METHOD OF SCATTERING PLANE SCANNING

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Abstract. The number density of interplanetary dust particles as a function of altitude above the symmetry plane can be determined from ground-based measurements by a new method.

After the latest spatial as well as ground-based photometric studies of the zodiacal light, the radial density variation of interplanetary dust in the ecliptic, as represented with heliocentric distance by a $r^{-\nu}$ law, seems to be established with remarkable accuracy: $\nu = 1.3 \pm 0.2$ for the inner part. The same cannot be said about the density distribution out of the ecliptic which remains an open question requiring further studies. (Dumont and Sánchez 1976; Leinert 1975; Weinberg and Sparrow 1978). In the present work we describe a method which could be used for deriving the form of the spatial distribution function from ground-based observations.

In Figure 1 an observer is located at point O in the ecliptic. If the scattering function $\sigma(\theta)$ does not depend on the heliocentric

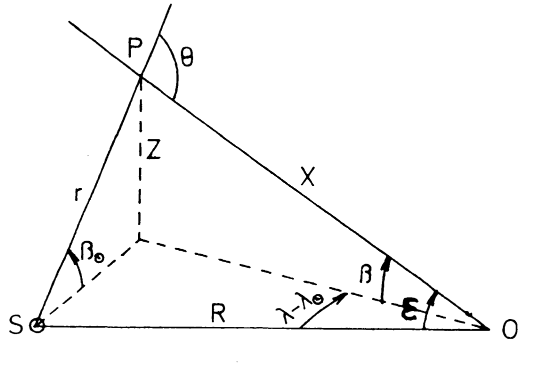

Fig. 1. Distances and angles for a scattering observation.

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the distance r, the observed intensity for a viewing direction passing through P is given by the well-known brightness integral:

$$I(\varepsilon,\beta,R) = F_o R_o^2 \int^{\infty} \frac{\sigma(\theta)\ n(r,\beta_\odot)}{r}\, dx \qquad (1)$$

being related to β, ε, and θ by

$$\sin\beta_\odot = \frac{\sin\beta\ \sin(\theta-\varepsilon)}{\sin\varepsilon} \qquad (2)$$

Now we make the following assumptions:
1. The spatial density distribution $n(r,\beta_\odot)$ can be written, following Hang (1958):

$$n(r,\beta_\odot) = n(r)\ f(\beta_\odot) \qquad (3)$$

with the condition $f(\beta_\odot=0) = 1$.
2. the radial function in the ecliptic can be expressed, according to the $r^{-\nu}$ law, as

$$n(r) = n_o \left(\frac{r}{r_o}\right)^{-\nu} \qquad (4)$$

(n_o = density at r_o = 1 AU)
Taking into account the former assumptions and changing the integration to θ, equation (1) takes the form:

$$I(\varepsilon,\beta,R) = \frac{F_o\ R_o^{(2+\nu)} n_o}{R^{(1+\nu)} \sin^{(1+\nu)}\varepsilon} \int_{\theta=\varepsilon}^{\pi} \sin^{\nu}\theta\ \sigma(\theta)\ f(\beta_\odot)\ d\theta \qquad (5)$$

With any assumed scattering function $\sigma(\theta)$ equation (5) is an integral equation of the first kind of the Volterra type, $f(\beta_\odot)$ being the unknown function. For solving the latter equation an approximate numerical approach will be used transforming it into a system of linear equations. The link between theory and observation is demonstrated in fig. 2. Starting from an initial elongation ε_1, a number p of density measurements, spaced by the same $\Delta\varepsilon$, out of the ecliptic and always in the same plane of scattering are performed. It is important to note that for all the points along the i labelled lines, the value of the function $f(\beta_\odot)$ is constant.

