BOOK REVIEWS

RUTHERFORD, D. E., *Introduction to Lattice Theory* (Oliver and Boyd, 1965), 117 pp., 35s.

An introduction to a subject may either concentrate upon some relatively small part of the subject and explore it leisurely and in some detail, or it may briefly touch upon many facets of the subject in an attempt to convey some idea of its range. In this book the author has followed the second course and in some thirty-seven sections has touched upon a considerable variety of interesting topics looked at in a lattice setting. These topics range over Boolean algebra, Boolean rings, propositional calculus, switching circuits, Boolean matrices and determinants, Brouwer algebra, intuitionistic logic, atomic lattices, Geometric lattices and lattice topology. Definitions are clear and concise, explanations lucid and proofs well presented. This book may be confidently recommended to all students entering on a study of mathematical logic.

R. L. GOODSTEIN

AKHIEZER, N. I., *The Classical Moment Problem* (Oliver and Boyd, 1965), x+252 pp., 70s.

The problem of moments has never quite been in the main stream of mathematical analysis. It has, however, interested some very distinguished mathematicians, and has substantial contacts with several other important topics (function-theory, continued fractions, orthogonal polynomials, operators in Hilbert spaces, ...). The classical one-dimensional power moment problem is to solve for σ the equation $\int t^n d\sigma(t) = c_n (n = 0, 1, 2, \ldots)$, where σ is to be a positive measure and the c_n are given constants. Here the integral is either over the whole real axis (Hamburger's problem), or a specified subset of it ((0, 1) Hausdorff; (0, ∞) Stieltjes). In the trigonometric moment problem the real axis is replaced by the unit circle, and negative as well as positive powers of t ($= \exp i\theta$) appear. The basic problem is an interesting and natural one; its ramifications are still far from being fully worked out.

For the most part the author confines himself to the one-dimensional power problem, with occasional excursions into more general situations. The five chapters are: Infinite Jacobi matrices and their associated polynomials; The power moment problem; Function theoretic methods in the moment problem; Inclusion of the power moment problem in the spectral theory of operators; Trigonometric and continuous analogues. For much of the book, a background of classical analysis such as used to be found in any honours degree course, together with some elementary ideas from functional analysis, should suffice. In the fourth chapter some acquaintance with spectral theory is required. The presentation of the material is straightforward, and an adequately equipped reader should encounter few difficulties.

There is naturally a substantial overlap between the present book and Shohat and Tamarkin's *The Problem of Moments* (American Mathematical Society, New York, 1943). Both contain, for example, a detailed study of the classical one-dimensional Hamburger problem. However, there is also much in Akhiezer that is not to be found in Shohat and Tamarkin, such as the treatment of the relation between the moment problem and spectral theory in Chapter 4.

Account is taken of work on the problem up to 1960 or so (the original Russian text appeared in 1961). A leading part in the investigation of the moment problem has been taken by Russian mathematicians, and this tradition has been fully maintained in recent years, Significant contributions have been made by the author himself, by M. G. Krein, and others.

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The book should be useful both as a convenient source of information about the present state of the problem and as a stimulus to further research. While it is true that abstract analysis (both harmonic and functional) has already had a significant influence on the development of the theory of the moment problem, it seems very likely that further fruitful interaction is possible here. For instance, the trigonometric moment problem seems to fit more easily into the general structure of abstract harmonic analysis than does the power problem; further work on abstract formulations of the power problem would evidently be of great interest. J. H. WILLIAMSON

YANO, K., *Differential Geometry on Complex and almost Complex Spaces* (Pergamon Press, 1965), xii + 323 pp., 90s.

The theory of complex and almost complex spaces belongs to one of the most attractive sections of modern Differential Geometry. Complex spaces first appeared in the early 1930's, in papers by Schouten and van Dantzig, and Kähler. Their studies had applications to algebraic varieties and these applications stimulated the interest of the differential geometers of the immediate post-war period. At that time, the emphasis in differential geometry was changing from local to global considerations and the development of the theory of complex spaces was influenced by this change. Nevertheless, tensors play an important part and a casual glance at the book under review is enough to show that it is necessary to have a working knowledge of the tensor calculus in order to understand the theory. The concept of an almost complex space arose in 1947 in the work of Weil and Ehresmann. The idea stems from the observation that a complex space admits a second order mixed tensor field whose square is minus unity, and that many properties of complex spaces depend only upon the existence of such a tensor. In an almost complex space, this property is taken as the starting point. A complex space is almost complex, but an almost complex space need not be complex, so that a wider class of spaces is being studied.

This book is indeed a masterly exposition of the subject, with an impressive wealth of detail. The patience needed to follow through the calculation in terms of tensor components should be rewarded by an appreciation of the beauty of the subject; there is much in complex and almost complex spaces that is mathematically attractive. Thus for the would-be differential geometer this book is a necessity; for others it could still be rewarding. The author has put in an immense amount of work. It might appear that he has left no avenue unexplored, but, as he himself states in the preface, the theory is a very fruitful one and he hopes that some of his readers will be encouraged to exploit this domain of differential geometry.

The first chapter is devoted to basic definitions concerning differentiable manifolds; it includes accounts of the concepts of Lie derivative and Killing vector. Green's theorem and some of its applications are discussed in Chapter II. These results are well worth a study in their own right, but are introduced here because of their wide applications in the present work. Complex manifolds are introduced in Chapter III. There are accounts of the basic properties of compex tensors and Hermitian and Kähler spaces. The study of Kähler spaces is continued in the next chapter, where the results of Chapter II together with Hodge's theorem on harmonic tensors are applied to prove a considerable body of theorems. The concept of an almost complex space appears in Chapter V. The question of the integrability of an almost complex structure (that is, whether the structure is induced by a complex structure) is studied early in Chapter V and several necessary and sufficient conditions are given. In particular, the important tensor introduced by Nijenhuis is discussed. Affine connexions in almost complex spaces are studied in the next chapter; an account of A. G. Walker's tensor differentiation is also given. Chapters VII and VIII are devoted to studies of almost Kähler spaces and almost Tachibana spaces respectively. Both types are