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OPTIMAL STOPPING PROBLEMS WITH A RANDOM TIME HORIZON

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The theory of optimal stopping provides a powerful set of tools for the study of the American contingent claim pricing problem in mathematical finance. We give a self-contained overview of the theory, including the complete proofs of existence and uniqueness theorems for the optimal stopping time in finite-time formulation. These theorems are developed with the goal of formulating the corresponding free-boundary problems for the valuation of atypical American contingent claims involving nonstopping times, which restrict the optimal stopping rules to be made before the last exit times of the price of the underlying assets at its running maximum or at any fixed level.

By exploiting the theory of enlargement of filtrations associated with random times, the original pricing problem with random times can be transformed into an equivalent optimal stopping problem with a semi-continuous, time-dependent gain function, whose partial derivative is singular at certain points. The difficulties in establishing the monotonicity of the optimal stopping boundary, the regularity of the value function and its differentiability in the boundary lie essentially in these somewhat unpleasant features of the gain function.

However, it turns out that a successful analysis of the continuation and stopping sets with proper assumptions can help us overcome these difficulties and further obtain the important properties possessed by the free boundary, which eventually leads us to the desired free-boundary problem. After this, we derive the nonlinear integral equations that characterise the free boundary and the value function. The solutions

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to these equations are examined in detail, and their financial justification is discussed briefly in the final chapter.

Some of this research has appeared in [1, 2].

References

[1] Z. Wu and L. Li, 'The American put option with a random time horizon', Preprint, 2023, arXiv:2211.13918.

[2] Z. Wu and L. Li, 'The Russian option with a random time horizon', Preprint, 2023, arXiv:2211.13917.

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