(ii) If a lump of the alloy containing say 2 lbs. of copper and 3 lbs. of zinc be fused with other 4 lbs of copper and other 4 lbs. of zinc into another lump, then the second lump is "more coppery" than the first. Hence

$$\frac{2}{3} < \frac{2+4}{3+4},$$

or

$$\frac{a}{b} < \frac{a+c}{b+c} \text{ if } a < b.$$

Similarly

$$\frac{a}{b} > \frac{a+c}{b+c} \text{ if } a > b.$$

Also we may illustrate the inequality between

$$\frac{a}{c} \text{ and } \frac{a-c}{b-c}.$$

(iii) If a lump of an alloy of copper and zinc containing a parts of copper and b parts of zinc be fused with a lump of a second alloy of copper and zinc containing c parts of copper and d parts of zinc, then the lump so formed will contain $(a+c)$ parts of copper and $(b+d)$ parts of zinc. If $a/b \neq c/d$, then the new alloy is "less coppery" than the one and "more coppery" than the other. That is $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$. Similarly, if we suppose n alloys to be fused into one, we see that

$$(a_1 + a_2 + \ldots + a_n)/(b_1 + b_2 + \ldots + b_n)$$

lies between the least and greatest of the fractions a_1/b_1, a_2/b_2, etc.

Graphical illustrations of these propositions are also instructive.

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Proof of a Property of Simson's Line.—The following is a slightly simplified version of a well-known proof of this theorem :—

The Simson's Line of P, with respect to $\triangle$ABC, bisects PO, where O is the orthocentre of the triangle.

Draw PQ perpendicular to BC, meeting BC in X, and circle ABC in Q. XY, the Simson's Line of P, meeting OA in Y, is parallel to QA.

If O_1 is the orthocentre of $\triangle PBC$, then

PO_1 = twice the distance of the circumcentre from BC,

$= AO$.

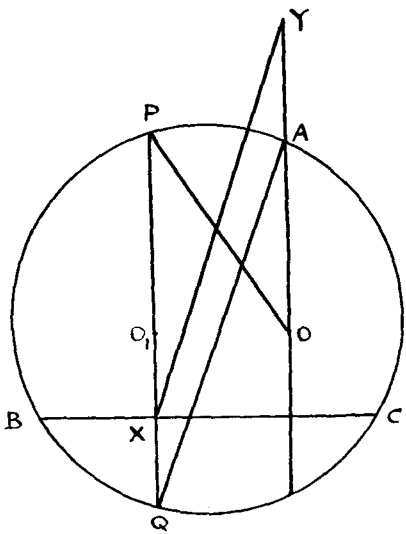
Also

$$O_1X = XQ = YA.$$

$\therefore$ $PX = YO$; and these are parallel lines.

$\therefore$ PXOY is a parallelogram.

$\therefore$ XY bisects PO.



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