Appendix A

Sparticle production cross sections

In this appendix, we list all $2 \to 2$ sparticle production subprocess cross sections, first for hadron colliders, and then for e^+e^- colliders.

A.1 Sparticle production at hadron colliders

A.1.1 Chargino and neutralino production

The production subprocess cross section for $d\bar{u} \to \widetilde{W}_i \widetilde{Z}_j$ has already been presented in Chapter 12, so will not be repeated here.

For $d\bar{d} \to \widetilde{W}_1 \overline{\widetilde{W}}_1$, we find (here, $\widetilde{W}_1 \equiv \widetilde{W}_1^-$ and $\overline{\widetilde{W}}_1 \equiv \widetilde{W}_1^+$)

$$\frac{d\sigma}{dz}(d\bar{d} \to \widetilde{W}_1 \overline{\widetilde{W}}_1) = \frac{p_{\widetilde{W}_1}}{192\pi \hat{s}^{3/2}} \left[M_\gamma + M_Z + M_{\tilde{u}} + M_{\gamma Z} + M_{\gamma \tilde{u}} + M_{Z\tilde{u}} \right], \tag{A.1}$$

where

$$\begin{aligned}
M_\gamma &= \frac{16e^4 Q_d^2}{\hat{s}} \left[E^2(1+z^2) + m_{\widetilde{W}_1}^2 (1-z^2) \right], \\
M_Z &= 16e^4 \cot^2\theta_W \hat{s} |D_Z(\hat{s})|^2 \Big\{ (x_c^2 + y_c^2)(\alpha_d^2 + \beta_d^2)[E^2(1+z^2) + m_{\widetilde{W}_1}^2(1-z^2)] \\
&\quad - 2y_c^2(\alpha_d^2 + \beta_d^2) m_{\widetilde{W}_1}^2 - 8x_c y_c \alpha_d \beta_d E p z \Big\}, \\
M_{\tilde{u}} &= \frac{e^4 \sin^4\gamma_R \hat{s}}{\sin^4\theta_W [E^2 + p^2 - 2Epz + m_{\tilde{u}_L}^2]^2} (E - pz)^2, \\
M_{\gamma Z} &= 32e^4 Q_d \cot\theta_W (\hat{s} - M_Z^2) |D_Z(\hat{s})|^2 \\
&\quad \times \Big\{ \alpha_d x_c [E^2(1+z^2) + m_{\widetilde{W}_1}^2(1-z^2)] - 2\beta_d y_c E p z \Big\}, \\
M_{\gamma \tilde{u}} &= \frac{4e^4 Q_d \sin^2\gamma_R}{\sin^2\theta_W} \frac{[(E-pz)^2 + m_{\widetilde{W}_1}^2]}{[E^2 + p^2 - 2Epz + m_{\tilde{u}_L}^2]}, \quad \text{and}
\end{aligned}$$

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$$M_{Z\tilde{u}} = \frac{4e^4 \cot\theta_W \sin^2\gamma_R}{\sin^2\theta_W}(\hat{s} - M_Z^2)(\alpha_d - \beta_d)\hat{s}|D_Z(\hat{s})|^2 \times \left\{ \frac{(x_c - y_c)[(E - pz)^2 + m_{\widetilde{W}_1}^2] + 2y_c m_{\widetilde{W}_1}^2}{E^2 + p^2 - 2Epz + m_{\tilde{u}_L}^2} \right\}.$$

The corresponding expression for $u\bar{u} \to \widetilde{W}_1\overline{\widetilde{W}}_1$ is obtained from the one above by making several changes. Of course, the contribution from $\tilde{u}$ exchange is replaced by that from $\tilde{d}$ exchange. To obtain the chargino production cross section from $u\bar{u}$ collisions, replace, (i) $\alpha_d \to \alpha_u$, $\beta_d \to \beta_u$, $Q_d \to Q_u$, $\tilde{u}_L \to \tilde{d}_L$, and $\gamma_R \to \gamma_L$ everywhere, (ii) $z \to -z$ in just $M_{\tilde{d}}$, $M_{\gamma\tilde{d}}$, and $M_{Z\tilde{d}}$, (iii) change the sign of $M_{\gamma\tilde{d}}$ and $M_{Z\tilde{d}}$, and finally, (iv) change $y_c \to -y_c$, in just $M_{Z\tilde{d}}$. The corresponding cross sections for $\widetilde{W}_2\overline{\widetilde{W}}_2$ production can be obtained from those for $\widetilde{W}_1\overline{\widetilde{W}}_1$ production by replacing $m_{\widetilde{W}_1} \to m_{\widetilde{W}_2}$, $\sin\gamma_R \to \cos\gamma_R$, $\sin\gamma_L \to \cos\gamma_L$, and $(x_c, y_c) \to (x_s, y_s)$.

The cross section for $d\bar{d} \to \widetilde{W}_1\overline{\widetilde{W}}_2$ is given by

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}z}(d\bar{d} \to \widetilde{W}_1\overline{\widetilde{W}}_2) = \frac{e^4 p}{192\pi\hat{s}^{1/2}}(M_Z + M_{\tilde{u}} + M_{Z\tilde{u}}), \tag{A.2}$$

where

$$M_Z = 4(\cot\theta_W + \tan\theta_W)^2|D_Z(\hat{s})|^2 \times \big[(x^2 + y^2)(\alpha_d^2 + \beta_d^2)(E^2 + p^2z^2 - \Delta^2 - \xi m_{\widetilde{W}_1}m_{\widetilde{W}_2}) + 2x^2\xi(\alpha_d^2 + \beta_d^2)m_{\widetilde{W}_1}m_{\widetilde{W}_2} - 8xy\alpha_d\beta_d Epz\big],$$

$$M_{\tilde{u}} = \frac{\sin^2\gamma_R\cos^2\gamma_R}{\sin^4\theta_W}\frac{[(E - pz)^2 - \Delta^2]}{[2E(E - \Delta) - 2Epz + m_{\tilde{u}_L}^2 - m_{\widetilde{W}_1}^2]^2},$$

$$M_{Z\tilde{u}} = -\frac{2\theta_y(\cot\theta_W + \tan\theta_W)\sin\gamma_R\cos\gamma_R(\hat{s} - M_Z^2)(\alpha_d - \beta_d)|D_Z(\hat{s})|^2}{\sin^2\theta_W} \times \frac{(x - y)[(E - pz)^2 - \Delta^2 - \xi m_{\widetilde{W}_1}m_{\widetilde{W}_2}] + 2x\xi m_{\widetilde{W}_1}m_{\widetilde{W}_2}}{[2E(E - \Delta) - 2Epz + m_{\tilde{u}_L}^2 - m_{\widetilde{W}_1}^2]},$$

where $\Delta = \frac{m_{\widetilde{W}_2}^2 - m_{\widetilde{W}_1}^2}{4E}$ and $\xi = (-1)^{\theta_{\widetilde{W}_1} + \theta_{\widetilde{W}_2} + 1}$. The corresponding expression for $u\bar{u} \to \widetilde{W}_1\overline{\widetilde{W}}_2$ is obtained from the one above by replacing, (i) $\alpha_d \to \alpha_u$, $\beta_d \to \beta_u$, $\tilde{u}_L \to \tilde{d}_L$, and $\gamma_R \to \gamma_L$, everywhere, (ii) $z \to -z$ in $M_{\tilde{d}}$ and $M_{Z\tilde{d}}$, and finally, (iii) just in $M_{Z\tilde{d}}$, $\theta_y \to \theta_x$, $\xi \to -\xi$, $(x - y) \to (x + y)$ (in the first term) and $x \to y$ (in the second term containing $m_{\widetilde{W}_1}m_{\widetilde{W}_2}$).

