MAGNETIC FIELDS AND CONVECTION

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SUMMARY

In a highly conducting plasma convection is hindered by the imposition of a magnetic field. Convection may set in as direct or overstable modes and behaviour near the onset of instability depends on the ratio of the magnetic to the thermal diffusivity. Vigorous convection produces local flux concentrations with magnetic fields that may be much greater than the equipartition value. The interaction between magnetic fields and convection can be observed in detail on the sun and is essential to any stellar dynamo.

1. INTRODUCTION

Magnetic fields - whether primeval or maintained by dynamo action - are ubiquitous. Any rotating, convecting star seems able to generate a magnetic field, though the interaction between convection, rotation and magnetic fields bristles with problems for the theorist. We can usefully distinguish between the problem of maintaining large scale fields by dynamo action, which will be discussed by Dr Childress, and that of the interaction between small scale convection and an imposed magnetic field. I shall assume that any convective timescale is short compared with the lifetime of large scale magnetic fields and I shall not concern myself with their origin.

The scale of ordinary laboratory experiments is too small for them to model hydromagnetic behaviour in astrophysical plasmas. However, the sun provides a marvellous laboratory where such phenomena can be observed. Sunspots are dark because normal convection is suppressed by the strong magnetic fields; on a smaller scale, it is now possible to resolve features a few hundred kilometres across and to follow the interaction between weak fields and granular convection. This increase in resolution has revealed more magnetic structures and stronger magnetic fields than had been expected.

The theoretical description of a convecting system is particularly rich when stabilizing and destabilizing effects compete in it (Spiegel 1972). Dr Huppert has reviewed thermohaline convection; the nonlinear Lorentz force makes magnetic convection yet more complicated. I shall first summarize the results of linear theory and then discuss various nonlinear problems: is motion steady or oscillatory? are there subcritical instabilities? how is energy transport affected by the field? what limits flux concentration between convection cells and how strong are the fields

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produced? Not all these questions are yet answered but nonlinear magnetic convection is gradually being understood. Finally, I shall try to relate this theory to solar magnetic fields and to some aspects of the dynamo problem.

2. LINEAR THEORY

In the absence of a magnetic field a stratified gas is stable to adiabatic perturbations if Schwarzschild's criterion is locally satisfied. The imposition of a uniform magnetic field inhibits the onset of convection: a plane, perfectly conducting layer is convectively stable if

$$\frac{d\ln T}{d\ln p} - \left(\frac{d\ln T}{d\ln p}\right)_{ad} < \frac{B_o^2}{B_o^2 + \gamma\mu p}$$

(Gough and Tayler 1966), where B_0 is the vertical component of the magnetic field, T is the temperature, p the pressure, γ the ratio of specific heats, μ the permeability and the adiabatic gradient $(d\ln T/d\ln p)_{ad} = (\gamma-1)/\gamma$ for a perfect gas. Strong magnetic fields can therefore hinder the onset of convection in a star, though the difference between the adiabatic and the radiative gradient is usually large enough for instability to occur before the latter is attained (Moss and Tayler 1969, 1970; Tayler 1971).

When the conductivity σ is finite, plasma can move across the lines of force and the stabilizing effect of the magnetic field is relaxed. What happens depends on the relative values of the magnetic diffusivity $\eta = (\mu\sigma)^{-1}$ and of the thermal and viscous diffusivities κ and ν. In typical stellar conditions, $\kappa \gg \eta \gg \nu$. The onset of instability in a Boussinesq fluid has been studied in detail (Thompson 1951; Chandrasekhar 1952, 1961; Danielson 1961; Weiss 1964a; Gibson 1966). For a plane layer of depth d the stabilizing effect of a uniform magnetic field is measured by the dimensionless Chandrasekhar number

$$Q = \frac{B_o^2 d^2}{\mu\rho\eta\nu} ,$$

which is the square of a Hartmann number and can be regarded as a "magnetic Rayleigh number" (Spiegel 1972). A configuration is defined by Q, by the Rayleigh number $R = g\alpha\beta d^4/\kappa\nu$, where α is the coefficient of thermal expansion and β the superadiabatic temperature gradient, and by the Prandtl number $\sigma = \nu/\kappa$ and the magnetic Schmidt (or Prandtl) number

