propositions are introduced in Chapter 2. Next, algebraic systems called ternary fields are used to provide coordinates in the plane and a study is made of the algebraic consequences of the geometrical situations. The remaining chapters deal with the extension to three dimensions, with the fundamental proposition of projective geometry and with the concept of order.

The book is generously provided with figures which, whilst not essential, are of considerable help to the reader in following the exacting logical arguments. There are very few references and no bibliography, the reader being referred for these to G. Pickert's comprehensive monograph " Projektive Ebenen " (Springer, Berlin, 1955). This is perhaps a slight defect in a book which is otherwise compact and self-contained. A small related complaint is that the name " Moulton plane " might have been attached to the model, denoted by M, used to establish the independence of Desargues' proposition from the axioms of incidence.

The geometrical propositions treated in the book are strictly confined to those relevant to its logical purpose. Thus conics are not mentioned (except for a sentence to that effect on p. 61). It cannot therefore be regarded as a textbook for a traditional course in Projective Geometry. This remark, however, is in no way a criticism: those who are interested in its important theme will find it very useful.

D. MONK

QUINE, WILLARD VAN ORMAN, *Set Theory and its Logic* (Harvard University Press, London: Oxford University Press, 1964), xv+359 pp., 48s.

This account of set theory and its foundations is sound, lucid and well produced. The author has a lively style and strikes the right balance in presenting his material, some of which is original. Few words are wasted, unnecessary details being omitted, yet important points are always clearly explained and are often presented in a new way. It is therefore perhaps the most concise and readable general survey of axiomatic set theory at present available.

The book is unique in first introducing the basic topics of abstract set theory within a framework which does not commit the author to any particular one of the usual axiomatic systems. This neutrality is preserved by adopting only very weak comprehension axioms (axioms asserting the existence of particular classes or sets) and, where stronger existence assumptions are needed, incorporating them as explicit hypotheses for particular theorems. Use of the axiom of choice is made explicit in the same way.

This approach has two advantages. It gives the reader a sound, unprejudiced basis for an appreciation and comparison of the main axiomatic systems of set theory and also emphasises the central problem of such theories—that of accepting comprehension axioms which are strong enough for all mathematical needs but which are weak enough to prevent the derivation of the well-known paradoxes which follow from the naïve comprehension schema

$$(\exists y)(\forall x)[x \in y \Leftrightarrow P(x)]$$

where $P(x)$ is any condition which can be formulated in the system without using the variable " y ". An interesting by-product is the author's treatment of the natural numbers and mathematical induction which, unlike the classical treatments, does not require the existence of infinite sets.

Part I of the book contains the rigorous development of elementary set theory and the arithmetic of natural numbers, the use of a notation for class abstracts (the author calls them " virtual classes " since the existence of a corresponding real class is not implied) making for conciseness and clarity of expression. Part II covers the following topics: Rational and Real numbers, constructed in such a way that the reals contain

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the rationals and not merely a subset isomorphic to the rationals (a modification of the Dedekind cut is used); Ordinal numbers (von Neumann's version), transfinite induction and recursion; Cardinal numbers, introduced (again following von Neumann) as initial ordinals; the Axiom of Choice and its equivalents. Part III is a description of, and an interesting discussion of the relations between, the main axiomatic systems of set theory—essentially those of Russell, Zermelo, von Neumann-Bernays and two systems of the author.

The book is practically self-contained, assuming some knowledge of logic (elementary quantification theory) but no previous knowledge of mathematics or set theory. This, together with its soundness and readability, makes it suitable reading not only for mathematics students (at graduate or undergraduate level), whether as part of an organised course on axiomatic set theory or not, but also for philosophers with an interest in the foundations of mathematics. An excellent index and system of numbering formulæ make it also a useful reference book. More advanced readers will regret the relegation of proof theory to footnotes and parentheses.

A. A. TREHERNE

HERVÉ, M., *Several Complex Variables: Local Theory* (Oxford University Press, 1963), 26s. 6d.

The Theory of Several Complex Variables has, in the past fifteen years, undergone a remarkable development as a result of the work of Oka, Cartan, Stein, Grauert and Remmert. No books on the subject however have appeared for very many years, and the time seems certainly ripe for more modern texts.

The present book by Professor Hervé is a modest but useful contribution. It is essentially self-contained and develops the local theory from its foundations. It begins with the classical results—Weierstrass preparation, Hartogs Theorem, etc. and ends up with detailed results on the structure of analytic sets, including the proof of the coherence of the sheaf of an analytic set.

Compared with the treatment of the same topics given by Cartan in his Séminaires 1951-52 the present exposition strikes the reviewer as a little heavy-handed. This is no doubt due to the author's rather concrete and classical approach, and may be compensated by the fact that little is required of the reader except diligence.

M. F. ATIYAH

HELGASON, S., *Differential Geometry and Symmetric Spaces* (Academic Press, 1962), 486 pp., 89s. 6d.

This book is the first to give a comprehensive account of Cartan's theory of symmetric spaces, i.e. Riemannian manifolds for which the curvature tensor is invariant under all parallel displacements, and of the more modern developments concerning functions defined on these spaces. The book is well written but, since the style is very compact, it will be difficult reading for anyone not already acquainted with the basic ideas of modern differential geometry and the theory of Lie groups. A reading of Lichnerowicz's little book on Tensor Calculus and the recent book by Flanders on Differential Forms would be an excellent propaedeutic to the serious study of the book. However, features of the book which add to its value as a textbook are the short summaries provided at the beginning of each chapter and the collections of problems at the end of each chapter. The bibliography, occupying 15 pages of text, is also an asset. An extraordinary amount of material is included in the book, and anyone prepared to work through it conscientiously will be richly rewarded not only by what he will learn about symmetric spaces but by the sound knowledge he