STUDY OF THE NORMAL VARIATIONAL EQUATION IN AN HOMOGENEOUS FIELD OF DEGREE FIVE

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We study the normal variational equation of degree five:

$$\frac{d^2\xi}{dt^2} + \lambda \cdot C_5^3(t) \cdot \xi = 0 \tag{1.1}$$

where λ is a parameter and $C_5(t)$ verifies

$$\begin{cases} \ddot{x} + x^4 = 0 , \\ x(0) = 1 , \quad x(0) = 0 , \end{cases} \qquad x = x(t) . \tag{1.2}$$

The solution $C_5(t)$ of (1.2) is the inverse function of

$$t = F(x) = \sqrt{\frac{5}{2}} \int_x^1 \frac{du}{\sqrt{1-u^5}} .$$

We prove that the pole t_5 of $C_5(t)$ defined by $C_5(t) \underset{t \to t_5}{\longrightarrow} -\infty$ is given by

$$t_5 = K \left[1 + \frac{1}{\cos\left(\frac{\pi}{5}\right)} \right] ,$$

where $K = \sqrt{\frac{5}{2}}\, I_1$, $I_1 = \int_0^1 \frac{du}{\sqrt{1-u^5}}$.

We give the development of $C_5(t)$ in the neighborhood of the pole t_5 by using the Siegel theorem (Yoshida, 1986):

$$C_5(t_5 + \tau) = \sqrt[3]{\frac{-10}{9}} \cdot \tau^{-2/3} \left[1 - x + a_1 x^2 + \cdots + a_n x^n + \cdots \right]$$

where $x = \frac{9^{5/3}}{13 \cdot 10^{5/3}} \cdot \tau^{10/3}$, $a_1, \cdots, a_n$ are constants to be determined. We raccord the solution $C_5(t)$ in the neighborhood of t_5 . We prove by using the lacets method that the solution $C_5(t)$ has two independent real periods and two independent imaginary periods:

$$\begin{cases} P = 8\sqrt{\frac{5}{2}} \cdot I_1 \cdot (1 - \alpha^2) , & Q = 4\sqrt{\frac{5}{2}} \cdot I_1 \cdot (1 + \alpha) , \\ P' = i\, 8\sqrt{\frac{5}{2}} \cdot I_1 \cdot \alpha \cdot \sqrt{1 - \alpha^2} , & Q' = i\, 4\sqrt{\frac{5}{2}} \cdot I_1 \cdot \sqrt{1 - \alpha^2} , \end{cases}$$

where $\alpha = \cos\left(\frac{\pi}{5}\right)$. By putting $z = [c_5(t)]^5$, we transform the N.V.E. (1.1) to the Gauss hypergeometric equation:

$$z(1-z)\frac{d^2y}{dz^2} + \left(\frac{4}{5} - \frac{13z}{10}\right)\frac{dy}{dz} + \frac{\lambda}{10} y = 0$$

where $y = y(z)$. We obtain the solutions of (1.1) in form of series.

J. Henrard and S. Ferraz-Mello (eds.), Impact of Modern Dynamics in Astronomy, 459–460.

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References

A. Makhlouf and F. Nahon: 1991, On a family of Hill's equation in the complex field, *Celest. Mech.*, **52**, 227–291.

A. Lainé: Précis d'analyse mathématique, Tome 1, p. 213.

H. Yoshida: 1986, Existence of exponentially unstable periodic solution and the non integrability of homogeneous Hamiltonian systems, *Physica D*, **21**

H. Yoshida: 1987, A criterium for the non existence of an additional integral in systems with an homogeneous potential, *Physica D*, **29**