The cross section for neutralino pair production is given by,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z}(q\bar{q} \to \widetilde{Z}_i\widetilde{Z}_j) = \frac{p}{48\pi\hat{s}^{3/2}}\Big\{|A_{\widetilde{Z}_i}^q|^2|A_{\widetilde{Z}_j}^q|^2 G_t(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_L}, z)$$

$$+|B^q_{\widetilde{Z}_i}|^2|B^q_{\widetilde{Z}_j}|^2 G_t(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_R}, z) + 4e^2|W_{ij}|^2(\alpha_q^2+\beta_q^2)|D_Z(\hat{s})|^2$$
$$\times\left[\hat{s}^2-(m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})^2+4(-1)^{\theta_i+\theta_j+1}\hat{s}m_{\widetilde{Z}_i}m_{\widetilde{Z}_j}+4\hat{s}p^2z^2\right]$$
$$-\frac{1}{2}e(\alpha_q-\beta_q)(\hat{s}-M_Z^2)|D_Z(\hat{s})|^2\left[\mathrm{Re}(W_{ij}A^{q*}_{\widetilde{Z}_i}A^q_{\widetilde{Z}_j})G_{st}(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_L}, z)\right.$$
$$\left.+(-1)^{\theta_i+\theta_j}\mathrm{Re}(W_{ij}A^q_{\widetilde{Z}_i}A^{q*}_{\widetilde{Z}_j})G_{st}(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_L}, -z)\right]-\frac{1}{2}e(-1)^{\theta_i+\theta_j+1}$$
$$\times(\alpha_q+\beta_q)(\hat{s}-M_Z^2)|D_Z(\hat{s})|^2\left[\mathrm{Re}(W_{ij}B^{q*}_{\widetilde{Z}_i}B^q_{\widetilde{Z}_j})G_{st}(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_R}, z)\right.$$
$$\left.\left.+(-1)^{\theta_i+\theta_j}\mathrm{Re}(W_{ij}B^q_{\widetilde{Z}_i}B^{q*}_{\widetilde{Z}_j})G_{st}(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}_R}, -z)\right]\right\}, \qquad \text{(A.3)}$$

where

$$p=\frac{\lambda^{1/2}(\hat{s}, m^2_{\widetilde{Z}_i}, m^2_{\widetilde{Z}_j})}{2\sqrt{\hat{s}}}, \qquad \text{(A.4a)}$$

$$G_t(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}}, z)$$
$$=\frac{1}{16}\left\{\left[\frac{\hat{s}^2-(m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})^2-4p\hat{s}^{3/2}z+4p^2\hat{s}z^2}{[\frac{1}{2}(\hat{s}-m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})-\sqrt{\hat{s}}pz+m^2_{\tilde{q}}]^2}\right.\right.$$
$$\left.+\frac{\hat{s}^2-(m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})^2+4p\hat{s}^{3/2}z+4p^2\hat{s}z^2}{[\frac{1}{2}(\hat{s}-m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})+\sqrt{\hat{s}}pz+m^2_{\tilde{q}}]^2}\right]$$
$$\left.-\frac{8(-1)^{\theta_i+\theta_j}m_{\widetilde{Z}_i}m_{\widetilde{Z}_j}\hat{s}}{[\frac{1}{2}(\hat{s}-m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})+\sqrt{\hat{s}}pz+m^2_{\tilde{q}}][\frac{1}{2}(\hat{s}-m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})-\sqrt{\hat{s}}pz+m^2_{\tilde{q}}]}\right\}, \qquad \text{(A.4b)}$$

and

$$G_{st}(m_{\widetilde{Z}_i}, m_{\widetilde{Z}_j}, m_{\tilde{q}}, z)$$
$$=\frac{\hat{s}^2-(m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})^2-4\hat{s}^{3/2}pz+4\hat{s}p^2z^2+4(-1)^{\theta_i+\theta_j+1}\hat{s}m_{\widetilde{Z}_i}m_{\widetilde{Z}_j}}{\frac{1}{2}(\hat{s}-m^2_{\widetilde{Z}_i}-m^2_{\widetilde{Z}_j})-\sqrt{\hat{s}}pz+m^2_{\tilde{q}}}. \qquad \text{(A.4c)}$$

A.1.2 Gluino and squark production

Next, we turn to production of strongly interacting sparticles. Gluino pair production takes place via either gluon–gluon annihilation, or via quark–antiquark annihilation. The subprocess cross sections are usually presented as differential distributions in

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the Mandelstam variable $\hat{t}$:

$$\frac{d\sigma}{d\hat{t}}(gg \to \tilde{g}\tilde{g}) = \frac{9\pi\alpha_s^2}{4\hat{s}^2}\left\{\frac{2(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})}{\hat{s}^2} + \frac{(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})-2m_{\tilde{g}}^2(m_{\tilde{g}}^2+\hat{t})}{(m_{\tilde{g}}^2-\hat{t})^2} + \frac{(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})-2m_{\tilde{g}}^2(m_{\tilde{g}}^2+\hat{u})}{(m_{\tilde{g}}^2-\hat{u})^2} + \frac{m_{\tilde{g}}^2(\hat{s}-4m_{\tilde{g}}^2)}{(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})} - \frac{(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})+m_{\tilde{g}}^2(\hat{u}-\hat{t})}{\hat{s}(m_{\tilde{g}}^2-\hat{t})} - \frac{(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{g}}^2-\hat{u})+m_{\tilde{g}}^2(\hat{t}-\hat{u})}{\hat{s}(m_{\tilde{g}}^2-\hat{u})}\right\}, \tag{A.5a}$$

and

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{g}\tilde{g}) = \frac{8\pi\alpha_s^2}{9\hat{s}^2}\left\{\frac{4}{3}\left(\frac{m_{\tilde{g}}^2-\hat{t}}{m_{\tilde{q}}^2-\hat{t}}\right)^2 + \frac{4}{3}\left(\frac{m_{\tilde{g}}^2-\hat{u}}{m_{\tilde{q}}^2-\hat{u}}\right)^2 + \frac{3}{\hat{s}^2}\left[(m_{\tilde{g}}^2-\hat{t})^2+(m_{\tilde{g}}^2-\hat{u})^2+2m_{\tilde{g}}^2\hat{s}\right] - 3\frac{\left[(m_{\tilde{g}}^2-\hat{t})^2+m_{\tilde{g}}^2\hat{s}\right]}{\hat{s}(m_{\tilde{q}}^2-\hat{t})} - 3\frac{\left[(m_{\tilde{g}}^2-\hat{u})^2+m_{\tilde{g}}^2\hat{s}\right]}{\hat{s}(m_{\tilde{q}}^2-\hat{u})} + \frac{1}{3}\frac{m_{\tilde{g}}^2\hat{s}}{(m_{\tilde{q}}^2-\hat{t})(m_{\tilde{q}}^2-\hat{u})}\right\}. \tag{A.5b}$$

