CORRESPONDENCE

(*To the Editors of the* Journal of the Institute of Actuaries)

DEAR SIRS,

Joint-life ultimate annuity values on the A 1949–52 *table*

Offices making net premium valuations on an ultimate basis by the A 1949–52 table will require ultimate annuity values for the calculation of joint-life net premiums and valuation factors. These are available only for equal ages in Vol. IV of the tables. It seems useful, therefore, to have a simple formula for deriving ultimate from select annuity values.

In the case of a mortality table under which the select period is one year, the following equations hold good:

(i) $$\frac{a_x}{a_{[x]}} = \frac{p_x}{p_{[x]}}$$

and $$\frac{a_{[x]} - a_x}{a_{[x]}} = 1 - \frac{p_x}{p_{[x]}} = \phi_x, \quad \text{say;}$$

(ii) $$\frac{a_{xy}}{a_{[xy]}} = \frac{p_{xy}}{p_{[xy]}}.$$

ϕ_x can be simply tabulated for the required range of ages and a table for ages 20–80 is appended.

Equation (ii) may be expressed in terms of ϕ as follows:

$$\frac{a_{xy}}{a_{[xy]}} = (1-\phi_x)(1-\phi_y) = 1-\phi_x-\phi_y+\phi_x\phi_y.$$

This formula, as I have remarked, applies where the select period is one year. In the A 1949–52 tables the select period is two years, and to obtain very accurate results where both lives are old the coefficient of $\phi_x\phi_y$ should be altered to about 1·8. Bearing in mind, however, the question of quick and convenient calculation, I suggest that the formula should be:

(iii) $$a_{xy} = a_{[xy]}(1-\phi_x-\phi_y+2\phi_x\phi_y).$$

In this formula the term containing $\phi_x\phi_y$ may be omitted if both lives are under 60 and it may be considered permissible for valuation purposes to omit it altogether and to use the simple formula:

(iv) $$a_{xy} = a_{[xy]}(1-\phi_x-\phi_y).$$

The use of this formula will tend somewhat to overstate the net liability, but the overall error should not be large unless there is a high proportion of old lives.

In the following table, ultimate joint-life annuity values for selected pairs of ages, calculated by formulae (iii) and (iv), are compared with the true values. The comparison is made mainly on the basis of 2½ % interest, but some 4 % values for equal ages were calculated and are included in the table.

Ages $x{:}y$	$2\frac{1}{2}$% values of a_{xy}			4% values of a_{xy}		
	Formula (iii)	True value	Formula (iv)	Formula (iii)	True value	Formula (iv)
20:20	26·089	26·088	26·089	20·036	20·035	20·036
50:50	13·900	13·901	13·900	12·027	12·028	12·027
60:60	9·265	9·266	9·261	8·342	8·343	8·338
70:70	5·353	5·353	5·332	4·988	4·990	4·968
80:80	2·623	2·613	2·544	2·508	2·501	2·433
22:79	4·831	4·831	4·831			
32:79	4·825	4·824	4·824			
42:79	4·778	4·777	4·776			
52:79	4·620	4·620	4·613			
62:79	4·262	4·261	4·244			

Table of ϕ factors

Age x	ϕ_x	Age x	ϕ_x
20	·0007	50	·0049
1	·0007	1	·0056
2	·0007	2	·0063
3	·0008	3	·0070
4	·0008	4	·0078
5	·0008	5	·0088
6	·0008	6	·0099
7	·0008	7	·0110
8	·0008	8	·0123
9	·0008	9	·0137
30	·0008	60	·0152
1	·0008	1	·0169
2	·0009	2	·0187
3	·0009	3	·0208
4	·0009	4	·0231
5	·0010	5	·0256
6	·0010	6	·0284
7	·0011	7	·0315
8	·0012	8	·0347
9	·0013	9	·0384
40	·0014	70	·0425
1	·0016	1	·0470
2	·0018	2	·0518
3	·0020	3	·0571
4	·0023	4	·0629
5	·0026	5	·0694
6	·0029	6	·0762
7	·0034	7	·0837
8	·0039	8	·0919
9	·0044	9	·1007
		80	·1103

Yours faithfully,
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