EVOLUTION OF A ROTATING STAR OF NINE SOLAR MASSES

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To compute the evolution of a rotating star, the following approximations were used in order to obtain some of the main effects of rotation with simple models of spherical symmetry:

(a) The angular velocity distribution was assumed to be spherically symmetric: $\omega = \omega(r)$, and the stars assumed to be spherical.

(b) In the (spherical) stellar structure equations only the radial component $c_r = \omega^2 r \sin^2 \vartheta$ of the centrifugal force $\mathbf{c}$ per gram was taken into account, and in order to keep spherical symmetry was replaced by its mean value $\bar{c}_r = (\frac{2}{3})\, \omega^2 r$ over a sphere. Oblateness effects were neglected.

One therefore gets the following equation for hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{4\pi r^4} + \frac{\omega^2}{6\pi r}, \tag{1}$$

which replaces the normal condition of hydrostatic equilibrium in our stellar evolution program described elsewhere (see Kippenhahn *et al.*, 1967).

If one computes time sequences of stellar models, a prescription of how the function $\omega(r)$ or $\omega(M_r)$ changes with time is necessary. The computations have been started on the main sequence. Until now we have used only solid body rotation for the initial rotation on the zero age main sequence. The change of the angular velocity distribution with time was computed under the following assumption: either the angular momentum has to be conserved locally (i.e. in spherical shells) or over a certain region in the star which is rotating as a solid body. If one prescribes in which regions one has local angular momentum conservation and in which regions one has $\omega = \text{const.}$ then the evolution of the angular velocity distribution of the model is determined.

We followed the evolution of a star of nine solar masses with a chemical composition of $X = 0.739$, $Z = 0.021$. Two cases of rotation were computed:

Case (α): Convective regions are in solid body rotation, angular momentum is conserved locally in radiative regions;

Case (β): Regions which are chemically homogeneous are in solid body rotation; angular momentum is conserved in regions in which the molecular weight increases inwards.

The latter case is based on the picture that in a rapidly rotating star the Eddington-Vogt circulation and the Goldreich-Schubert-Fricke instability (Goldreich and Schubert, 1967; Fricke, 1968) produce a sufficiently strong mixing of angular momentum and since both types of motion are hindered by μ-gradients (Mestel, 1953; Goldreich and Schubert, 1967), the regions of varying molecular weight are the only regions where the angular momentum has to be conserved locally.

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For the main sequence model the maximum angular velocity possible for solid body rotation was used in both cases. The computations were terminated in a phase, where a carbon-oxygen core is surrounded by a helium burning shell which produces most of the energy radiated from the surface of the star. For comparison the evolution of the nonrotating model was also computed.

In the case of rotation the time scales for nuclear burning at the center of the star

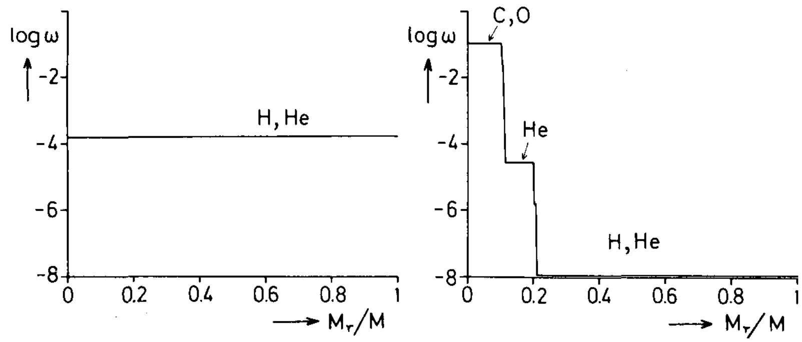

Fig. 1. The angular velocity distribution as a function of the depth in a star of 9 $M_\odot$. In the *left diagram* the star is in the zero-age main sequence stage rotating with break-up velocity. The angular velocity is assumed to be constant. In the *right diagram* the angular velocity distribution in an evolved stage is given. The star is now a red supergiant, a carbon-oxygen core has already been formed, which is surrounded by a helium envelope which again is surrounded by an envelope consisting of the original hydrogen-rich mixture. The evolution of the ω-distribution has been computed according to the assumptions of our case (β). ω varies only in regions of variable chemical composition. In the evolved stage the star has a rapidly spinning core with a rotational period of about 60 sec. With this period the star has reached a stage where the centrifugal force balances gravity in the equatorial plane at the boundary of the carbon-oxygen core.

were longer: about 4% for hydrogen burning and about 12% for helium burning, with slight differences between cases (α) and (β) (see Figure 1).

Two problems follow from these computations. In case (β) the rotation law near the main sequence will be governed by the fact that the convective core and the envelope, both in solid body rotation, are separated by a zone of varying molecular weight, which prevents the transport of angular momentum from the core into the envelope. But the structural changes in the envelope are such that the ratio of centrifugal to gravitational force at the equator increases with time, causing a mass loss due to rotation in and close to the main sequence band, since the star was assumed to be fully rotating at the beginning. This might give an explanation for Be-stars like Pleione without the assumption of solid body rotation for the entire star. The question is how much mass will be lost before the star becomes a red giant.

The second problem is raised by the formation of rapidly rotating cores, as indicated in Figure 1. After central helium burning the material in the carbon-oxygen core reaches an angular velocity such that in the equatorial plane the centrifugal force balances gravity. This is true for both cases, although the details of the rotation law for these two series differ from each other. This raises the question of what happens if the core of an evolved star spins faster and faster.

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Discussion

Deutsch: Is it possible to scale these solutions to approximate stars starting from lesser rotational velocities?

Thomas: It may be possible, but in case (β) I would prefer to re-do the computations near the main sequence with a smaller starting value for the angular velocity.

Roxburgh: The solutions you have can be scaled at least approximately since the effect of rotation on evolution is small and the change in angular velocity can be calculated from the spherical evolution, with the assumptions you have made.

Ostriker: Some of the very evolved models ($\alpha_c \simeq 0.1$) would appear to be unstable to the Kelvin modes which are thought to lead to fission. This leads to the intriguing possibility of the creation of white dwarf-like binary stars *within* red giants!

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