Gluinos can also be produced in association with squarks. The subprocess cross section is independent of whether the squark is the right- or the left-type:

$$\frac{d\sigma}{d\hat{t}}(gq \to \tilde{g}\tilde{q}) = \frac{\pi\alpha_s^2}{24\hat{s}^2}\frac{\left[\frac{16}{3}(\hat{s}^2+(m_{\tilde{q}}^2-\hat{u})^2)+\frac{4}{3}\hat{s}(m_{\tilde{q}}^2-\hat{u})\right]}{\hat{s}(m_{\tilde{g}}^2-\hat{t})(m_{\tilde{q}}^2-\hat{u})^2} \times\left((m_{\tilde{g}}^2-\hat{u})^2+(m_{\tilde{q}}^2-m_{\tilde{g}}^2)^2+\frac{2\hat{s}m_{\tilde{g}}^2(m_{\tilde{q}}^2-m_{\tilde{g}}^2)}{(m_{\tilde{g}}^2-\hat{t})}\right). \tag{A.6}$$

There are many different subprocesses for production of squark pairs. Since left- and right-squarks generally have different masses and different decay patterns, we present the differential cross section for each subprocess separately. (In early literature, these subprocesses were usually summed over squark types.) The results

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are:

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(gg \to \tilde{q}_i\bar{\tilde{q}}_i) = \frac{\pi\alpha_s^2}{4\hat{s}^2}&\left\{\frac{1}{3}\left(\frac{m_{\tilde{q}}^2+\hat{t}}{m_{\tilde{q}}^2-\hat{t}}\right)^2+\frac{1}{3}\left(\frac{m_{\tilde{q}}^2+\hat{u}}{m_{\tilde{q}}^2-\hat{u}}\right)^2\right.\\
&+\frac{3}{32\hat{s}^2}\left(8\hat{s}(4m_{\tilde{q}}^2-\hat{s})+4(\hat{u}-\hat{t})^2\right)+\frac{7}{12}\\
&-\frac{1}{48}\frac{(4m_{\tilde{q}}^2-\hat{s})^2}{(m_{\tilde{q}}^2-\hat{t})(m_{\tilde{q}}^2-\hat{u})}\\
&+\frac{3}{32}\frac{\left[(\hat{t}-\hat{u})(4m_{\tilde{q}}^2+4\hat{t}-\hat{s})-2(m_{\tilde{q}}^2-\hat{u})(6m_{\tilde{q}}^2+2\hat{t}-\hat{s})\right]}{\hat{s}(m_{\tilde{q}}^2-\hat{t})}\\
&+\frac{3}{32}\frac{\left[(\hat{u}-\hat{t})(4m_{\tilde{q}}^2+4\hat{u}-\hat{s})-2(m_{\tilde{q}}^2-\hat{t})(6m_{\tilde{q}}^2+2\hat{u}-\hat{s})\right]}{\hat{s}(m_{\tilde{q}}^2-\hat{u})}\\
&\left.+\frac{7}{96}\frac{\left[4m_{\tilde{q}}^2+4\hat{t}-\hat{s}\right]}{m_{\tilde{q}}^2-\hat{t}}+\frac{7}{96}\frac{\left[4m_{\tilde{q}}^2+4\hat{u}-\hat{s}\right]}{m_{\tilde{q}}^2-\hat{u}}\right\}, \qquad \text{(A.7a)}\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(q_1\bar{q}_2 \to \tilde{q}_{1L}\bar{\tilde{q}}_{2R}) &= \frac{d\sigma}{dt}(q_1\bar{q}_2 \to \tilde{q}_{1R}\bar{\tilde{q}}_{2L})\\
&= \frac{2\pi\alpha_s^2}{9\hat{s}^2}\frac{m_{\tilde{g}}^2\hat{s}}{(\hat{t}-m_{\tilde{g}}^2)^2}, \qquad \text{(A.7b)}\end{aligned}$$

$$\frac{d\sigma}{d\hat{t}}(q_1\bar{q}_2 \to \tilde{q}_{1L(R)}\bar{\tilde{q}}_{2L(R)}) = \frac{2\pi\alpha_s^2}{9\hat{s}^2}\frac{-\hat{s}\hat{t}-(\hat{t}-m_{\tilde{q}_{L(R)}}^2)^2}{(\hat{t}-m_{\tilde{g}}^2)^2}, \qquad \text{(A.7c)}$$

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(q_1q_2 \to \tilde{q}_{1L}\tilde{q}_{2R}) &= \frac{d\sigma}{d\hat{t}}(q_1q_2 \to \tilde{q}_{1R}\tilde{q}_{2L})\\
&= \frac{2\pi\alpha_s^2}{9\hat{s}^2}\frac{-\hat{s}\hat{t}-(\hat{t}-m_{\tilde{q}_L}^2)(\hat{t}-m_{\tilde{q}_R}^2)}{(\hat{t}-m_{\tilde{g}}^2)^2}, \qquad \text{(A.7d)}\end{aligned}$$

$$\frac{d\sigma}{d\hat{t}}(q_1q_2 \to \tilde{q}_{1L(R)}\tilde{q}_{2L(R)}) = \frac{2\pi\alpha_s^2}{9\hat{s}^2}\frac{m_{\tilde{g}}^2\hat{s}}{(\hat{t}-m_{\tilde{g}}^2)^2}, \qquad \text{(A.7e)}$$

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{q}_L\bar{\tilde{q}}_R) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{q}_R\bar{\tilde{q}}_L) = \frac{2\pi\alpha_s^2}{9\hat{s}^2}\frac{m_{\tilde{g}}^2\hat{s}}{(\hat{t}-m_{\tilde{g}}^2)^2}, \qquad \text{(A.7f)}$$

$$\begin{aligned}\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{q}_{L(R)}\bar{\tilde{q}}_{L(R)}) &= \frac{2\pi\alpha_s^2}{9\hat{s}^2}\left\{\frac{1}{(\hat{t}-m_{\tilde{g}}^2)^2}+\frac{2}{\hat{s}^2}-\frac{2/3}{\hat{s}(\hat{t}-m_{\tilde{g}}^2)}\right\}\\
&\times\left[-\hat{s}\hat{t}-(\hat{t}-m_{\tilde{q}_{L(R)}}^2)^2\right], \qquad \text{(A.7g)}\end{aligned}$$

