BOOK REVIEWS

BOURBAKI, N., *Éléments de Mathématique XXIII, Les Structures Fondamentales de L'Analyse* Livre II, *Algèbre*, Chapitre 8, *Modules et Anneaux Semi-Simples* (Hermann, Paris, 1958), 189 pp., 38s. 3d.

This chapter of Book II of the above series is divided into thirteen sections, and it is assumed throughout these that the rings have identity elements and that the modules are unitary.

The first section is concerned with commutative properties of rings and modules —centralisers, etc. The chain conditions of Artin and Noether are introduced in § 2, and in the following four sections the notions of " simplicity ", " semi-simplicity " and " radical " are considered. The radical and semi-simplicity of tensor products are discussed in § 7. §§ 8, 9 deal with applications of the preceding theory to composite extensions of fields and endomorphisms of vector spaces. § 10 is concerned with simple subrings of a given ring and with isomorphisms of simple rings, while these results are applied in § 11 to establish Wedderburn's theorem on division rings and to characterise the quaternions. The last two sections deal with norms and traces and with representation theory.

There is an Appendix containing a discussion of algebras which may not have an identity element, and this is followed by an interesting Historical Note in which the development of the subject through its main stages is traced.

The various concepts introduced in the text are well illustrated by means of examples, and there is an impressive collection of exercises at the end of each section. Nevertheless, the beginner will find this book difficult because of the practice of proceeding from the very general to the particular. On account of its unifying effect, however, this treatment should appeal to the reader with some experience.

A. GEDDES

LEVI, H., *Elements of Algebra* (Chelsea Publishing Co., New York, 3rd ed. 1960), 160 pp., \$3.25.

The aim, which Professor Levi had in mind when writing this book, was to provide a serious text on the foundations of algebra for university students whose course was not primarily mathematical in character, but from which the normal mathematical student could learn a great deal.

Using the theory of sets, the author develops the cardinal numbers, defines number systems in general, and leads to the construction of the integers, rational numbers and finally the real number system. At the same time algebraic expressions, polynomials and equations are clearly defined, whilst the book closes with a chapter on the Peano Axioms and the definitions of groups, rings and fields.

This is a book which should be available to the university student and the mathematical specialist at the training colleges. The printing and binding is up to the usual high standard of American publications.

G. ALLMAN

NATANSON, I. P., *Theory of Functions of a Real Variable* (Frederick Ungar Publishing Co., New York, 1955), 277 pp., \$6.50.

This is a translation by Leo F. Boron of the first nine chapters of a longer book by the author, published in Russian in 1941. It consists mainly of an account of the classical theory of the Lebesgue integral for functions of a real variable. The author assumes only a knowledge of elementary undergraduate analysis and begins with a chapter on infinite sets and cardinal numbers and another on open and closed sets of real numbers. Then the theory of Lebesgue measure for bounded sets is developed (by means of outer and inner measures). The Lebesgue integral of a bounded measurable function is defined by subdivision of the range and the definition is extended in the usual way to unbounded functions. The original Russian text contained no treatment of measure or integrals on unbounded sets, but, in this American edition, the editor, Edwin Hewitt, has added appendices to the appropriate chapters in order to rectify this omission. There is a chapter on the space L^2 and the book ends with two chapters on functions of bounded variation, absolute continuity and the differentiation properties of the integral.

Throughout, the book is very easy to read; proofs are given clearly and in full. The author has not confined himself to the bare bones of the subject, but has clad them with a wealth of additional material. As well as the main subjects outlined above, many other topics are introduced. For example, proofs are given of Weierstrass's approximation theorem, Helly's choice principle and Riesz's representation theorem for continuous linear forms on the space C. Within the limitations imposed by restriction to real variables only, the contents form an excellent account of integration theory, and a useful introduction to functional analysis.

A. P. ROBERTSON

ZAANEN, A. C., *An Introduction to the Theory of Integration* (North Holland Publishing Company, Amsterdam, 1958), ix+254 pp., 50s.

It is now customary to develop the general theory of integration either by means of measure theory or by extending an elementary integral to a larger class of functions. The author, feeling that it is important to be familiar with both approaches, has combined them in this book. After an introductory chapter containing some set theory and topology, the book starts with measure theory. Given a measure on a semi-ring of sets, it is shown how to extend it (by means of the corresponding exterior measure and Carathéodory's definition of measurability) to a larger σ-ring of sets. This extension process is then used to develop the theory of the Daniell integral. One starts with a vector lattice of bounded functions on which there is a positive linear functional, continuous under monotonic convergence to zero. The ordinate sets of the positive functions in the lattice generate a semi-ring on which the functional is a measure, and its extension is made to yield the classes of measurable and integrable functions. If the vector lattice contains $\min(f, 1)$ whenever it contains f, every integral so obtained corresponds to a measure in the usual way.

Subsequent chapters deal with such topics as Fubini's theorem, the Radon-Nikodym theorem and differentiation of the integral; in addition to the usual differentiation theory for the Lebesgue integral on a Euclidean space, there is an account of differentiation of set functions relative to a monotone sequence of nets. The author has also included certain parts of functional analysis which are relevant to integration theory; the sections on Banach spaces and Hilbert space form a useful introduction to these subjects, and they are applied to the study of the spaces L_p and the Fourier transformation in L_2. The book ends with a chapter on ergodic theory.

Numerous exercises are scattered throughout the book; many contain further results in the theory and are accompanied by condensed solutions. A reader with the minimum preparation may find the going hard in places, but he will be doubly rewarded, by having mastered the most useful parts of integration theory and also by becoming acquainted with some other important branches of modern analysis.

A. P. ROBERTSON

NIVEN, I., *Irrational Numbers* (Carus Mathematical Monographs No. 11, 1956), 164 pp., 24s.

This book covers a wide field, beginning with Cantor series and the countability of the rationals in Chapter I, then giving an elementary treatment of trigonometric and

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