WAVELET-BASED RESAMPLING TECHNIQUES

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Resampling methods were first developed in the late 1970s with the aim of obtaining estimates of the error in a statistic calculated from a sample. A necessary restriction was that the observations in the sample had to be independent, and various procedures have since been proposed to adapt the methods to dependent observations.

Wavelet transforms that permit data to be analysed from a frequency or time viewpoint simultaneously have also been developed over a similar period. In simple terms, a wavelet transform produces coefficients that are differences between adjacent averages over increasing scales, and it has been noted that these differences are less correlated than the original data. The potential exists, therefore, for the wavelet transform to be applied to dependent data and the resulting coefficients, if deemed to be independent, can be resampled. Applying the inverse transform to the resampled coefficients may produce a surrogate set of data that shares similar characteristics with the original sample.

The aims of the thesis can be expressed thus:

(1)   *to apply wavelet transforms, or variations thereof, to various types of dependent data and examine the resulting coefficients for independence*;

(2)   *if the coefficients are independent, to apply the inverse transform and investigate whether the resulting surrogate data could have come from the population that produced the original sample.*

The types of dependent data considered are time series, restricted to AR(1) and MA(1) processes, one-dimensional point processes and two-dimensional point processes, both unmarked and marked.

The tone of the thesis is investigative, examining various methods and modifications that may be suitable, rather than seeking to justify one particular method.

The discrete wavelet packet transform, combined with wavelet lifting, was used to decompose time series and one-dimensional point processes into components whose elements were judged to be independent. These elements were resampled and the inverse transform subsequently yielded surrogate series. The surrogates of AR(1) and MA(1) time series appeared to be similar to the data only when the parameter was positive. The diagnostics for surrogates of homogeneous, nonhomogeneous and clustered one-dimensional point processes indicated that the surrogates resembled the data.

The wavelet lifting philosophy was used as a basis for developing an algorithm for two-dimensional point processes. Briefly, at each stage of the decomposition, the difference between the position coordinates of a point and the mean position of the point's connected vertices is calculated. The resulting vector of differences is checked for independence and, if satisfied, is resampled. Recomposition yields the surrogate point patterns. Variations of the method were developed for clustered processes as well as qualitatively and quantitatively marked processes. In most instances the method or some variation thereof was able to produce surrogates that could have come from the same population as the data.

The wavelet transform thus provides a basis for developing algorithms that can produce surrogates of some types of time series and point processes.

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