THEORIES OF LARGE SCALE STRUCTURE

S. F. Shandarin
Institute for Physical Problems
ul. Kosyggina 2, Moscow 117334, USSR

This talk is concerned with one of the most important class of theories of Large Scale Structure (hereafter LSS) based on two principal assumptions. It is supposed that (i) the main process is gravitational instability in expanding Universe and (ii) the primordial perturbations are small Gaussian density fluctuations. Both assumptions are natural in inflationary model of the very early universe as well as in cosmological models dominated by Dark Matter (DM) in the form of Weakly Interacting Massive Particles (WIMPs). Other possible models of LSS formation are discussed by N.Turok, J.Ostriker and A.Dekel.

The advantage of theories of this kind is very well specified initial conditions uniquely determined by the spectrum of primordial density fluctuations. In principle all statistical characteristics of the observed LSS can be derived from the primordial spectrum. However this problem is extremely difficult mathematically.

It is worth to remind that the simplest inflationary model predicts fractal spectrum (so called Harrison -Peebles-Zeldovich spectrum) for primordial perturbations independently on the kind of WIMPs making up DM. However the following evolution of the density perturbations depends on the kind of WIMPs that results in different kinds of spectrum say after decoupling epoch. This in turn results in different scenarios of LSS formation. The well known extremes are a top-bottom scenario in the Hot Dark Matter (HDM) model and a bottom-top scenario in the Cold Dark Matter (CDM) model. Terms HDM and CDM (proposed by Dick Bond) specify the largest scale of perturbations erased by free streaming at relativistic stage. In the HDM model this scale is about the size of superclusters and therefore the first objects formed at the non-linear stage are pancakes of similar sizes. In this scenario galaxies form later by fragmentation of the pancakes.

J. Audouze et al. (eds.), Large Scale Structures of the Universe, 273–280.

In the CDM model the free streaming scale is much less than globular star clusters therefore the formation of LSS goes in process similar to hierarchical clustering.

By the present the most reliable estimates of statistical characteristics of LSS (e.g. two and three point correlation functions, percolation properties etc) prdicted in different scenarios have been made by means of numerical simulation. The main problem the numerical simulations encounter is large dynamical range (ratio L/l where L and l are the largest and smallest linear scales in numerical simulations) needed for adequate simulation of LSS formation. As I was told at Balatonfured by M.Davis the record dynamical range $L/l \sim 600$ had been achieved by M.Davis, G.Efstathiou, C.Frenk and S.White in their recent simulations in the frame of CDM model. In typical simulations L/l is about ten times less. Additional problems arise due to the restricted namber of mass points (typically about $2 \cdot 10^5$ particles) one can use in the simulations.

Keeping this in mind it is worth to try to develop an approximate analitical approach even paying the price of reasonable simplification of the problem. One of the most successful approach of this kind is well known Zeldovich's approximate solution for non-linear gravitational instability in zero-temperature dust (1970). This solution proves very useful for analysis of LSS formation in HDM (massive neutrino) model (Shandarin, 1983; Shandarin, Doroshkevich, Zeldovich, 1983). Unfortunately its application to formation of LSS in CDM model is much more restricted. The reason is that as has been said the LSS formation in CDM model has many essential features of hierarchical clustering that cannot be described adequately in the frame of Zeldovich's approximation. The main disadvantage of Zeldovich's solution is that it fails to follow the motion of collisionless matter after pancake formation. If one formally extrapolates Zeldovich's solution he comes to conclusion that thickness of pancakes grows unlimitedly. However 1D numerical simulations (Doroshkevich et al, 1980) showed that the motion of collisionless matter inside pancakes becomes oscillatary rather than progressive one. This slows down the growth of the pancake thickness substantially. It is reasonable to think that in realistic 3D geometry the matter oscillating across the pancake keeps progressively moving along it. This results in the formation of filaments and clumps. 3D numerical simulations give support to this assumption.