$$\tau = \frac{\nu}{\eta} .$$

If, for simplicity, we adopt "free" boundary conditions (Chandrasekhar 1961, Gibson 1966) then the linear modes have the form

$$w = W(z)\, f(x,y)\, e^{st} ,$$

where $W(z) = W_0 \sin\pi z$ $(0 \leqslant z \leqslant d)$ and

$$\nabla^2 f = -\frac{a^2}{d^2} f ,$$

with W_0 and a constant, referred to cartesian co-ordinates with the z-axis vertical. If $\kappa \leqslant \eta$ $(\sigma \geqslant \tau)$ linear instability sets in as in ordinary Rayleigh-Bénard convection. The growth rate s is real and instability sets in as a direct mode, corresponding to an exchange of stabilities, when $R = R^{(e)}$. (Semantics are succinctly summarized by Spiegel, 1972.) For large Q, $R^{(e)}$ is a minimum when the dimensionless horizontal wavenumber

$$a \approx \left(\tfrac{1}{2}\pi^2 Q\right)^{\frac{1}{6}} ;$$

so convection first appears in vertically elongated cells at $R = R_c^{(e)} \approx \pi^2 Q$.

Standing hydromagnetic waves in an unstratified fluid produce oscillations which are damped by ohmic and viscous dissipation. When $\kappa > \eta$ these oscillations may be destabilized by the thermal stratification (Cowling 1976a), so that convection sets in as overstable oscillations when $R = R^{(o)}$. For sufficiently large Q, overstability first occurs in elongated cells, when

$$R = R_c^{(o)} \approx \frac{\sigma^2(1+\tau)}{\tau^2(1+\sigma)} \pi^2 Q .$$

When $\sigma < \tau$, therefore, $R_c^{(o)} < R_c^{(e)}$ and instability first appears as overstable oscillations. At $R = R_c^{(o)}$ there are two complex conjugate growth rates but as the Rayleigh number is raised $|Im(s)|$ decreases until for some $R = R^{(i)}$ the growth rates are purely real. Thus convective instability sets in with direct modes at $R = R^{(i)}$. As $Q \to \infty$, for $\sigma \ll \tau \ll 1$ $(\kappa \gg \eta \gg \nu)$ $R_c^{(o)} \approx \left(\frac{\sigma}{\tau}\right)^2 \pi^2 Q = \left(\frac{\eta}{\kappa}\right)^2 \pi^2 Q$ and the minimum value of $R^{(i)}$ is $R_c^{(i)} \approx (\sigma/\tau)\pi^2 Q$ (Danielson 1961; Weiss 1974a); thus $R_c^{(o)} \ll R_c^{(i)} \ll R_c^{(e)}$. For $R^{(i)} < R < R^{(e)}$ there are two distinct positive real growth rates. One of these changes sign when $R = R^{(e)}$ but this exchange of stabilities has no physical significance.

So far we have considered only free boundary conditions. Analogous results hold also for other boundary conditions (Chandrasekhar 1961, Gibson 1966). In particular, the effect of superposing a stable layer on top of the unstable region has been investigated by Musman (1967) and Savage (1969). The treatment has also been extended to include some effects of compressibility (Kato 1966; Syrovatsky and Zhugzhda 1967, 1968; Saito and Kato 1968). If the Alfvén speed is small compared with the sound speed, slow magnetosonic oscillations become overstable; if the Alfvén speed is large, the fast magnetosonic mode can be destabilized (Cowling 1976b).