If we measure the intensity along any of the observation directions (label j), equation (5) can be written

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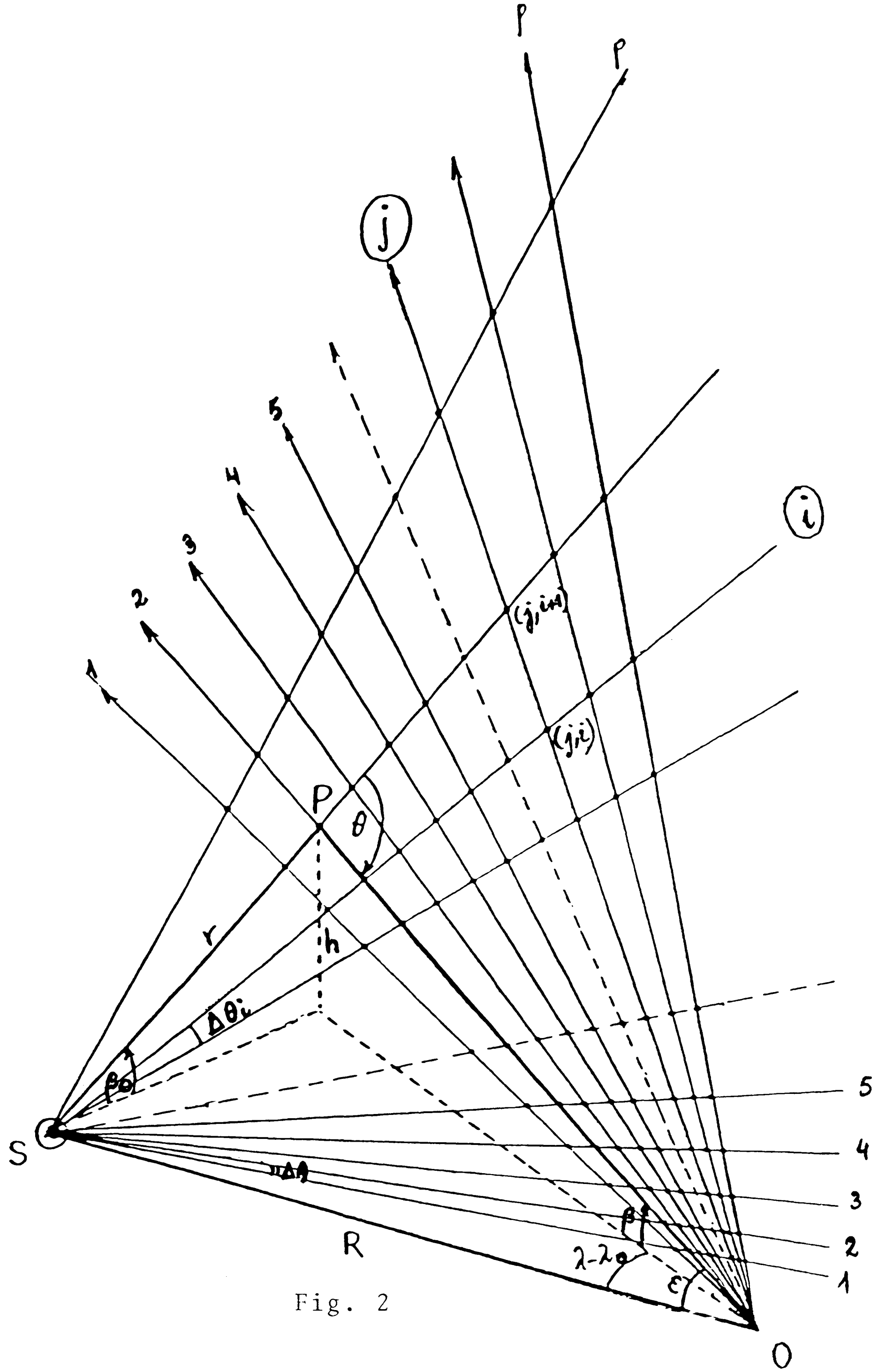

Fig. 2

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$$I(\varepsilon,\beta)_j = \frac{F_\circ R_\circ n_\circ}{\sin^{(1+\nu)}\varepsilon_j}\left\{\lim_{\Delta\theta\to 0}\sum_{\theta_1=\varepsilon_j}^{\pi}\left[\sin^{\nu}\theta_i\ \sigma(\theta_i)\right]_j f(\beta\ )_i\ \Delta\theta\right\} \qquad (6)$$

(for $R = R_o$).