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$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{q}'_{L(R)}\bar{\tilde{q}}'_{L(R)}) = \frac{4\pi\alpha_s^2}{9\hat{s}^4}\left[-\hat{s}\hat{t} - (\hat{t} - m^2_{\tilde{q}'_{L(R)}})^2\right], \tag{A.7h}$$

$$\frac{d\sigma}{d\hat{t}}(qq \to \tilde{q}_L\tilde{q}_L) = \frac{d\sigma}{d\hat{t}}(qq \to \tilde{q}_R\tilde{q}_R)$$
$$= \frac{\pi\alpha_s^2}{9\hat{s}^2}m^2_{\tilde{g}}\hat{s}\left\{\frac{1}{(\hat{t} - m^2_{\tilde{g}})^2} + \frac{1}{(\hat{u} - m^2_{\tilde{g}})^2} - \frac{2/3}{(\hat{t} - m^2_{\tilde{g}})(\hat{u} - m^2_{\tilde{g}})}\right\}, \tag{A.7i}$$

and

$$\frac{d\sigma}{d\hat{t}}(qq \to \tilde{q}_L\tilde{q}_R)$$
$$= \frac{2\pi\alpha_s^2}{9\hat{s}^2}\left\{\frac{[-\hat{s}\hat{t} - (\hat{t} - m^2_{\tilde{q}_L})(\hat{t} - m^2_{\tilde{q}_R})]}{(\hat{t} - m^2_{\tilde{g}})^2} + \frac{[-\hat{s}\hat{u} - (\hat{u} - m^2_{\tilde{q}_L})(\hat{u} - m^2_{\tilde{q}_R})]}{(\hat{u} - m^2_{\tilde{g}})^2}\right\}. \tag{A.7j}$$

Because there are essentially no third generation quarks in the proton, these cross sections for gluino and squark production are fixed by SUSY QCD, and depend only on the masses of the squarks and gluinos: in particular, cross sections for producing third generation squarks do not depend on the intra-generational mixing. In this case, the squark type should be read as 1 or 2 instead of L or R. Refer back to the exercise in Section 8.4.1 in this connection.

A.1.3 Gluino and squark associated production

Gluinos and squarks may also be produced in association with charginos and neutralinos. Here, the subprocess cross sections are given by:

$$\frac{d\sigma}{d\hat{t}}(\bar{u}g \to \widetilde{W}_i\bar{\tilde{d}}_L) = \frac{\alpha_s}{24\hat{s}^2}|A^u_{\widetilde{W}_i}|^2\psi(m_{\tilde{d}_L}, m_{\widetilde{W}_i}, \hat{t}), \tag{A.8}$$

$$\frac{d\sigma}{d\hat{t}}(dg \to \widetilde{W}_i\tilde{u}_L) = \frac{\alpha_s}{24\hat{s}^2}|A^d_{\widetilde{W}_i}|^2\psi(m_{\tilde{u}_L}, m_{\widetilde{W}_i}, \hat{t}), \tag{A.9}$$

$$\frac{d\sigma}{d\hat{t}}(qg \to \widetilde{Z}_i\tilde{q}) = \frac{\alpha_s}{24\hat{s}^2}\left(|A^q_{\widetilde{Z}_i}|^2 + |B^q_{\widetilde{Z}_i}|^2\right)\psi(m_{\tilde{q}}, m_{\widetilde{Z}_i}, \hat{t}), \tag{A.10}$$

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \widetilde{Z}_i\tilde{g}) = \frac{\alpha_s}{18\hat{s}^2}\left(|A^q_{\widetilde{Z}_i}|^2 + |B^q_{\widetilde{Z}_i}|^2\right)\left[\frac{(m^2_{\widetilde{Z}_i} - \hat{t})(m^2_{\tilde{g}} - \hat{t})}{(m^2_{\tilde{q}} - \hat{t})^2}\right.$$
$$\left. + \frac{(m^2_{\widetilde{Z}_i} - \hat{u})(m^2_{\tilde{g}} - \hat{u})}{(m^2_{\tilde{q}} - \hat{u})^2} - \frac{2(-1)^{\theta_i+\theta_{\tilde{g}}}m_{\tilde{g}}m_{\widetilde{Z}_i}\hat{s}}{(m^2_{\tilde{q}} - \hat{t})(m^2_{\tilde{q}} - \hat{u})}\right], \tag{A.11}$$

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and

$$\frac{d\sigma}{d\hat{t}}(\bar{u}d \to \widetilde{W}_i\tilde{g}) = \frac{\alpha_s}{18\hat{s}^2}\left[|A^u_{\widetilde{W}_i}|^2\frac{(m^2_{\widetilde{W}_i}-\hat{t})(m^2_{\tilde{g}}-\hat{t})}{(m^2_{\tilde{d}_L}-\hat{t})^2}\right.$$
$$+|A^d_{\widetilde{W}_i}|^2\frac{(m^2_{\widetilde{W}_i}-\hat{u})(m^2_{\tilde{g}}-\hat{u})}{(m^2_{\tilde{u}_L}-\hat{u})^2}$$
$$\left.+\frac{2(-1)^{\theta_{\tilde{g}}}\mathrm{Re}(A^u_{\widetilde{W}_i}A^d_{\widetilde{W}_i})m_{\tilde{g}}m_{\widetilde{W}_i}\hat{s}}{(m^2_{\tilde{d}_L}-\hat{t})(m^2_{\tilde{u}_L}-\hat{u})}\right], \tag{A.12}$$

where

$$\psi(m_1, m_2, t) = \frac{s+t-m_1^2}{2s} - \frac{m_1^2(m_2^2-t)}{(m_1^2-t)^2} + \frac{t(m_2^2-m_1^2)+m_2^2(s-m_2^2+m_1^2)}{s(m_1^2-t)}. \tag{A.13}$$

A.1.4 Slepton and sneutrino production

The subprocess cross section for $\tilde{\ell}_L\bar{\tilde{\nu}}_L$ production is given by

$$\frac{d\sigma}{d\hat{t}}(d\bar{u} \to \tilde{\ell}_L\bar{\tilde{\nu}}_L) = \frac{g^4|D_W(\hat{s})|^2}{192\pi\hat{s}^2}\left(\hat{t}\hat{u} - m^2_{\tilde{\ell}_L}m^2_{\tilde{\nu}_L}\right). \tag{A.14}$$

The production of $\tilde{\tau}_1\bar{\tilde{\nu}}_\tau$ is given as above, replacing $m_{\tilde{\ell}_L} \to m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_L} \to m_{\tilde{\nu}_\tau}$, and multiplying by an overall factor of $\cos^2\theta_\tau$. Similar substitutions hold for $\tilde{\tau}_2\bar{\tilde{\nu}}_\tau$ production, except the overall factor is $\sin^2\theta_\tau$.