3. NONLINEAR CONVECTION

In a Boussinesq fluid the magnetic field satisfies the induction equation

$$\frac{\partial \underline{B}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{B}) + \eta \nabla^2 \underline{B}$$

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while the vorticity $\underline{\omega}$ and the temperature T are governed by the equations

$$\frac{\partial \underline{\omega}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{\omega}) - \frac{1}{\rho} \nabla \wedge (\underline{B} \wedge \underline{j}) - \alpha \nabla T \wedge \underline{g} + \nu \nabla^2 \underline{\omega},$$

$$\frac{\partial T}{\partial t} = - \underline{u} . \nabla T + \kappa \nabla^2 T ,$$

and

$$\nabla . \underline{u} = 0 , \qquad \nabla \underline{B} = 0 .$$

Here $\underline{j} = \mu^{-1} \nabla \wedge \underline{B}$ is the electric current, $\underline{u}$ is the velocity and ρ the density. For two-dimensional convection, with $\underline{u}$ and $\underline{B}$ confined to the xz-plane and independent of y, $\underline{B}$ can be described by a flux function (the y-component of the vector potential) A such that

$$\underline{B} = \left(-\frac{\partial A}{\partial z}, 0, \frac{\partial A}{\partial x}\right), \qquad \mu j = - \nabla^2 A ,$$

and

$$\frac{\partial A}{\partial t} = - \underline{u} . \nabla A + \eta \nabla^2 A ,$$

while the vorticity equation reduces to

$$\frac{\partial \omega}{\partial t} = - \underline{u} . \nabla \omega + \frac{1}{\rho} \underline{B} . \nabla j - g \alpha \frac{\partial T}{\partial x} + \nu \nabla^2 \omega .$$

The most convenient boundary condition on the field is obtained by setting $B_x = 0$ at z = 0, d. This is somewhat artificial but corresponds to the free boundary conditions adopted for linear theory.

Near the exchange of stabilities ($R = R^{(e)}$) finite amplitude solutions can be constructed using modified perturbation theory. Veronis (1959) observed in a footnote that subcritical instabilities were possible when $\kappa > \eta$. Busse (1975) has considered a two-dimensional model in which the magnetic field affects the amplitude, but not the form, of the motion. For R near R_c, the critical Rayleigh number in the absence of a magnetic field, he combined a perturbation expansion for the velocity with a computed solution for the distorted magnetic field. He showed that when $\kappa \gg \eta$ stationary convection is possible with $R_c < R \ll R^{(e)}$. In these solutions the magnetic Reynolds number

$$R_m = \frac{U d}{\eta}$$

(where U is a typical velocity) is large and the magnetic field is confined to narrow regions so that its overall stabilizing effect is correspondingly reduced; an analogous argument applies to the thermohaline problem that Dr Huppert has described. Since subcritical convection appears when $\sigma \ll \tau$ and $R^{(o)} \ll R^{(e)}$ this technique cannot rigorously establish whether steady finite amplitude solutions are possible before the onset of overstability. However, Busse's results do suggest that such subcritical instabilities may occur and Proctor (private communication) has developed a simplified model of magnetic convection which shows steady motion when R is close to R_c.

Perturbation methods are reliable only while the Péclet number ($U d/\kappa$) is low and the convective energy transport relatively small. The efficiency of convection is measured by the Nusselt number $N = (F - \kappa\beta_{ad})/\kappa\beta$, where F is the total thermometric flux and β_{ad} the adiabatic temperature gradient. The effect of a magnetic field on the Nusselt number was investigated by van der Borght, Murphy and Spiegel (1972), using the mean field approximation (Spiegel 1971). They considered only steady convection which, in this approximation, is independent of both σ and τ . For a fixed Rayleigh number, N decreased monotonically and smoothly with increasing Q until the exchange of stabilities was reached; thereafter convection was completely suppressed. Van der Borght (1974) has also attempted to describe the time dependent problem.

