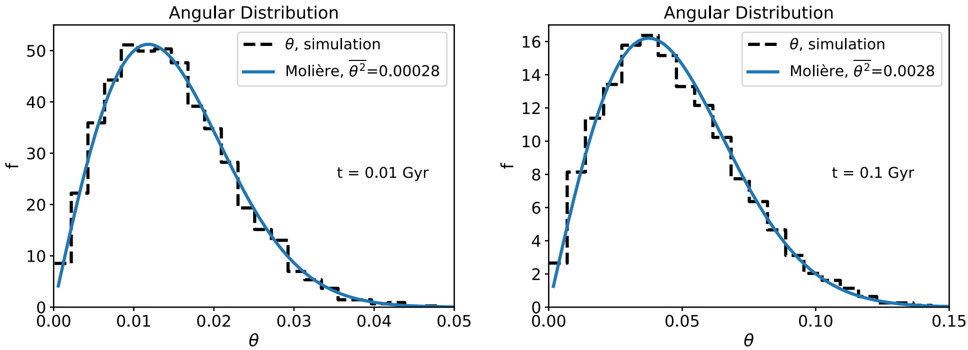

Figure 1. The angular deflection test problem is shown. The distribution of the deflection angle is given for two different times that the particle is travelling within the target. This corresponds to different target thicknesses, i.e. the left-hand panel gives a target that is thinner than the one of the right-hand panel. The black curve shows the simulation result and the blue one gives the exact solution. This figure is a reproduction of Fig. 3 of Fischer et al. (2021a).

with each other. From this, we can conclude that the implementation of our scheme works as expected. Hence, we turn our focus to an astrophysical motivated problem in the next section.

3. Merging Galaxy Clusters and SIDM

In this section, we study mergers of galaxy clusters since they are observationally and theoretically well-studied systems. The most famous system in this context is probably the Bullet Cluster, which has also been studied in the context of SIDM (Randall et al. 2008; Robertson et al. 2017a,b). If DM undergoes self-interactions, one would expect that the DM haloes of the clusters behave differently than their galaxies. In particular, if self-interactions are frequent, a drag force that decelerates the DM component may arise. The galaxies are not affected by the drag force and thus an offset between the DM and the galactic component can occur. As claimed by Kim et al. (2017) small offsets can also arise in the case of isotropic scattering if the DM density is large enough. Similar to Kim et al. (2017) we study equal mass mergers with head-on collisions. The haloes follow an NFW profile and have a virial mass of $10^{15}\mathrm{M}_{\odot}$. In Fig. 2, we show the position of the DM density peaks along the merger axis of our simulations with various DM models. In particular, we simulate collisionless DM, rSIDM, and fSIDM employing several cross-sections. The scattering of the SIDM models is elastic and velocity-independent. Note that to match rSIDM and fSIDM, we use the momentum transfer cross-section. For more details see Fischer et al. (2021a). If the cross-section is increasing, the merger time becomes smaller and for a very large cross-section, the DM haloes coalesce on contact. During the infall phase the peak position is mostly unaffected by the self-interactions, but at about the first pericentre passage, differences arise. If one compares rSIDM to fSIDM, differences appear to be small, i.e. the DM models behave similarly. For a cross-section of $1.5\,\mathrm{cm}^2/\mathrm{g}$, the plot shows the largest difference between the models. However, we are interested in the DM-galaxy offsets. The simulation contains collisionless particles that follow the same NFW profile as the DM component to mimic the galaxies of the cluster. At the centre of each halo, we placed a more massive particle to mimic the brightest cluster galaxy (BCG). In Fig. 3, we show the offset between the DM component and the BCG position as a function of time relative to the first pericentre passage. For a given cross-section, the offset is much larger for the fSIDM runs. First, the DM peaks are in between the galaxy peaks, and at a later time, it is vice versa (the sign of the offset changes). We measure the maximum offset for the latter case and show it for even more

https://doi.org/10.1017/S1743921322001247 Published online by Cambridge University Press

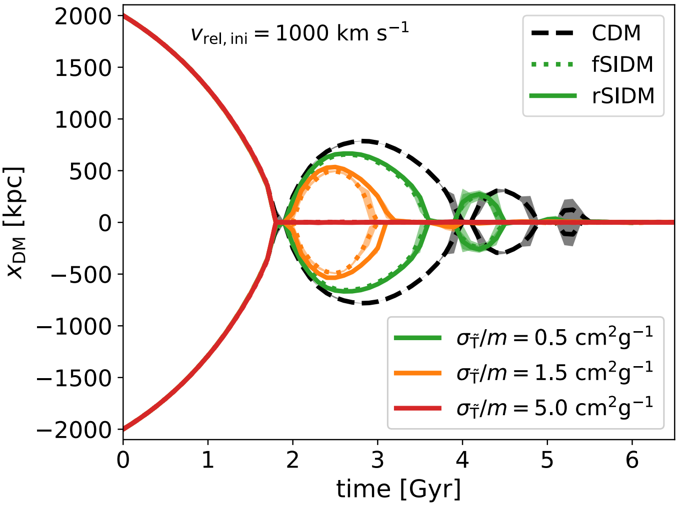

Figure 2. The peak position of the DM haloes as a function of time for several cross-sections is shown. This figure is a reproduction of the lower panel of Fig. 8 of Fischer et al. (2021a).