One possibility to describe this process is to consider the motion of dust with small viscosity. At every place where the velocity field is smooth the viscosity practically does not influence the motion of the matter

https://doi.org/10.1017/S0074180900136149 Published online by Cambridge University Press

but viscosity prevents overturning of streams and formation of multistream regions. As a result very thin films of very dense matter form instead of pancakes. However what is important they have about the same masses as pancakes do.

After very short description of general physical idea let us discuss the mathematical aspect of the problem.

First let us introduce coordinates commoving to general expantion of the Universe

$$x_i = r_i/a(t)$$

and peculiar velocity

$$V_i = u_i - \dot{a}/a \, r_i = a \frac{dx_i}{dt},$$

here $a(t)$ is a scale factor, $\dot{a} = da/dt$.

In this variables the equations governing the evolution of density perturbations are as follows (e.g. Peebles, 1980)

$$\frac{dV_i}{dt} + \frac{\dot{a}}{a} V_i = - \frac{1}{a} \nabla_x \varphi ,$$
$$\nabla_x^2 \varphi = 4 \pi \, G a^2 (\rho (x_i,t) - \bar{\rho}(t)), \qquad (1)$$
$$\frac{\partial \rho}{\partial t} + 3 \frac{\dot{a}}{a} \rho + \frac{1}{a} \nabla_x (\rho V_i) = 0,$$

here φ is perturbation of gravitational potential, $\rho(x_i,t)$ is the density of mass, and $\bar{\rho}(t)$ is the mean density.

The next step is a very important assumption that Zeldovich's solution approximately describes the motion of matter outside pancakes as well as of the pancakes as a whole even after the formation of them. In comoving coordinates Zeldovich's approximation takes the following form

$$x_i = q_i + b(t) \cdot s_i(q_j), \qquad (2)$$

where x_i and q_i are Eulerian and Lagrangian coordinates, $b(t)$ is a function of time describing the evolution of growing mode of density perturbations in linear regime, and $s_i(q_j) = \nabla_q S_o(q_j)$ is a potential vector field specifying the spatial distribution of density at the linear stage.

One can reduce the system of three equations to the system of two equations calculating approximately the gravitational acceleration from Zeldovich's solution

https://doi.org/10.1017/S0074180900136149 Published online by Cambridge University Press

$$\frac{dV_i}{dt} = \left(\frac{\dot{a}}{a} + \frac{\ddot{b}}{\dot{b}} \right) V_i .$$

Introducing new variables $n(x_i,t) = a^3 \rho(x_i,t)$, $v_i(x_j,t) = \frac{1}{a\cdot\dot{a}} V_i(x_j,t)$ and excluding the equation for gravitational potential one easily obtains the system of two equations approximately describing the evolution of density perturbations

$$\frac{\partial v_i}{\partial a} + v_k \frac{\partial v_i}{\partial x_k} = -\left(\frac{\ddot{a}}{\dot{a}} - \frac{\ddot{b}}{\dot{b}} \right) v_i , \qquad (3)$$
$$\frac{\partial n}{\partial a} + \frac{\partial (n\, v_i)}{\partial x_i} = 0,$$

here-after we will use $a(t)$ istead of t.

In $\Omega = 1$ cosmology the right hand side term of the first equation equals zero exactly bec use of in this case $a = b$. However if $\Omega < 1$ this term is negative that can be interpreted as friction due to too fast expantion of the open Universe. For simplicity we shall discuss only the flat Universe $\Omega = 1$.

An evident solution of (3) is (2) with $b = a$ and $s_i(q_j)$ being interpreted as initial velocity field $v_{oi} = s_i(q_j)$. This is neither surprising nor new and reflects only the selfconsistency of the using Zeldovich's solution.