Fully nonlinear two-dimensional computations show a different range of behaviour (Weiss 1975). For free boundary conditions with $\sigma = 1$, $\tau \gg 1$ there is a general tendency to generate nonlinear oscillations. When $\tau = 5$, convection first appears as overstable oscillations in accordance with linear theory. If Q is then decreased, while R is kept fixed, these oscillations are stabilized at some small but finite amplitude and convection remains comparatively inefficient. As Q is further reduced, $R^{(i)}$ becomes less than R and linear theory predicts direct, exponentially growing modes. These appear in the numerical solutions but eventually develop into periodic nonlinear oscillations. The Lorentz force is quadratic in $\underline{B}$ and linear theory underestimates the restoring force. Hence nonlinear magnetic convection differs from thermohaline convection, where the stabilizing force remains linear. If Q is decreased yet further the oscillations develop into irregular aperiodic motion and, eventually, into steady convection. The time-averaged Nusselt number rises monotonically as Q decreases but there are no noticeable discontinuities, nor could any hysteresis be detected.

Dr. Galloway will describe his numerical study of axisymmetric convection in a magnetic field. The results are qualitatively similar, though he found some hysteresis, indicating different solutions when Q was decreased from the critical value and subsequently increased. So far, however, numerical experiments have provided no evidence of any jump in the Nusselt number or of any metastable conducting state associated with subcritical convection. Computations on thermohaline convection (Huppert and Moore 1977) demonstrate that such phenomena can occur. Busse's finite amplitude results and Proctor's simple model both suggest that, with suitably chosen parameters, metastable magnetic configurations should exist. Further computations, with a wider range of diffusivities, are needed to establish whether subcritical convection can be found. It is obviously important to determine what parameter ranges allow metastable states and whether linear stability theory has any relevance to convection in a strong magnetic field in a star.

4. FLUX CONCENTRATION

In the limit when Q is sufficiently small the magnetic field is weak and the Lorentz force has no dynamical effect. The velocity $\underline{u}$ can then, in principle,

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be derived from some theory of ordinary Rayleigh-Bénard convection. If $\underline{u}$ is then fed into the induction equation the kinematically distorted magnetic field can be calculated. This has been done for various plausible velocity fields (Parker 1963; Clark 1965, 1966; Weiss 1966; Clark and Johnson 1967; Busse 1975). When the magnetic Reynolds number is high, magnetic flux is rapidly swept to the edges of convection cells to form ropes. Within a cell, the field is wound up until the lines of force eventually reconnect and magnetic flux is expelled. If $R_m >> 1$ the pattern of motion must persist for many turnover times before the expulsion process is completed. However, ropes are formed between the cells by the time they have turned over once. Within these ropes the field strength has an approximately Gaussian profile and the peak field

$$\left.\begin{array}{ll} B^* \approx R_m^{\frac{1}{2}} B_o & \text{in two dimensions} \\ \approx R_m B_o & \text{in three dimensions,} \end{array}\right\}$$

where B_o is the average initial field (or the field in the absence of convection).

Astrophysical length scales are large and R_m is big enough for enormous magnetic fields to be produced locally if concentrations were purely kinematic. Eventually, the $\underline{j} \wedge \underline{B}$ force in the flux rope must become powerful enough to halt the concentration: amplification of the field is then dynamically limited. But it is not immediately obvious what limiting field strength can be produced. Partly on dimensional grounds, it has been popularly supposed that the local field strength cannot exceed the equipartition field B_e, where

$$\frac{B_e^2}{2\mu} = \frac{1}{2}\rho U^2 .$$

The principal argument for this limit depends on considering pressure fluctuations associated with convection but in a Boussinesq fluid the pressure can be eliminated from the equations and the equipartition limit should therefore be irrelevant.

Busse (1975) showed that for small amplitude two-dimensional convection $B^* \propto (\sigma\tau)^{1/4}$. Hence B^*/B_e could be made arbitrarily large by a suitable choice of τ. The full two-dimensional problem has been investigated numerically for convection driven by imposed horizontal temperature gradients (Peckover and Weiss 1977) and by heating from below (Weiss 1975), and Dr. Galloway has computed solutions for axisymmetric convection. The maximum value of the peak field B^* can be estimated by a simple argument. The two-dimensional results show that kinematic amplification is halted when ohmic dissipation in the flux rope becomes comparable with viscous dissipation throughout the convection cell. It follows that the maximum field $B^*_{max} \propto \tau^{1/4}$, a result confirmed by the computations. In three dimensions a similar argument yields a maximum field $B^*_{max} \propto \tau$ (Galloway et al. 1977). By choosing τ sufficiently large, B^* can be made much greater than B_e and solutions have been obtained with $B^*/B_e \approx 5$ (though the particular value has no significance).