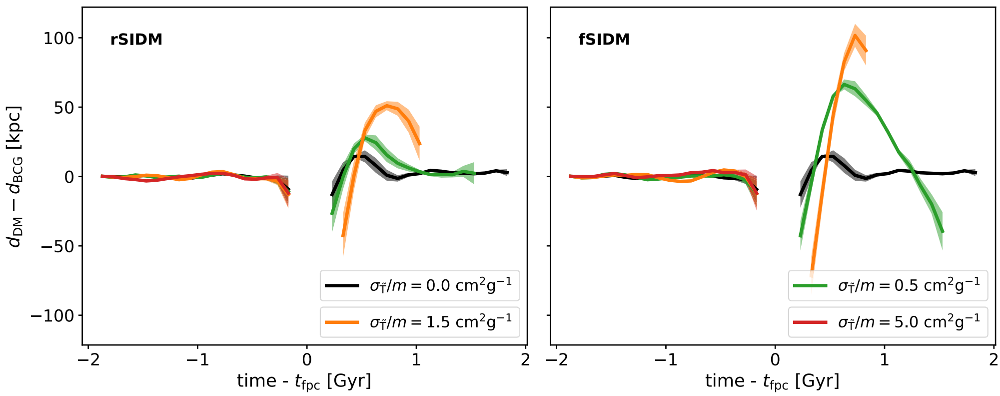

Figure 3. DM-galaxy offset as a function of time relative to the first pericentre passage ($t_{\rm fpc}$). The offset is positive when the galaxy peaks are in between the DM peaks and negative otherwise. When the DM peaks are close, peak finding becomes inaccurate and we do not display the offsets. This figure is a reproduction of the lower panels of Fig. 9 of Fischer et al. (2021a).

cross-sections in Fig. 4. For large cross-sections, the offsets become zero as the DM haloes coalesce on contact, thus the type of offset shown here can no longer occur. For small cross-sections, the offset increases with cross-section and the maximum offset is much larger for fSIDM than for rSIDM. It becomes clear that very large offsets can only be explained by fSIDM but not by rSIDM.

4. Conclusions

From the work presented here, we can draw two main conclusions.

First, it is possible to model DM with frequent self-interactions within N-body simulations. The presented numerical scheme relies on an effective drag force arising from self-interactions. Moreover, we have demonstrated through a test problem similar to Rutherford's experiment that the numerical scheme allows to accurately simulate small-angle scattering.

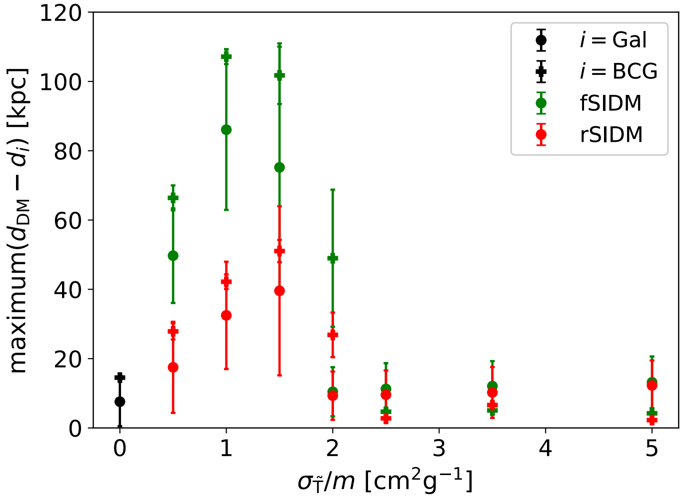

Figure 4. Maximum DM-galaxy offset as a function of the cross-section. Only offsets where the galaxy peaks are in between the DM peaks are considered. We display offsets for rSIDM (red) and fSIDM (green) measured relative to the peak of the galaxy distribution and the BCG position. This figure is a reproduction of Fig. 10 in Fischer et al. (2021a).

Secondly, we found that rSIDM and fSIDM show different phenomenologies. In particular, we have demonstrated that the size of the offset between galaxies and DM in galaxy cluster mergers depends crucially on the shape of the differential cross-section. It is worth mentioning that this is not only the case for a fairly large cross-section, but also true for a strength of self-interactions that is within the current observational bounds, e.g. from dark matter density cores. Although there have been observational claims of fairly large offsets in the literature (e.g. Harvey et al. 2015), this can not be taken as evidence for fSIDM as a more thorough analysis of observations does not confirm these large offsets (Wittman et al. 2018). In the future, a combination of multiple measurements, e.g. offsets and dark matter cores, could possibly allow to constrain the shape of the differential cross-section, i.e. discriminate between rSIDM and fSIDM.