The subprocess cross section for $\tilde{\ell}_L\bar{\tilde{\ell}}_L$ production is given by

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{\ell}_L\bar{\tilde{\ell}}_L)$$
$$= \frac{e^4}{24\pi\hat{s}^2}\left(\hat{t}\hat{u} - m^4_{\tilde{\ell}_L}\right)\left\{\frac{q_\ell^2q_q^2}{\hat{s}^2} + (\alpha_\ell-\beta_\ell)^2(\alpha_q^2+\beta_q^2)|D_Z(\hat{s})|^2\right.$$
$$\left.+\frac{2q_\ell q_q\alpha_q(\alpha_\ell-\beta_\ell)(\hat{s}-M_Z^2)}{\hat{s}}|D_Z(\hat{s})|^2\right\}. \tag{A.15a}$$

The cross section for sneutrino production is given by the same formula, but with α_ℓ, β_ℓ, q_ℓ, and $m_{\tilde{\ell}_L}$ replaced by α_ν, β_ν, 0, and $m_{\tilde{\nu}_L}$, respectively. The cross section for $\tilde{\tau}_1\bar{\tilde{\tau}}_1$ production is obtained by replacing $m_{\tilde{\ell}_L} \to m_{\tilde{\tau}_1}$ and $\beta_\ell \to \beta_\ell\cos 2\theta_\tau$. The cross section for $\tilde{\ell}_R\bar{\tilde{\ell}}_R$ production is given by substituting $\alpha_\ell - \beta_\ell \to \alpha_\ell + \beta_\ell$ and $m_{\tilde{\ell}_L} \to m_{\tilde{\ell}_R}$ in (A.15a). The cross section for $\tilde{\tau}_2\bar{\tilde{\tau}}_2$ production is obtained from the formula for $\tilde{\ell}_R\bar{\tilde{\ell}}_R$ production by replacing $m_{\tilde{\ell}_R} \to m_{\tilde{\tau}_2}$ and $\beta_\ell \to \beta_\ell\cos 2\theta_\tau$. Finally,

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the cross section for $\tilde{\tau}_1\bar{\tilde{\tau}}_2$ production is given by

$$\frac{d\sigma}{d\hat{t}}(q\bar{q} \to \tilde{\tau}_1\bar{\tilde{\tau}}_2) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \bar{\tilde{\tau}}_1\tilde{\tau}_2)$$
$$= \frac{e^4}{24\pi\hat{s}^2}(\alpha_q^2 + \beta_q^2)\beta_\ell^2 \sin^2 2\theta_\tau |D_Z(\hat{s})|^2(\hat{u}\hat{t} - m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2). \quad \text{(A.15b)}$$

A.2 Sparticle production at e^+e^- colliders

The degree of longitudinal beam polarization has been parametrized as

$$P_L(e^-) = f_L - f_R, \quad \text{where} \quad \text{(A.16a)}$$
$$f_L = \frac{n_L}{n_L + n_R} = \frac{1 + P_L}{2} \quad \text{and} \quad \text{(A.16b)}$$
$$f_R = \frac{n_R}{n_L + n_R} = \frac{1 - P_L}{2}. \quad \text{(A.16c)}$$

Here, $n_{L,R}$ are the number of left-(right-)polarized electrons in the beam, and $f_{L,R}$ is the corresponding fraction. Thus, a 90% right-polarized beam would correspond to $P_L(e^-) = -0.8$, and a completely unpolarized beam corresponds to $P_L(e^-) = 0$.

We present in this section relevant $2 \to 2$ SM and SUSY cross sections, retaining information on the polarization of the incoming beams. The calculations can be performed using usual techniques, but in addition inserting projection operators $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$ to select the desired left- or right-polarized initial state particles.[1]

We begin by listing lowest order Standard Model cross sections for right- or left-polarized incoming electrons and positrons. For SM fermion pair production, we have:

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to f\bar{f}) = \frac{N_f}{4\pi}\frac{p}{E}\Phi_{fR/L}(z) \quad \text{(A.17a)}$$

where $z = \cos\theta$ (θ is the angle between the final state particle and the electron), E is the beam energy, and p is the magnitude of the momentum of the final state SM fermions, $f = \mu,\ \tau,\ \nu_\mu,\ \nu_\tau$, and q, and:

$$\Phi_{fR/L}(z) = e^4\left[\frac{q_f^2}{s^2}\left(E^2(1+z^2) + m_f^2(1-z^2)\right) + \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right.$$
$$\left.\times\left(\left[(\alpha_f^2 + \beta_f^2)(E^2 + p^2z^2) \pm 4\alpha_f\beta_f Epz + (\alpha_f^2 - \beta_f^2)m_f^2\right]\right)\right.$$

[1] We assume that we are at high energy $E \gg m_e$ so that the difference between chirality and helicity can be safely ignored.

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$$- \frac{2(\alpha_e \pm \beta_e)(s - M_Z^2)q_f}{s[(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2]} \left(\alpha_f \left[E^2(1+z^2) + m_f^2(1-z^2)\right] \right.$$
$$\left. \left. \pm 2\beta_f E p z\right) \right]. \qquad \text{(A.17b)}$$

The upper (lower) signs are for $\Phi_{f\mathrm{R}}$ ($\Phi_{f\mathrm{L}}$). For electron pair production t-channel photon exchange contributions must be included. For Z pair production, we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z}(e_{\mathrm{R/L}}\bar{e}_{\mathrm{L/R}} \to ZZ)$$
$$= \frac{\mathrm{e}^4(\alpha_e \pm \beta_e)^4 p}{4\pi s\sqrt{s}} \left[\frac{u(z)}{t(z)} + \frac{t(z)}{u(z)} + \frac{4M_Z^2 s}{u(z)t(z)} - M_Z^4\left(\frac{1}{t^2(z)} + \frac{1}{u^2(z)}\right)\right], \qquad \text{(A.18)}$$

where s, $t(z)$, and $u(z)$ are the Mandelstam variables. For W^+W^- production we have,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z}(e_{\mathrm{R/L}}\bar{e}_{\mathrm{L/R}} \to W^+W^-) = \frac{\mathrm{e}^4 p}{16\pi s\sqrt{s}\sin^4\theta_{\mathrm{W}}}\Phi_{WW\mathrm{R/L}}(z), \qquad \text{(A.19a)}$$

where

$$\Phi_{WW\mathrm{R}}(z) = \frac{4(\alpha_e + \beta_e)^2\tan^2\theta_{\mathrm{W}}|D_Z|^2}{s^2}\left[U_T(z)(p^2 s + 3M_W^4) + 4M_W^2 p^2 s^2\right], \qquad \text{(A.19b)}$$

and

$$\Phi_{WW\mathrm{L}}(z) = \frac{U_T(z)}{s^2}\left[3 + 2(\alpha_e - \beta_e)\tan\theta_{\mathrm{W}}(s - 6M_W^2)\mathrm{Re}(D_Z)\right.$$
$$\left. + 4(\alpha_e - \beta_e)^2\tan^2\theta_{\mathrm{W}}(p^2 s + 3M_W^4)|D_Z|^2\right] + \frac{U_T(z)}{t^2(z)}$$
$$+ 8(\alpha_e - \beta_e)\tan\theta_{\mathrm{W}} M_W^2 \mathrm{Re}(D_Z) + 16(\alpha_e - \beta_e)^2\tan^2\theta_{\mathrm{W}} M_W^2 p^2 |D_Z|^2$$
$$+ 2\left[1 - 2(\alpha_e - \beta_e)\tan\theta_{\mathrm{W}} M_W^2 \mathrm{Re}(D_Z)\right]\left[\frac{U_T(z)}{st(z)} - \frac{2M_W^2}{t(z)}\right], \qquad \text{(A.19c)}$$

with $U_T(z) = u(z)t(z) - M_W^4$, and $D_Z = (s - M_Z^2 + \mathrm{i}M_Z\Gamma_Z)^{-1}$.

