Integrate

$$\int uvdx = u\int vdx - \int\left\{\frac{du}{dx}\int vdx\right\}dx \;\ldots\ldots\ldots\ldots\ldots(2)$$

which is the usual formula.

If, however, in (1) $u\int vdx$ be a constant its differential coefficient is zero, and on integration in (2) *it remains zero.* In other words the formula for integration by parts no longer holds true, if the parts be so chosen, that the first term of the result, which is usually memorised as "the one part into the integral of the other," is a constant. This exception is not noted in the text-books; and the error to which it gives rise is not in all cases so evident, as in the example chosen above.

J. R. MILNE.

Vectors as applied to Problems in Velocity.—A velocity is a vector quantity, for it possesses magnitude and direction. It is, of course, necessary to make clear in every case what given point (moving or not) is considered as fixed for the time being, to which the velocity of the moving point (so-called, though it may only be relatively moving) is referred.

If P is the point considered fixed, and the point Q is moving relatively to P, we may denote the velocity of Q relative to P by the symbol $V_{Q/P}$.

The velocity of P relative to Q will be expressed by $V_{P/Q}$. Obviously $\overline{V}_{P/Q} = -\overline{V}_{Q/P}$.

In kinematics two kinds of problem present themselves. In the first and simplest it is required to find the resultant of a number of mutually independent velocities, all of which are referred to the same fixed point, usually the Earth.

Suppose we have n velocities, each referred to the same point E, viz., $V_{1/E}$, $V_{2/E}$, ..., $V_{n/E}$. Then if $V_{0/E}$ represents the resultant velocity we have $\overline{V}_{0/E} = \overline{V}_{1/E} + \overline{V}_{2/E} + \ldots + \overline{V}_{n/E}$, the summation being carried out by the method of the Vector Polygon.

Such problems are so simple that they hardly demand any special notation. But let us now consider the case when the velocities involved are mutually dependent.

Example 1. A boat (B) is heading at right angles to the current (C) of a river with a velocity of 8 miles an hour. The current is flowing at the rate of 4 miles an hour. Find the resultant velocity of the boat. Denoting the Earth by E, we have $V_{B/C} = 8$, $V_{C/E} = 4$; and we have to find $V_{B/E}$. Draw (and note the letters used) BC, CE

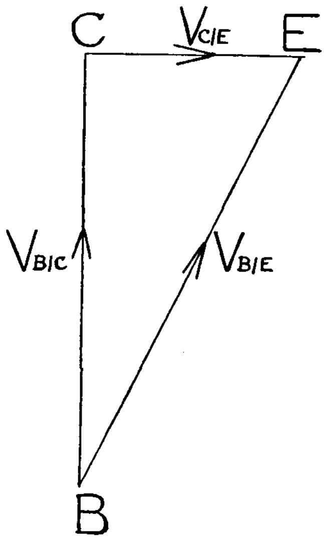

Fig. 1.

to represent $V_{B/C}$, $V_{C/E}$ in magnitude and direction. Then BE represents $V_{B/E}$ in magnitude and direction.

We have therefore the important theorem

$$\overline{V}_{B/E} = \overline{V}_{B/C} + \overline{V}_{C/E},$$

which may be extended indefinitely. Thus

$$\overline{V}_{A/X} = \overline{V}_{A/B} + \overline{V}_{B/C} + \ldots + \overline{V}_{V/W} + \overline{V}_{W/X}.$$

Example 2. Two trains, A and B, are moving with given velocities in given directions. Find the apparent velocity of A as seen from B. We have $\overline{V}_{A/B} = \overline{V}_{A/E} + \overline{V}_{E/B}$.

Now $\overline{V}_{E/B} = -\overline{V}_{B/E}$ or $= \overline{V}_{B/E}$ reversed, and is therefore known.

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Draw AE, EB to represent $V_{A/E}$, $V_{E/B}$ (*i.e.* $V_{B/E}$ reversed), in magnitude and direction.

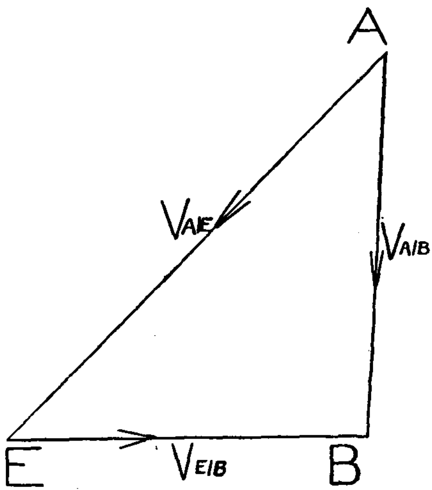

Fig. 2.

Then AB represents $V_{A/B}$ in magnitude and direction.

This method seems in some ways better than the application to both A and B of a velocity equal and opposite to that of B. In particular it has the advantage that by it problems involving both resultants and relative velocities can be solved at once, instead of having to be dealt with step by step.

Example 3. Rain (R), which is falling with a vertical velocity 20, is carried to the left by the wind (W) with a velocity 10. A man (M) walks to the right with a velocity 8. Find the velocity and direction with which the rain would strike him.

Here $V_{R/W} = 20$, $V_{W/E} = 10$, $V_{M/E} = 8$.

$$\text{Now } \overline{V}_{R/M} = \overline{V}_{R/W} + \overline{V}_{W/E} + \overline{V}_{E/M}$$

$$= \overline{V}_{R/W} + \overline{V}_{W/E} + \overline{V}_{M/E} \text{ reversed.}$$

Draw RW, WE, EM to represent in magnitude and direction $V_{R/W}$, $V_{W/E}$, $V_{E/M}$. Then RM represents in magnitude and direction the velocity of the rain relative to the man.

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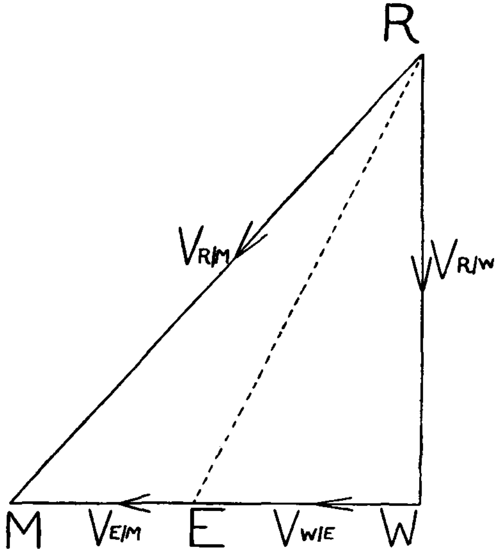

Fig. 3.

Notice that, though we did not set out to find it, we have found $V_{R/E}$, which is completely represented by RE. Thus the advantage of the notation employed for the ends of the vectors, as well as for the vectors themselves, will be seen.

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