NEW CAUSTIC PHENOMENA IN DOUBLE-PLANE LENSING

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Consider two point masses m_1 and m_2 on distinct planes with respective shears γ_1, γ_2 and continuous matter having densities κ_1 and κ_2. It is assumed that the lens equation is as follows:

$$\begin{aligned}\mathbf{y} &= \mathbf{x}_1 - [m_1 \frac{\mathbf{x}_1}{|\mathbf{x}_1|^2} + \kappa_1 \mathbf{x}_1 + \gamma_1(-u_1, v_1)] \\ &\quad - [m_2 \frac{\mathbf{x}_2(\mathbf{x}_1) - \mathbf{d}}{|\mathbf{x}_2(\mathbf{x}_1) - \mathbf{d}|^2} + \kappa_2 \mathbf{x}_2(\mathbf{x}_1) + \gamma_2(u_2, -v_2)],\end{aligned}$$

where

$$\mathbf{x}_2(\mathbf{x}_1) = \mathbf{x}_1 - \beta \left[m_1 \frac{\mathbf{x}_1}{|\mathbf{x}_1|^2} + \kappa_1 \mathbf{x}_1 + \gamma_1(-u_1, v_1) \right]$$

and β is a measure of the distance between the two planes. This lens system is governed by nine parameters: $\mathbf{d} = (\delta_1, \delta_2)$, β, m_1, m_2, κ_1, κ_2, γ_1, and γ_2. Figure 1 displays the caustics for the values $\mathbf{d} = (0.0675, 0)$, $\beta = 0.6$, $m_1 = m_2 = 1$, $\kappa_1 = 1.9995$, $\kappa_2 = 3$, $\gamma_1 = 0.2$, and $\gamma_2 = 0.3$. None of these caustics can be generated by single-plane multiple point-mass lenses (due to the violations of convexity) or a double-plane two point-mass lens (since the latter lens cannot produce lips and swallowtails — Erdl & Schneider (1993)). Also, note the occurrences of two cusps of the second kind (Petters 1995), which are impossible in single-plane, multiple point-mass gravitational lensing.

C. S. Kochanek and J. N. Hewitt (eds), Astrophysical Applications of Gravitational Lensing, 283–284.

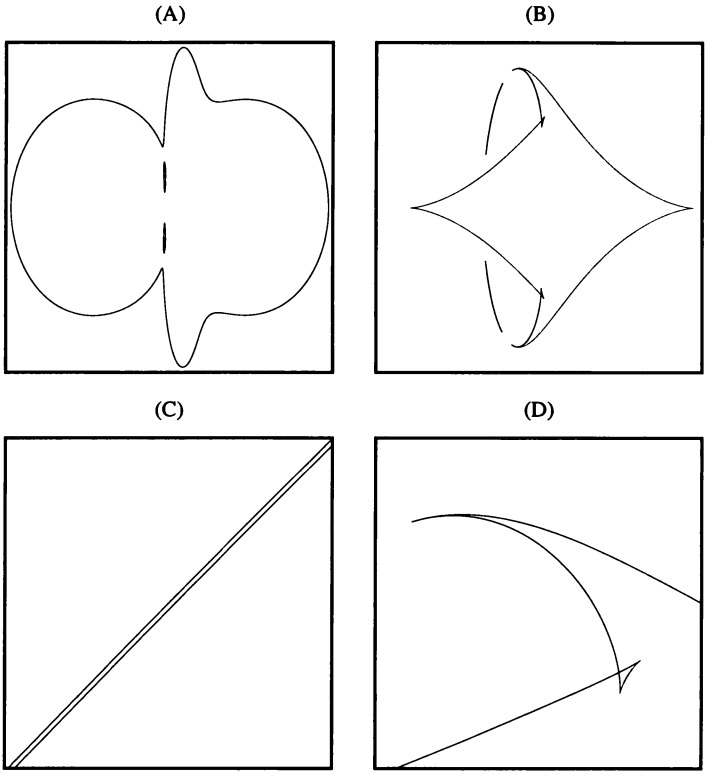

Figure 1. A critical curve and caustic for the double-plane lens with continuous matter and shear. (A) A critical curve with three connected components is transformed (B) into a caustic with lips and swallowtail singularities, and a cusp of the second kind. (C) A close-up of the lips caustic, showing that it is not a double line. (D) A close-up of the cusp of the second kind.

A more extended treatment of caustics due to the double-plane two point-mass lens with continuous matter and shear will appear in the paper Petters & Wicklin (1995).

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References

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