The reader will have noticed that we have not listed cross sections from the $e_{\mathrm{L/R}}\bar{e}_{\mathrm{L/R}}$ initial states. This is because these cross sections vanish in the chiral limit, i.e. when the electron mass is neglected. The reason is that gauge interactions couple members of the electroweak doublets (singlets) to one another, but never a doublet to a singlet. The same reasoning, therefore, applies to SUSY processes that involve only s-channel photon and Z exchange. The reader should understand that these unlisted cross sections also vanish.

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For lowest order MSSM Higgs boson production, we have

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to Zh) = \frac{p}{16\pi\sqrt{s}} \frac{e^4 \sin^2(\alpha+\beta)}{\sin^2\theta_W \cos^2\theta_W} \times \frac{(\alpha_e \pm \beta_e)^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}(M_Z^2 + E_Z^2 - p^2 z^2). \quad \text{(A.20a)}$$

To obtain the corresponding cross section for ZH production, replace $\sin^2(\alpha+\beta)$ with $\cos^2(\alpha+\beta)$. The angular distribution for the production of h (or H) bosons in association with A is given by,

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to hA) = \frac{p^3}{16\pi\sqrt{s}} \frac{e^4 \cos^2(\alpha+\beta)}{\sin^2\theta_W \cos^2\theta_W} \frac{(\alpha_e \pm \beta_e)^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}(1-z^2); \quad \text{(A.20b)}$$

for HA production, replace $\cos^2(\alpha+\beta)$ with $\sin^2(\alpha+\beta)$. Finally, the differential cross section for H^+H^- production is given by,

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to H^+H^-) = \frac{e^4}{4\pi}\frac{p^3}{\sqrt{s}}(1-z^2) \times \left[\frac{1}{s^2} + \left(\frac{2\sin^2\theta_W - 1}{2\cos\theta_W \sin\theta_W}\right)^2 \frac{(\alpha_e \pm \beta_e)^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \frac{1}{s}\left(\frac{2\sin^2\theta_W - 1}{\cos\theta_W \sin\theta_W}\right)\frac{(\alpha_e \pm \beta_e)(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]. \quad \text{(A.20c)}$$

For sfermion pair production ($\tilde{f}_i\bar{\tilde{f}}_i$, with $f = \mu,\ \nu_\mu,\ \nu_\tau,\ u,\ d,\ c,\ s$, and $i = \text{L}$ or R), we find:

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to \tilde{f}_i\bar{\tilde{f}}_i) = \frac{N_f}{256\pi}\frac{p^3}{E^3}\Phi_{\tilde{f}_i R/L}(z), \quad \text{(A.21a)}$$

where

$$\Phi_{\tilde{f}_i R/L}(z) = e^4(1-z^2) \times \left[\frac{8q_f^2}{s} + \frac{2A_{f_i}^2(\alpha_e \pm \beta_e)^2 s - 8(\alpha_e \pm \beta_e)q_f A_{f_i}(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right], \quad \text{(A.21b)}$$

and $A_{f_{L,R}} = 2(\alpha_f \mp \beta_f)$. The cross sections for producing $\tilde{f}_L\tilde{f}_R$ pairs are zero since both the photon and the Z boson only couple to pairs of like type (LL or RR) sfermions. In the corresponding expressions for third generation sfermions, we need

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to include intragenerational mixing. For the case of $\tilde{t}_1\bar{\tilde{t}}_1$ production, we have $A_{t_1} = 2(\alpha_t - \beta_t)\cos^2\theta_t + 2(\alpha_t + \beta_t)\sin^2\theta_t$; for $\tilde{t}_2\bar{\tilde{t}}_2$ production, simply switch $\cos^2\theta_t$ with $\sin^2\theta_t$. Since Z couples to $\tilde{t}_1\tilde{t}_2$ pairs we also have,

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to \tilde{t}_1\bar{\tilde{t}}_2) = \frac{48\pi\alpha^2}{\sqrt{s}} \frac{(\alpha_e \pm \beta_e)^2\beta_t^2\cos^2\theta_t\sin^2\theta_t}{[(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2]} p^3(1 - z^2). \tag{A.21c}$$

The differential cross sections for stau and sbottom pair production are given by analogous formulae.

For selectron pair production, we find

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to \tilde{e}_L\bar{\tilde{e}}_L) = \frac{1}{256\pi}\frac{p^3}{E^3}\Phi_{\tilde{e}_L R/L}(z), \tag{A.22a}$$

where

$$\Phi_{\tilde{e}_L R}(z) = \Phi_{\tilde{\mu}_L R}(z), \tag{A.22b}$$

and

$$\begin{aligned}\Phi_{\tilde{e}_L L}(z) = \Phi_{\tilde{\mu}_L L}(z) &+ \sum_{i=1}^{4} \frac{2|A^e_{\tilde{Z}_i}|^4 s(1 - z^2)}{[2E(E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_i}]^2} \\ &- 8e^2(1 - z^2)\sum_{i=1}^{4}\frac{|A^e_{\tilde{Z}_i}|^2}{[2E(E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_i}]} \\ &\times \left[1 + \frac{(\alpha_e - \beta_e)^2 s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right] \\ &+ \sum_{i<j=1}^{4} \frac{4|A^e_{\tilde{Z}_i}|^2|A^e_{\tilde{Z}_j}|^2 s(1 - z^2)}{[2E(E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_i}][2E(E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_j}]}.\end{aligned} \tag{A.22c}$$

Similarly,

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R} \to \tilde{e}_R\bar{\tilde{e}}_R) = \frac{1}{256\pi}\frac{p^3}{E^3}\Phi_{\tilde{e}_R R/L}(z), \tag{A.23a}$$

where

$$\Phi_{\tilde{e}_R L}(z) = \Phi_{\tilde{\mu}_R L}(z), \tag{A.23b}$$

and

$$\Phi_{\tilde{e}_R R}(z) = \Phi_{\tilde{e}_L L}(z) \quad \text{but with the substitutions,}$$

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$$A^e_{\tilde{Z}_i} \to B^e_{\tilde{Z}_i},\ m_{\tilde{e}_L} \to m_{\tilde{e}_R}, \quad \text{and} \quad (\alpha_e - \beta_e) \to (\alpha_e + \beta_e). \tag{A.23c}$$

