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GENERALIZED CONVEXITY AND DUALITY IN MATHEMATICAL PROGRAMMING

TERENCE WEIR

Consider the constrained minimization problem

(P): minimize $f(x)$ subject to $g(x) \geq 0$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^m$ are differentiable functions. P. Wolfe considered the following dual problem to (P) when f is convex and g concave

(D): maximize $f(u) - y^t g(u)$ subject to $\nabla f(u) = \nabla y^t g(u)$, $y \geq 0$

and established weak and strong duality theorems under suitable assumptions. Several authors proved converse duality results for convex primal programs (P) and Wolfe's dual (D). O.L. Mangasarian first showed that some of these converse duality results may be extended to primal problems (P) with f pseudoconvex and g quasiconcave. However, by means of a counterexample, he also showed that the weak and strong duality theorems for (P) and (D) are not amenable to such extensions.

In this thesis a duality theory is developed with emphasis on weakened convexity. In Chapter 2, several duals to (P) (different from the Wolfe dual) are given and duality theorems for these new duals are established under weakened convexity conditions. For example, it is shown that the problem

maximize $f(u)$ subject to $\nabla y^t g(u) = \nabla f(u)$, $y^t g(u) \leq 0$, $y \geq 0$,

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is a dual to (P) if f is pseudoconvex and $y^t g$ is quasiconcave. Several examples are given where the Wolfe dual fails but where the new duals are applicable. Duals with weakened convexity conditions are also given for minimization problems with both inequality and equality constraints. As well, converse duality results are established. Also duality theorems with weakened convexity conditions are given for a new pair of symmetric dual problems.

In Chapter 3 duality for fractional programs is discussed. Consider the problem of minimizing $f(x)/g(x)$ subject to $h(x) \leq 0$ where f and g are differentiable functions from $\mathbb{R}^n$ into $\mathbb{R}$ and h is a differentiable function from $\mathbb{R}^n$ into $\mathbb{R}^m$. Under the assumptions that f is convex and nonnegative, g is concave and positive (or f convex and g linear and positive) a number of duality results can be found in the literature. Here a number of different duals are given that allow the weakening of these convexity requirements. A specific example is given where h is quasiconvex and not convex for which standard fractional programming duality does not apply but for which the duals proposed here do hold. Weak, strong and converse dual theorems are given for these new dual programs.

Chapter 4 is devoted to second and higher order duality. Previous results in these areas are extended and new duals with weakened convexity conditions are given. A specific example is given showing a possible application of second order duality. As well, the concept of partial second order duality is introduced and several partial second order duals given.

In Chapter 5 some of the results of Chapter 2 are extended to problems with polyhedral cone constraints in real space, and we also briefly discuss duality using the generalized convex (or invex) functions recently investigated by M.A. Hanson.

In the final chapter of this thesis, duality for continuous time programming problems, variational problems and control problems are discussed. New duals with weakened convexity conditions are given for these problems.

22 Daniel Street, Burwood,
Victoria 3125, Australia.