The second very important step is the inserting of a viscosity term in the first equation of system (3) that results in

$$\frac{\partial v_i}{\partial a} + v_k \frac{\partial v_i}{\partial x_k} = \nu \Delta v_i . \qquad (4)$$

The particular form of the viscosity term has been chosen to obtain well known Burgers' equation that has the analitical solution in the case of potential motion (Burgers, 1940, 1974)

$$v_i(x_j,a) = \frac{\int \frac{x_i - q_i}{a} \exp\left(-\frac{1}{2\nu} G(x_j,q_j,a)\right) d^3q}{\int \exp\left(-\frac{1}{2\nu} G(x_j,q_j,a)\right) d^3q} , \qquad (5)$$

where $G(x_j,q_j,a) = S_o(q_j) + \frac{(x_j - q_j)^2}{2\,a}$ and $S_o(q_j)$ is the

https://doi.org/10.1017/S0074180900136149 Published online by Cambridge University Press

potential of the initial velocity field $v_i(x_j,0) = \nabla S_o(q_j)$ (at $a = 0$ $x_j = q_j$). By the way S_o is proportional to gravitational potential at the linear stage.

To obtain solution (5) one needs to make Hopf - Cole substitution: $v_i(x_j,a) = \nabla_x S(x_j,a)$ and $S(x_j,a) = - 2\nu \ln U(x_j,a)$ that transfers equation (4) into linear one $\partial U/\partial a = \nu \Delta U$.

The first ideas of using Burgers' equation for the problem of LSS formation were published by Gurbatov, Saichev and Shandarin (1984, 1985).

It is interesting that in limiting case of $\nu \rightarrow 0$ one can reduce solution (5) to very simple form

$$v_i(x_j,a) = \frac{x_i - q_i}{a} , \qquad (6)$$

which is equivalent to Zeldovich's solution (2) with $b = a$ at $\Omega = 1$.

However we know that Zeldovich's solution predicts the formation of pancakes where three stream flows of matter arise. It must be stressed that the system with arbitrary small viscisity described by equation (4) behaves in a drastically different manner than pure collisionless matter. Viscosity makes impossible the formation of multistream flows independently on ν . Its value determines the thickness of dense layers: the less ν the thiner the layers. At $\nu \rightarrow 0$ layers, filaments and clumps turn into infinitely thin films, threads and points respectively. We shall not discuss the inner structure of layers at finite viscosity ν . Instead of let us consider the general evolution of matter distribution predicted by Burgers' model.

To analyse solution (5) at $\nu \rightarrow 0$ it is helpful to use the following geometrical interpretation. Directly from equation (5) one can see that the greatest input into integrals in (5) (at given x_i and a) is given by point q_i where $G(x_i,q_i,a)$ considered as a function of q_i has the deepest minimum. On the other hand at the same points paraboloid $P(x_i,q_i,a) = - (x_i - q_i)^2/2a + H$ touches hypersurface $S_o(q_i)$ for the first time when H grows from $- \infty$. At the linear stage of the density perturbation growth a is small and paraboloid P is very narrow (asymptotically a needle at $a \rightarrow 0$) therefore it can touch hypersurface S_o at every point. This is interpreted as follows. At small a a point with Lagrangian coordinates of the point of touching arrives at the point in Eulerian space having the coordinates of the paraboloid apex. Thus at small a one can find Eulerian position of every point from Lagrangian space by moving paraboloid P keeping its touching hypersurface S_o.

Later when a is larger paraboloid P becomes wider and it cannot touch some points of hypersurface $S_o(q_i)$ without intersection the hypersurface at other points but according to this interpretation this is not permitted. At this time there are places where paraboloid P touches S_o at two points simultaneousely. In these positions the apex of the paraboloid indicates the Eulerian coordinates of dense films (or infinitely thin pancakes). There are also positions where the paraboloid touches S_o at three or four points simultaneously. In these cases the apex indicates filaments or clusters respectively.

With growth of a the paraboloid becomes even wider that results in approaching the touching points closer to minima of S_o. Asymptotically at large a touching points practically coinside with deepest minima of S_o that control the positions of the clumps of mass. At this stage most of the matter is contained in clumps which are moving and merging that makes them more massive.

