BENDING WAVES ON CURRENT SHEETS

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ABSTRACT. Current sheets are found to be subject to bending waves described by a dispersion relation indicating that these are, essentially, modified surface Alfvén waves. Applications to the observed magnetic polarity sectors in the solar wind and to other astrophysical environments, such as planetary magnetospheres, are suggested.

1. INTRODUCTION

Recent in situ measurements of the solar wind plasma have demonstrated the need for a new theoretical description of both its large scale structure and of its microscopic properties. This would be in contrast to the "classical" model (Parker 1958) of the solar wind which assumes that the wind is the result of a spherically symmetric expansion of the coronal plasma, and that at the base of the corona the transport of thermal energy can be described by the collisional electron thermal conductivity.

Here we start from the well known observation of a sector structure in the magnetic field polarity and recall that, except for occasional short intervals in which the polarity is not well defined, two, four, or occasionally six major sectors are observed per solar rotation. As a result, a model where a current sheet surrounds the Sun and is inclined roughly 15° to the solar equator (Smith 1979; but see also Hoeksema et al. 1982 and Bruno et al. 1982) has been proposed. We note the following points: (1) The tilted current sheet model is only a small departure from a situation where no tilt is present, since the required inclination is small. (2) The sector structure is a large scale phenomenon where the polarity reversal is observed an even number of times (two, sometimes four, more rarely six times). (3) The sector structure is related to a current sheet, since it is not observed above a certain heliographic latitude; it is a very gentle disturbance on the current sheet. Therefore, we propose that the large scale topology of the interplanetary magnetic fields should be explained in terms of large scale bending waves of relatively small amplitude which are naturally excited on an otherwise axisymmetric current sheet disk.

Our point of view is supported by a strong analogy that the above men-

M. R. Kundu and G. D. Holman (eds.), Unstable Current Systems and Plasma Instabilities in Astrophysics, 491–496.

tioned preliminary data on the disk structure of the solar wind have with the geometry of galactic disks. In fact a promising theory for warps in self-gravitating disks has been developed and warps have been suggested to be the manifestation of large scale bending waves (Bertin and Mark 1980; Bertin and Casertano 1982; see, also, Hunter and Toomre 1969). The theory of bending waves has found recently an attractive application to the (driven) corrugations of Saturn's rings (Shu et al. 1983).

2. BENDING WAVES ON A CURRENT DISK

We consider an equilibrium axisymmetric current disk characterized by a thickness δ_z, a mass density ρ_D, and an outward streaming velocity $\underline{u}$. For mathematical purposes it may be convenient to ignore the physical thickness δ_z and work on an infinitesimally thin disk model. If we refer to the cylindrical coordinates (R,ϕ,z), this is described by $\rho_D(R,z) = \sigma(R)\delta(z)$, and $\underline{J}_D = \underline{j}(R)\delta(z)$, where $\sigma(R)$ and $\underline{j}(R)$ are surface mass and line current densities. Here $\delta(z)$ is the standard delta-function. Close to the current sheet the magnetic field $\underline{B}_D$ is perpendicular to the local current streamlines $\underline{j}$ and its strength B_D is approximately given by $B_D = (2\pi/c)j$.

We are interested in perturbations that bend the current disk from its planar equilibrium configuration. These waves, produce components of the transverse perturbed magnetic field $(\tilde{B}_z)$ that are odd in z. We recall that, in a finite thickness model configuration, even modes have different physical characteristics and produce local current enhancements without breaking the planar symmetry of the current layer. In the infinitesimally thin disk model the bent sheet can be described by the local height $h(R,\phi,t)$ of the bent disk from the equatorial plane. In particular, we look for normal modes of the form $h(R,\phi,t) = \mathrm{Re}\{\tilde{h}(R)\exp[i(\omega t - m\phi)]\}$.

The equation of motion for the bent disk can be written as

$$\sigma \frac{D^2 h}{Dt^2} = F_z \tag{1}$$

where the total (convective) derivative is defined by $D/Dt = i[\omega - (m/R)u_\phi + ku_R]$. We have introduced a radial wavenumber $k = k(R)$, that is appropriate for those perturbations that can be studied by a WKBJ analysis, as we assume $|Rk| > 1$.