Now, we consider the p different observing directions. Since $f(\beta_\odot)$ is expected to be a decreasing function of $\beta_\odot$, for the spacing of the p lines i, we introduce a scaling parameter $\alpha \geq 1$, such that

$$\begin{aligned} \Delta\theta_1 &= \Delta\theta \\ \Delta\theta_2 &= \alpha\Delta\theta \\ &\cdots\cdots \\ \Delta\theta_p &= \alpha^{(p+1)}\Delta\theta \end{aligned} \qquad (7)$$

In this way, the following matrix equation can be written

$$\frac{\sin^{(1+\nu)}\varepsilon_j}{F_\circ R_\circ n_\circ} I(\varepsilon,\beta)_j = \left[\sin^{\nu}\theta_i\ \sigma(\theta_i)\right]_j f(\beta_\odot)_i \alpha^{(i-1)}\ \Delta\theta \qquad (8)$$

For a given scattering function $\sigma(\theta)$, the form of the distribution function $f(\beta_\odot)$ can be obtained by inversion of the matrix $[\sin^{\nu}\theta_i\sigma(\theta_i)]_j$. The validity of the above described method is ultimately conditioned by its selfconsistency. By this we mean that once the function $f(\beta_\odot)$ is determined (after a certain number of scannings), its insertion in equation (6) (where θ can actually reach π), should reproduce the observed intensity in any direction.

Equation (6) has been applied to the ecliptic, $30° \leq \varepsilon \leq 70°$, as a check of the method, using the scattering function obtained by Mujica (Mujica et al., this volume); the results are presented in Figure 3.

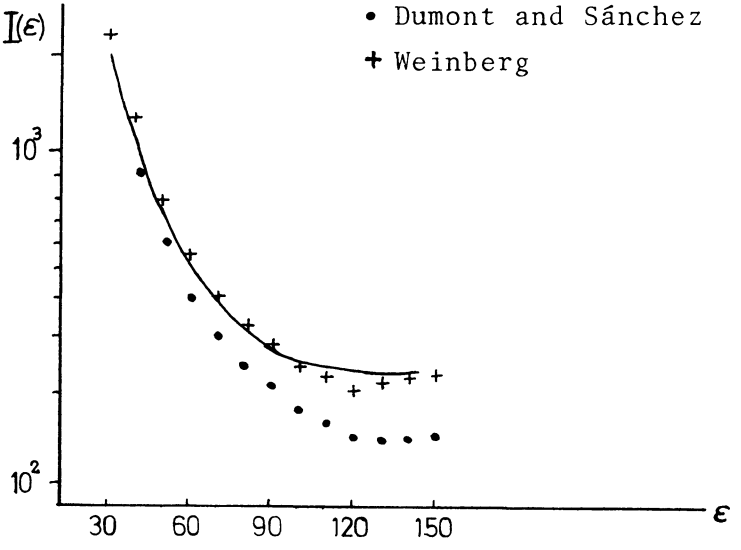

Fig. 3

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DISCUSSION

Dumont: I believe that your suggestion is one of the best possible for determining the law of variation of the dust density out of the ecliptic at least from earthbound measurements. Until we have data from the out-of-ecliptic mission, I completely agree that new ground-based observations, in addition to the existing ones, must be planned and/or reduced by such a method in order to improve our poor knowledge of the off-ecliptic distribution of scatterers.
Sánchez: Yes; we think the same and we are planning observations to be carried out as soon as our new spectropolarimeter is working.

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