It is interesting that the approach based on Burgers' equation predicts that the evolution of the mass clumps is governed by negative peaks (deepest peaks are negative) of potential S_o. It is worth to remind that at present the hypothesis according to which the highest peaks of filtered linear density fluctuations determine the evolution of galaxies and clusters of galaxies is very popular (Peacock, and Heavens, 1985; Bardeen et al, 1986). Formally the difference between two approaches can be described as difference in kind of filtering. In the former case the spectrum of potential P is $\Delta^2(k) \propto k^{-4}\,\delta^2(k)$, where $\delta^2(k)$ is the spectrum of density perturbations. In the latter case filtered spectrum is $\delta_f^2(k) \propto \delta^2(k)\,\exp(-k^2/k_o^2)$ with parameter k_o or similar. Both kinds of filtering give more weight to the longwave part of the initial spectrum of the density perturbations and reduce the influence of the shortwave part of the spectrum. Another difference is the role of the peaks.

To illustrate the new model we give the asymptotic law for the evolution of the scale of clustering in a simple case of power law spectrum of linear density perturbations

$$\delta^2(k) \propto k^n, \text{ at } k \to 0.$$

Burgers' model predicts that the well known from linear theory law $M \propto a^{(\frac{6}{n+3})}$ is valid only in the cases of rather shallow spectra $-1 \leq n \leq 1$ (Gurbatov, Saichev, Shandarin, 1984, 1985). For spectral indexes $n > 1$ the scale of clustering grows with a limit law

$$M \propto a^{3/2}$$

independently on n, which is in conflict with the generally accepted result. The discussed model predicts stronger non-linear effects for initial spectra with indexes $1 < n <$

Summerizing this short discussion of a new approach to the problem of evolution of density perturbations at the non-linear stage and formation of LSS I would like to stress its advantages and disadvantages.

The proposed model gives a total qualitative picture of gravitational instability from linear stage up to infinite future (assuming that $\Omega = 1$). If the spectrum of linear density fluctuations has a cutoff at some scale the formation of LSS begins from the formation of pancakes of this scale. Later filaments and clusters form. In the course of time the mass moves from pancakes to filaments and from filaments to clumps the typical mass of which grows with time. Finally practically all the mass concentrates in clusters that continue to move and merge. In the frame of the proposed model one can in principle to calculate positions, peculiar velocities and masses of clusters.

The main disadvantage of the model is that at present form it cannot describe the inner structure neither pancakes nor filaments and clusters.

In fact the dynamics of the model does not depend on the assumption that initial perturbations are Gaussian thus it can be applied to any type of initial conditions provided that the main process is gravitational instability.

An important advantage of the model is its possibility to analyse the statistics of clumps of matter at arbitrary time without calculation of intermediate steps.

REFERENCES

Bardeen J.M., Bond J.R., Kaiser N., Szalay A.S. 1986, Ap.J. 304,15.

Burgers J.M. 1940, Proc.Roy.Neth.Acad.Sci. 43,2.

Burgers J.M. 1974, The Nonlinear Diffusion Equation, Reidel.

Doroshkevich A.G., Kotok E.V., Novikov I.D., Polyudov A.N. Sigov Yu.S., Shandarin S.F. 1980,MNRAS 192,321.

https://doi.org/10.1017/S0074180900136149 Published online by Cambridge University Press

Gurbatov S.N., Saichev A.I., Shandarin S.F. 1984 Preprint No 155, Inst.Appl.Math., Moscow.
Gurbatov S.N., Saichev A.I., Shandarin S.F. 1985 Dokl. Sov. Acad. 285,323.
Peacock J.A., Heavens A.F. 1985, MNRAS 217,805.
Peebles P.J.E. 1980, The Large Scale Structure of the Universe. Princeton Univ.Press.
Shandarin S.F. 1983, in "The Origin and Evolution of Galaxies". eds. B.J.T.Jones & J.E.Jones, Reidel, p.171.
Shandarin S.F., Doroshkevich A.G., Zeldovich Ya.B. 1983, Sov.Phys.Usp. 26,46.
Zeldovich Ya.B. 1970 Astron. Astrophys. 5,20.

https://doi.org/10.1017/S0074180900136149 Published online by Cambridge University Press