The force F_z arises in part from the action of the current sheet on itself. The reason is that bending the current disk modifies the associated magnetic field. We have to evaluate this field $\tilde{B}$ at $z = h(R,\phi,t)$ where the current j is displaced. The relevant integrals are found to be of the same form as the integral that describes self-gravitating disks. As a result we can make use of the asymptotic theory of bending waves in galaxies given by Bertin and Mark (1980) and find, in the WKBJ limit ($|kR| > 1$ and $m = O(1)$)

$$\tilde{F}_z = \frac{1}{c}(j_x\tilde{B}_y - j_y\tilde{B}_x) \simeq -\frac{2\pi}{c^2} j^2|k|\tilde{h} = -\frac{B_D^2}{2\pi}|k|\tilde{h} \quad , \tag{2}$$

This corresponds to a restoring force. Inserting Eq. (2) into Eq. (1) we find the relevant dispersion relation:

$$(\omega - \frac{m}{R} u_\phi + ku_R)^2 = \frac{B_D^2}{2\pi} |k| = \frac{v_A^2}{R\delta_z} |kR| \quad (3)$$

where $v_A = B_D/\sqrt{4\pi\rho}$ is an appropriate Alfvén speed constructed with the typical plasma density of the sheet and the magnetic field B_D generated just outside the current layer. Here δ_z is defined by $\sigma = 2\delta_z\rho$. This dispersion relation governs the flapping of the current sheet and describes essentially a surface Alfvén wave.

We expect that the dispersion relation given above should include, on the right hand side, an additional term ω_s^2 which depends on the fields and currents in the equilibrium, and describes the effects of a restoring magnetic force on the current sheet due to the action of fields not generated by the local current $\underline{J}_D$.

We now consider the case of low frequency bending waves, $\omega \sim m\Omega_s$, corresponding to disturbances that are almost neutral in the frame rotating at the angular velocity Ω_s of the Sun, and therefore rotate rigidly with angular velocity close to Ω_s in the inertial frame. Since we require $|kR| > 1$, we can take $|\omega - (m/R)u_\phi| << ku_R$. If we assume further $\omega_s^2 < v_A^2/(R\delta_z)$, we find $k \sim k_o$ with

$$|k_o(R)| = \frac{v_A^2}{u_R^2\delta_z} \quad (4)$$

This expression for k_o indicates the scalelength of low frequency bending waves. Our WKBJ analysis requires $|Rk_o| > 1$. Finally, from the dispersion relation we find $c_g = -\partial\omega/\partial k \sim u_R/2$. Therefore for these waves, the group velocity indicates that they naturally propagate outwards at half the radial speed of the wind. This suggests that if quasi-stationary bending structures are observed in the Ω_s-rotating frame they must have a good source of energy upstream, i.e. close to the symmetry axis.

3. EXCITATION MECHANISMS FOR BENDING WAVES

We may point to three forms of excitation that can apply to our case and may be not mutually exclusive:

3.1 Driving at the source

The Sun, and in particular the solar corona, is observed to possess irregular magnetic structures that undoubtedly are related to a slowly evolving, moderately nonaxisymmetric system of currents. Obviously such a source, at the center of the current disk, affects the dynamics of the current sheet far from the Sun. This is somewhat reminiscent of the processes that can drive spiral structures in galaxies that possess a well developed central bar (see Lin and Bertin 1981). Of course the lower multipoles (m = 1,2,3) are the first to be induced in this situation. Therefore it is not surprising to have two (a tilt), four, or six sector structures among the common cases in the solar wind.