Once the magnetic field becomes dynamically significant, vorticity is generated in the flux ropes, where there is a local balance between the magnetic and viscous terms in the vorticity equation. The buoyancy force generates vorticity with one sense, while the Lorentz force generates vorticity with the opposite sense and viscosity maintains a balance. The resultant vorticity distribution corresponds to a velocity field with a reduction in the transverse flow that concentrates the flux. A simple physical description confirms that two-dimensional amplification is halted as ohmic dissipation reduces the overall flow. In three dimensions motion can be excluded from the flux rope slightly earlier (Galloway, private communication).

As B_o is further increased, the flux ropes grow broader and develop a different structure. The field within a rope is more nearly uniform, dropping abruptly near its boundary. The ensuing current sheath produces a Lorentz force which prevents the motion from entering the flux tube. Ultimately the layer separates into convecting cells, from which magnetic flux has been expelled, and stagnant flux ropes in the interstices between them.

5. SOLAR MAGNETIC FIELDS

Cowling (1953, 1976a), Sweet (1971) and Mullan (1974b) have reviewed magnetic fields in the sun. The discussion of flux concentration relates most directly to intense, small scale magnetic fields in the photosphere (Weiss 1977). Over the last eight years ground-based observers have succeeded in resolving magnetic structures with a scale smaller than that of the granulation and fields of up to about 1500 G (Schröter 1971; Harvey 1971, 1977; Dunn and Zirker 1973; Mehltretter 1974; Stenflo 1976). These features are formed between granules and have lifetimes similar to those of individual granules. The fields are much larger than the local equipartition field ($B_e \approx 500$ G) and the magnetic pressure alone is almost sufficient to balance the external gas pressure. Such high fields can only be contained by that gas pressure (Parker 1976a). A full theory of convective transport in strong magnetic fields is needed to explain the formation of these flux ropes but a crude extrapolation from the Boussinesq results indicates that the field can be amplified to reach the strengths observed (Galloway et al. 1977).

On a larger scale, magnetic flux is swept aside by supergranules and concentrated at their boundaries to form a network in which most of the small scale features are located. Irregular small scale fields have recently been detected within the network (Harvey 1977) but the flux involved is relatively slight. As more flux is brought together the magnetic field interferes with convection so that the gas is cooled. Dark pores or sunspots then appear between the supergranules. The magnetic flux that emerges through a sunspot is presumably assembled into a rope deep in the convective zone, though supergranules certainly play a part in the formation of a spot. Conversely, though small flux ropes can be shifted to fit the pattern of supergranular convection, large sunspots are anchored deeper down and long-lived, stable convection cells may form around or near them (Harvey and Harvey 1973; Livingston and Orrall 1974; Meyer et al. 1974).

In the umbra of a sunspot the magnetic field is nearly vertical and strong enough to suppress convective instability. Following a suggestion by Biermann

(1941), Cowling (1953, 1976 a,b) argued that motion across the magnetic field is inhibited, and that convection is limited to predominantly vertical, oscillatory motion, in slender elongated cells. Theoretical models of sunspots (Chitre 1963; Deinzer 1965; Chitre and Shaviv 1967; Yun 1970) show that radiation alone cannot supply the energy emitted from the umbra and microturbulent velocities (Beckers 1976) provide some observational evidence for convection. (Umbral dots are too sporadic to be an essential feature of the transport process.) Various attempts have been made both to relate linear stability theory to umbral and penumbral structure in sunspots (Danielson 1961, 1965, 1966; Weiss 1964b, 1969; Musman 1967; Saito and Kato 1968; Danielson and Savage 1968; Savage 1969; Mullan and Yun 1973; Moore 1973) and also to study overstability in an isolated magnetic flux tube (Parker 1974b, B. Roberts 1976, Defouw 1977).