Acknowledgements

It is a pleasure to acknowledge all of my collaborators who contributed to the work I have described here. They are Marcus Brüggen, Kai Schmidt-Hoberg, Klaus Dolag, Felix Kahlhoefer, Antonio Ragagnin and Andrew Robertson. This work is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2121 "Quantum Universe" – 390833306.

References

Bullock, J. S. & Boylan-Kolchin, M. 2017, Small-Scale Challenges to the ΛCDM Paradigm. *ARAA*, 55(1), 343–387.

Burkert, A. 2000, The Structure and Evolution of Weakly Self-interacting Cold Dark Matter Halos. *APJL*, 534(2), L143–L146.

Dodelson, S. & Widrow, L. M. 1994, Sterile neutrinos as dark matter. *PRL*, 72(1), 17–20.

https://doi.org/10.1017/S1743921322001247 Published online by Cambridge University Press

Fischer, M. S., Brüggen, M., Schmidt-Hoberg, K., Dolag, K., Kahlhoefer, F., Ragagnin, A., & Robertson, A. 2021,a N-body simulations of dark matter with frequent self-interactions*. *Monthly Notices of the Royal Astronomical Society*, stab1198.

Fischer, M. S., Brüggen, M., Schmidt-Hoberg, K., Dolag, K., Ragagnin, A., & Robertson, A. 2021,b Unequal-mass mergers of dark matter haloes with rare and frequent self-interactions. *Monthly Notices of the Royal Astronomical Society*, 510b(3), 4080–4099.

Harvey, D., Massey, R., Kitching, T., Taylor, A., & Tittley, E. 2015, The nongravitational interactions of dark matter in colliding galaxy clusters. *Science*, 347(6229), 1462–1465.

Hu, W., Barkana, R., & Gruzinov, A. 2000, Fuzzy Cold Dark Matter: The Wave Properties of Ultralight Particles. *PRL*, 85(6), 1158–1161.

Kahlhoefer, F., Schmidt-Hoberg, K., Frandsen, M. T., & Sarkar, S. 2014, Colliding clusters and dark matter self-interactions. *MNRAS*, 437(3), 2865–2881.

Kim, S. Y., Peter, A. H. G., & Wittman, D. 2017, In the wake of dark giants: new signatures of dark matter self-interactions in equal-mass mergers of galaxy clusters. *MNRAS*, 469(2), 1414–1444.

Kummer, J., Brüggen, M., Dolag, K., Kahlhoefer, F., & Schmidt-Hoberg, K. 2019, Simulations of core formation for frequent dark matter self-interactions. *MNRAS*, 487(1), 354–363.

Moliere, G. 1948, Theorie der streuung schneller geladener teilchen ii mehrfach-und vielfach-streuung. *Zeitschrift für Naturforschung A*, 3(2), 78–97.

Oman, K. A., Marasco, A., Navarro, J. F., Frenk, C. S., Schaye, J., & Benítez-Llambay, A. r. 2019, Non-circular motions and the diversity of dwarf galaxy rotation curves. *MNRAS*, 482(1), 821–847.

Randall, S. W., Markevitch, M., Clowe, D., Gonzalez, A. H., & Bradač, M. 2008, Constraints on the Self-Interaction Cross Section of Dark Matter from Numerical Simulations of the Merging Galaxy Cluster 1E 0657-56. *ApJ*, 679(2), 1173–1180.

Robertson, A., Massey, R., & Eke, V. 2017,a Cosmic particle colliders: simulations of self-interacting dark matter with anisotropic scattering. *MNRAS*, 467a(4), 4719–4730.

Robertson, A., Massey, R., & Eke, V. 2017,b What does the Bullet Cluster tell us about self-interacting dark matter? *MNRAS*, 465b(1), 569–587.

Rocha, M., Peter, A. H. G., Bullock, J. S., Kaplinghat, M., Garrison-Kimmel, S., Oñorbe, J., & Moustakas, L. A. 2013, Cosmological simulations with self-interacting dark matter – i. constant-density cores and substructure. *Monthly Notices of the Royal Astronomical Society*, 430(1), 81–104.

Spergel, D. N. & Steinhardt, P. J. 2000, Observational evidence for self-interacting cold dark matter. *Physical Review Letters*, 84(17), 3760–3763.

Springel, V. 2005, The cosmological simulation code GADGET-2. *MNRAS*, 364(4), 1105–1134.

Springel, V., White, S. D. M., Jenkins, A., Frenk, C. S., Yoshida, N., Gao, L., Navarro, J., Thacker, R., Croton, D., Helly, J., & et al. 2005, Simulations of the formation, evolution and clustering of galaxies and quasars. *Nature*, 435(7042), 629–636.

Tulin, S. & Yu, H.-B. 2018, Dark matter self-interactions and small scale structure. *PHYSREP*, 730, 1–57.

Wittman, D., Golovich, N., & Dawson, W. A. 2018, The Mismeasure of Mergers: Revised Limits on Self-interacting Dark Matter in Merging Galaxy Clusters. *ApJ*, 869(2), 104.