3.2 Internal mechanisms of instability

Another possibility is that the solar wind current disk be internally un-

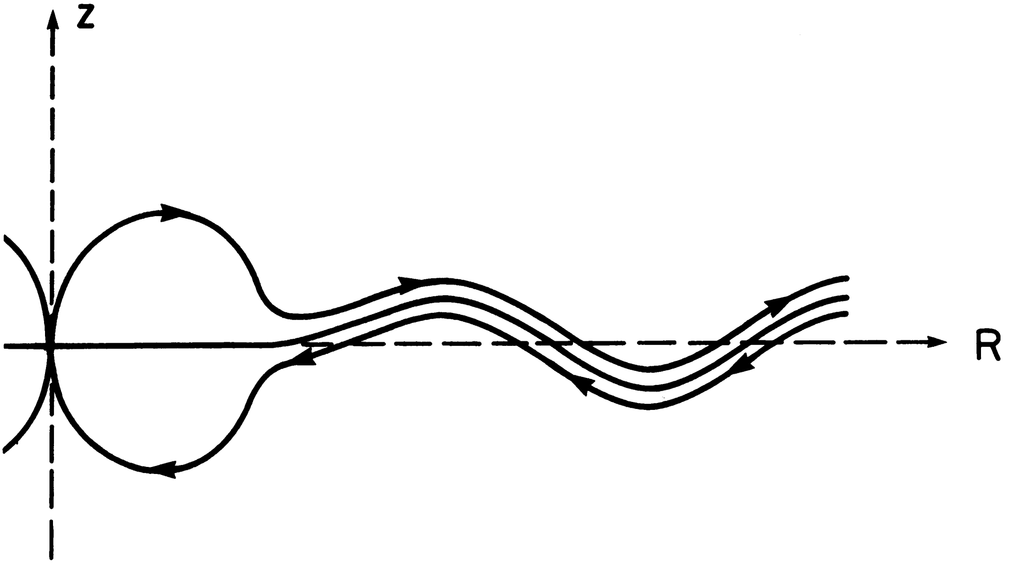

Fig. 1: Qualitative representation of bending wave in the current disk of the solar wind.

https://doi.org/10.1017/S0074180900076002 Published online by Cambridge University Press

stable against bending waves. It may well be that, for the excitation of these bending waves, an important role is played by the internal structural and dynamic properties of the current disk, such as the distortion of the distribution function of electrons and ions from a Maxwellian or the gradient of the current density, that go beyond our simplified MHD analysis. Indeed a few excitation mechanisms based on "microscopic" dynamics could be identified, such as magnetic instabilities which rely on temperature anisotropy for their excitation. Our previous studies of magnetic configurations relevant for the solar wind environment suggest that the above processes are quite sensitive to detailed properties of the particle distribution function (Bertin and Coppi 1980). In particular there are modes (Coppi 1982) involving magnetic reconnection around the neutral sheet that possess a symmetry which would naturally couple them with the bending waves studied above. Quite likely bending waves excited by internal mechanisms are the essence of what Alfvén suggested (1977) and is known under the name of "ballerina effect". To our knowledge, Alfvén sketched only briefly the concept of the flapping of the current sheet but gave no quantitative explanation of his suggestion, nor did he identify the relevant restoring forces.

3.3 Wind Driving

If a current disk is embedded in a medium characterized by varying stream velocities we expect a flapping instability similar to that of Kelvin-Helmohltz to develop. This situation would apply to the solar wind current disk, if available and future data should indicate sizeable velocity gradients across the disk. A circumstance of this type is more likely to occur in planetary magnetotails, such as those that are known to associate with the Earth, Jupiter, and Saturn. In this latter case the velocity gradient is naturally provided by the relative motion between the planet magnetosphere and the solar wind. The geometry would be different from that used above but similar physical concepts should apply.

This work was sponsored in part by the Consiglio Nazionale delle Ricerche of Italy and by A.F. Geophysics Laboratory, (Bedford, Massachusetts).

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DISCUSSION

Migliuolo: Since the mode varies with radius (k_r), this model predicts different polarizations of B at different distances from the sun. Hence we should observe it with Helios and other spacecraft.

Coppi: You have a good point. It should be possible to verify this model by correlating simultaneous measurements of the solar wind magnetic field at widely separated distances. However I do not know how yet to resolve certain ambiguities that an experiment of this type seems to involve.

Vlahos: You have made the commment several times in this conference that a Maxwellian distribution may not be a good approximation for astrophysical calculations of microinstability analysis. My question is, how far from a Maxwellian distribution do you have to be to alter the results significantly?

Coppi: In order to excite the reconnecting type of modes I discussed, a small temperature anisotropy ($0 < T_{\perp}/T_{\parallel} - 1 < 1$) is sufficient.

Kennel: Would you repeat the arguments about the direction of energy flow in your mode? Is it a negative energy mode?

Coppi: The spiral wave that we have found has positive energy in the frame that has been considered. The direction of the energy flow is outward.

https://doi.org/10.1017/S0074180900076002 Published online by Cambridge University Press