For $\tilde{e}_L\tilde{e}_R$ production, we have,[2]

$$\frac{d\sigma}{dz}(e_R\bar{e}_L \to \tilde{e}_L\bar{\tilde{e}}_R) = \frac{d\sigma}{dz}(e_L\bar{e}_R \to \tilde{e}_R\bar{\tilde{e}}_L) = 0, \tag{A.24a}$$

while

$$\frac{d\sigma}{dz}(e_L\bar{e}_L \to \tilde{e}_L\bar{\tilde{e}}_R) = \frac{1}{32\pi s}\frac{p}{E}\left[\sum_{i=1}^{4}\frac{|A^e_{\tilde{Z}_i}|^2|B^e_{\tilde{Z}_i}|^2 m^2_{\tilde{Z}_i}}{[E_{\tilde{e}_L} - pz + a_{\tilde{Z}_i}]^2}\right.$$
$$\left. + \sum_{i<j=1}^{4}\frac{2m_{\tilde{Z}_i}m_{\tilde{Z}_j}\mathrm{Re}(A^e_{\tilde{Z}_i}A^{e*}_{\tilde{Z}_j}B^{e*}_{\tilde{Z}_i}B^e_{\tilde{Z}_j})}{[E_{\tilde{e}_L} - pz + a_{\tilde{Z}_i}][E_{\tilde{e}_L} - pz + a_{\tilde{Z}_j}]}\right], \tag{A.24b}$$

where $a_{\tilde{Z}_i} = \frac{m^2_{\tilde{Z}_i} - m^2_{\tilde{e}_L}}{2E}$, and

$$\frac{d\sigma}{dz}(e_R\bar{e}_R \to \tilde{e}_R\bar{\tilde{e}}_L) = \frac{1}{32\pi s}\frac{p}{E}\left[\sum_{i=1}^{4}\frac{|A^e_{\tilde{Z}_i}|^2|B^e_{\tilde{Z}_i}|^2 m^2_{\tilde{Z}_i}}{[E_{\tilde{e}_R} - pz + a_{\tilde{Z}_i}]^2}\right.$$
$$\left. + \sum_{i<j=1}^{4}\frac{2m_{\tilde{Z}_i}m_{\tilde{Z}_j}\mathrm{Re}(A^e_{\tilde{Z}_i}A^{e*}_{\tilde{Z}_j}B^{e*}_{\tilde{Z}_i}B^e_{\tilde{Z}_j})}{[E_{\tilde{e}_R} - pz + a_{\tilde{Z}_i}][E_{\tilde{e}_R} - pz + a_{\tilde{Z}_j}]}\right], \tag{A.24c}$$

where now $a_{\tilde{Z}_i} = \frac{m^2_{\tilde{Z}_i} - m^2_{\tilde{e}_R}}{2E}$.

For $\tilde{\nu}_e$ pair production, we find

$$\frac{d\sigma}{dz}(e_R\bar{e}_L \to \tilde{\nu}_e\bar{\tilde{\nu}}_e) = \frac{d\sigma}{dz}(e_R\bar{e}_L \to \tilde{\nu}_\mu\bar{\tilde{\nu}}_\mu), \tag{A.25a}$$

while

$$\frac{d\sigma}{dz}(e_L\bar{e}_R \to \tilde{\nu}_e\,\bar{\tilde{\nu}}_e)$$
$$= \frac{p^3E}{8\pi}(1-z^2)\left[\frac{4e^4(\alpha_\nu - \beta_\nu)^2(\alpha_e - \beta_e)^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right.$$
$$+ \frac{g^4\sin^4\gamma_R}{[2E(E - pz) + m^2_{\tilde{W}_1} - m^2_{\tilde{\nu}_e}]^2} + \frac{g^4\cos^4\gamma_R}{[2E(E - pz) + m^2_{\tilde{W}_2} - m^2_{\tilde{\nu}_e}]^2}$$
$$- \frac{4e^2g^2(\alpha_\nu - \beta_\nu)(\alpha_e - \beta_e)(s - M_Z^2)\sin^2\gamma_R}{[(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2][2E(E - pz) + m^2_{\tilde{W}_1} - m^2_{\tilde{\nu}_e}]}$$

[2] As long as we neglect electron Yukawa couplings, neutralinos couple doublets (singlets) to doublets (singlets). Thus $\tilde{e}_L\bar{\tilde{e}}_R$ production occurs only from the $e_{L/R}\bar{e}_{L/R}$ initial state via t-channel neutralino exchange, in sharp contrast to other SUSY processes that we have seen. We will leave it to the reader to work out in advance those polarizations of the initial electron and positron beams that contribute to chargino and neutralino pair production processes which, we recall, have both s- and t-channel contributions.

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$$-\frac{4e^2g^2(\alpha_\nu-\beta_\nu)(\alpha_e-\beta_e)(s-M_Z^2)\cos^2\gamma_R}{[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2][2E(E-pz)+m_{\widetilde{W}_2}^2-m_{\tilde{\nu}_e}^2]}$$
$$\left.+\frac{2g^4\sin^2\gamma_R\cos^2\gamma_R}{[2E(E-pz)+m_{\widetilde{W}_1}^2-m_{\tilde{\nu}_e}^2][2E(E-pz)+m_{\widetilde{W}_2}^2-m_{\tilde{\nu}_e}^2]}\right]. \quad \text{(A.25b)}$$

The differential cross sections for neutralino pair production are given by,

$$\frac{d\sigma}{dz}(e_{R/L}\bar{e}_{L/R}\to \tilde{Z}_i\tilde{Z}_j)=\frac{p}{8\pi s\sqrt{s}}\left(M_{\tilde{e}\tilde{e}R/L}+M_{ZZR/L}+M_{Z\tilde{e}R/L}\right), \quad \text{(A.26)}$$

with

$$M_{\tilde{e}\tilde{e}R}=2|B^e_{\tilde{Z}_i}|^2|B^e_{\tilde{Z}_j}|^2G_t(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_R},z),$$
$$M_{\tilde{e}\tilde{e}L}=2|A^e_{\tilde{Z}_i}|^2|A^e_{\tilde{Z}_j}|^2G_t(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_L},z),$$

$$M_{ZZR/L}=$$

$$\frac{4e^2|W_{ij}|^2(\alpha_e\pm\beta_e)^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[s^2-(m_{\tilde{Z}_i}^2-m_{\tilde{Z}_j}^2)^2-4(-1)^{\theta_i+\theta_j}sm_{\tilde{Z}_i}m_{\tilde{Z}_j}+4sp^2z^2\right],$$

$$M_{Z\tilde{e}R}=\frac{-e(-1)^{(\theta_i+\theta_j+1)}(\alpha_e+\beta_e)(s-M_Z^2)}{2[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}$$
$$\times\Big[\mathrm{Re}(W_{ij}B^{e*}_{\tilde{Z}_i}B^e_{\tilde{Z}_j})G_{st}(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_R},z)$$
$$+(-1)^{(\theta_i+\theta_j)}\mathrm{Re}(W_{ij}B^e_{\tilde{Z}_i}B^{e*}_{\tilde{Z}_j})G_{st}(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_R},-z)\Big],$$

and

$$M_{Z\tilde{e}L}=\frac{-e(\alpha_e-\beta_e)(s-M_Z^2)}{2[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}$$
$$\times\Big[\mathrm{Re}(W_{ij}A^{e*}_{\tilde{Z}_i}A^e_{\tilde{Z}_j})G_{st}(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_L},z)$$
$$+(-1)^{\theta_i+\theta_j}\mathrm{Re}(W_{ij}A^e_{\tilde{Z}_i}A^{e*}_{\tilde{Z}_j})G_{st}(m_{\tilde{Z}_i},m_{\tilde{Z}_j},m_{\tilde{e}_L},-z)\Big].$$

The functions G_t and G_{st} are defined in (A.4b) and (A.4c), respectively.