Parker (1974a,b, 1975a, 1976a,b) has recently emphasized the importance of mechanical energy transported by transverse hydromagnetic waves, which may escape either upwards or downwards from the umbra. He suggests that thermal energy which would otherwise have reached the photosphere is carried away by these waves, which are so efficiently coupled to subphotospheric convection (cf. Mullan 1974a) that they refrigerate the sunspot. Cooling by Alfvén waves requires extreme efficiency and this mechanism has been criticized by Cowling (1976b). Moreover, the corona absorbs only a comparatively small amount of energy and excess X-ray emission is associated with active regions, not specifically with sunspots. So magnetic inhibition of convection still provides the most obvious explanation for the cooling of pores and spots.

Unlike the umbra, the penumbra of a sunspot is essentially inhomogeneous, and the radial filaments are correlated with convective motion (Beckers and Schröter 1969, Schröter 1971). According to linear theory convection in rolls lying in the plane of $\underline{B}$ is affected only by the vertical component of the field. The inclination of the magnetic field increases across the penumbra until it becomes almost horizontal at the edge of the spot. Danielson (1961) and Pikel'ner (1961) therefore suggested that the filamentary structure is caused by convection in horizontal rolls and this explanation is qualitatively convincing. Linear theory indicates that the penumbra may be convectively unstable, to direct rather than to overstable modes (Danielson 1961; Musman 1967; Saito and Kato 1968; Savage 1969). Nonlinear results imply that convective transport would then be significant, though motion might still be periodic (Weiss 1975). These theoretical models are obviously oversimplified. In particular, the boundary conditions are too stringent: one might, for instance, expect that vigorously convecting plasma from below the penumbra would be able to penetrate through the shallow magnetically dominated region (Meyer et al. 1977). A more complete theory should also explain the Evershed outflow as a consequence of convection (Galloway 1975).

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6. CONVECTION AND DYNAMO THEORY

There is some observational evidence that other stars with outer convection zones have magnetic cycles like the sun (Wilson 1976) and flux concentration is inevitable in any stellar dynamo that is driven by convection. Turbulent motion tends to remove magnetic fields from a convective zone: flux tubes may emerge from the surface and be carried off by a stellar wind or they may be expelled downwards into the radiative zone. Systematic differences in the velocity, caused for example by the radial density gradient, may pump flux preferentially in one direction (Moore and Proctor 1977). A more important topological effect was pointed out by Drobyshevsky and Yuferev (1974). In three dimensional convection with upward motion at cell centres, the sinking fluid forms a continuous network, while regions of rising fluid are separated from each other. Since a flux tube can wind continuously through downward moving gas there is a tendency to pump flux downwards and to concentrate the field at the base of a convecting layer. Topological pumping competes with magnetic buoyancy (Parker 1955, 1975b; Gilman 1970; Unno and Ribes 1976). If the field in the flux rope has the equipartition value then the rope floats upward relative to the ambient gas at about the Alfvén speed, which is equal to the downward convective velocity. So the net motion of the flux tube cannot readily be estimated, though it seems unlikely that flux can remain within the star unless it is surrounded by sinking gas.

A proper description of the inhomogeneous magnetic field must be included in any realistic dynamo model. The theory of turbulent dynamos has often been reviewed (eg. Parker 1970; P. H. Roberts 1971; Vainshtein and Zel'dovich 1972; Gubbins 1974; Mestel and Weiss 1974; Moffatt 1977). Without systematic helicity, homogeneous turbulence is unlikely to maintain a field (Moffatt 1977), and helicity is caused by rotation. The Coriolis force, like the Lorentz force, tends to inhibit convection but these constraints may be relaxed if both are simultaneously present (Malkus 1959; Chandrasekhar 1961; Eltayeb and Roberts 1970; Eltayeb 1972, 1975; van der Borght and Murphy 1973; Roberts and Stewartson 1974, 1975). Attempts to solve the full hydromagnetic dynamo problem will be discussed by Dr. Childress.

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