For chargino pair production we have,

$$\frac{d\sigma}{dz}(e_L\bar{e}_R\to \widetilde{W}_1\overline{\widetilde{W}}_1)=\frac{1}{64\pi s}\frac{p}{E}\big(M_{\gamma\gamma L}+M_{ZZL}+M_{\gamma ZL}$$
$$+M_{\tilde{\nu}\tilde{\nu}L}+M_{\gamma\tilde{\nu}L}+M_{Z\tilde{\nu}L}\big), \quad \text{(A.27a)}$$

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and

$$\frac{d\sigma}{dz}(e_R\bar{e}_L \to \widetilde{W}_1\overline{\widetilde{W}}_1) = \frac{1}{64\pi s}\frac{p}{E}\left(M_{\gamma\gamma R} + M_{ZZR} + M_{\gamma ZR}\right), \quad \text{(A.27b)}$$

with

$$M_{\gamma\gamma L} = M_{\gamma\gamma R} = \frac{16e^4}{s}\left[E^2(1+z^2) + m_{\widetilde{W}_1}^2(1-z^2)\right],$$

$$M_{ZZR/L} = \frac{16e^4\cot^2\theta_W s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\Big[(x_c^2+y_c^2)(\alpha_e \pm \beta_e)^2 \times$$

$$[E^2(1+z^2) + m_{\widetilde{W}_1}^2(1-z^2)] - 2y_c^2(\alpha_e \pm \beta_e)^2 m_{\widetilde{W}_1}^2 \mp 4x_c y_c(\alpha_e \pm \beta_e)^2 Epz\Big],$$

$$M_{\gamma ZR/L} =$$

$$\frac{-32e^4(\alpha_e \pm \beta_e)\cot\theta_W(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left\{x_c[E^2(1+z^2) + m_{\widetilde{W}_1}^2(1-z^2)] \mp 2y_c Epz\right\},$$

$$M_{\tilde{\nu}\tilde{\nu}L} = \frac{2e^4\sin^4\gamma_R}{\sin^4\theta_W}\frac{s(E-pz)^2}{[E^2+p^2-2Epz+m_{\tilde{\nu}}^2]^2},$$

$$M_{\gamma\tilde{\nu}L} = \frac{-8e^4\sin^2\gamma_R}{\sin^2\theta_W}\frac{[(E-pz)^2 + m_{\widetilde{W}_1}^2]}{[E^2+p^2-2Epz+m_{\tilde{\nu}}^2]},$$

and

$$M_{Z\tilde{\nu}L} = \frac{8e^4(\alpha_e-\beta_e)\cot\theta_W\sin^2\gamma_R}{\sin^2\theta_W}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} \times \left[\frac{(x_c-y_c)[(E-pz)^2+m_{\widetilde{W}_1}^2] + 2y_c m_{\widetilde{W}_1}^2}{E^2+p^2-2Epz+m_{\tilde{\nu}}^2}\right].$$

To obtain the differential cross section for $\widetilde{W}_2\overline{\widetilde{W}}_2$ production, we simply replace x_c with x_s, y_c with y_s, $\sin\gamma_R$ with $\cos\gamma_R$, and $m_{\widetilde{W}_1}$ with $m_{\widetilde{W}_2}$ in the corresponding expression for $\widetilde{W}_1\overline{\widetilde{W}}_1$ production. Finally for $\widetilde{W}_1\overline{\widetilde{W}}_2$ production we have,

$$\frac{d\sigma}{dz}(e_L\bar{e}_R \to \widetilde{W}_1\overline{\widetilde{W}}_2) = \frac{e^4}{64\pi}\frac{p}{E}\left[M_{ZZL} + M_{\tilde{\nu}\tilde{\nu}L} + M_{Z\tilde{\nu}L}\right], \quad \text{(A.27c)}$$

and

$$\frac{d\sigma}{dz}(e_R\bar{e}_L \to \widetilde{W}_1\overline{\widetilde{W}}_2) = \frac{e^4}{64\pi}\frac{p}{E}M_{ZZR}, \quad \text{(A.27d)}$$

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where

$$M_{ZZ\mathrm{R/L}} = \frac{4(\alpha_e \pm \beta_e)^2(\cot\theta_\mathrm{W} + \tan\theta_\mathrm{W})^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\Big[(x^2 + y^2)(E^2 + p^2z^2 \\ -\Delta^2 - \xi m_{\widetilde{W}_1} m_{\widetilde{W}_2}) + 2x^2\xi m_{\widetilde{W}_1} m_{\widetilde{W}_2} \mp 4xyEpz\Big]$$

$$M_{\tilde{\nu}\tilde{\nu}\mathrm{L}} = \frac{2\sin^2\gamma_\mathrm{R}\cos^2\gamma_\mathrm{R}}{\sin^4\theta_\mathrm{W}}\frac{[(E - pz)^2 - \Delta^2]}{[2E(E - \Delta) - 2Epz + m_{\tilde{\nu}}^2 - m_{\widetilde{W}_1}^2]^2},$$

and

$$M_{Z\tilde{\nu}\mathrm{L}} = \frac{-4\theta_y(\alpha_e - \beta_e)(\cot\theta_\mathrm{W} + \tan\theta_\mathrm{W})\sin\gamma_\mathrm{R}\cos\gamma_\mathrm{R}(s - M_Z^2)}{\sin^2\theta_\mathrm{W}[(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2]} \\ \times \frac{(x - y)[(E - pz)^2 - \Delta^2 - \xi m_{\widetilde{W}_1} m_{\widetilde{W}_2}] + 2x\xi m_{\widetilde{W}_1} m_{\widetilde{W}_2}}{[2E(E - \Delta) - 2Epz + m_{\tilde{\nu}}^2 - m_{\widetilde{W}_1}^2]},$$

with

$$\Delta = \frac{(m_{\widetilde{W}_2}^2 - m_{\widetilde{W}_1}^2)}{4E} \text{ and } \xi = (-1)^{\theta_{\widetilde{W}_1} + \theta_{\widetilde{W}_2} + 1}.$$

The cross sections for unpolarized or partially polarized beams can be obtained using,

$$\sigma = f_\mathrm{L}(e^-)f_\mathrm{L}(e^+)\sigma_\mathrm{LL} + f_\mathrm{L}(e^-)f_\mathrm{R}(e^+)\sigma_\mathrm{LR} \\ + f_\mathrm{R}(e^-)f_\mathrm{L}(e^+)\sigma_\mathrm{RL} + f_\mathrm{R}(e^-)f_\mathrm{R}(e^+)\sigma_\mathrm{RR}, \quad \text{(A.28)}$$

where f_L and f_R have been defined at the start of this subsection, and σ_{ij} $(i, j = \mathrm{L}, \mathrm{R})$ refers to the cross section from $e_i^- e_j^+$ annihilation.

https://doi.org/10.1017/9781009291248.019 Published online by Cambridge University Press