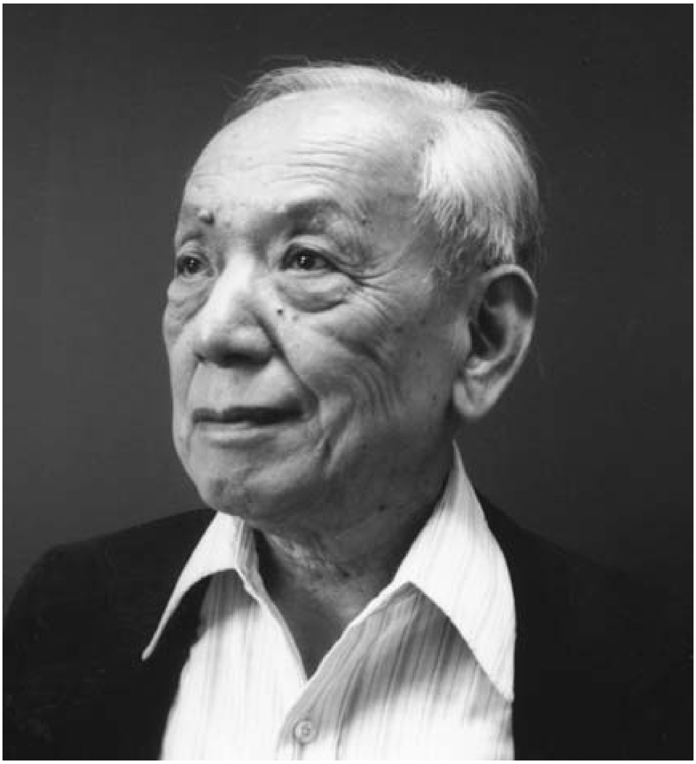
Bull. London Math. Soc. 38 (2006) 507–519 © 2006 London Mathematical Society
doi:10.1112/S0024609306018595

OBITUARY

SHIING-SHEN CHERN 1911–2004



Arlene Baxter

Shiing-Shen Chern was a towering figure in mathematics, both for his contributions to differential geometry and as a source of inspiration and encouragement for all mathematicians, and particularly those in China. Born in the final year of the Qing dynasty, and educated at a time when China was only beginning to set up Western-style universities, he lived to preside over the 2002 International Congress of Mathematicians in Beijing. He was a co-founder of MSRI in Berkeley and its first Director in 1981, and also set up the Nankai Institute for Mathematics in 1985. The London Mathematical Society elected him as an Honorary Member on 16th May 1986.

1. *Life*

Shiing-Shen Chern was born on October 26th, 1911 in Jia Xin, Chekiang Province, in China. His father practised law and worked for the government. At Fu Luen Middle School in Tsientsin he first showed his mathematical ability by doing all the exercises in classical English textbooks on algebra and trigonometry, and then went at the age of fifteen to Nankai University — a one-man Department run by Li-Fu Chiang, a student of Coolidge. As a result he studied a great deal of geometry, reading Coolidge, Salmon, Castelnuovo and Staude. He then became a postgraduate in 1930 at Tsinghua University in Beijing (or Peiping as it was then called) and came under the influence of Dan Sun, one of the few mathematicians in China writing research papers. During this period he became seriously interested in Sun's subject, projective differential geometry, and studied in detail the works of Wilhelm Blaschke. It was also at Tsinghua that he met his wife Shih-Ning, the daughter of a professor. After Blaschke paid a visit to Tsinghua in 1932 and lectured on differential-geometric invariants, Chern won a fellowship to study with him in Hamburg for two years. In 1936 he received his DSc there for work on the theory of webs. While in Hamburg, he also attended the lectures of Kähler on what is now called Cartan–Kähler theory, and the following year he spent in Paris studying with Cartan himself.

The year in Paris was formative, because his interaction with Cartan introduced Chern to notions which are now standard in differential geometry, but which few people understood then. As he later remarked: "Without the notation and terminology of fibre bundles, it was difficult to explain these concepts in a satisfactory way", but clearly Chern absorbed Cartan's methods, as his papers of the time show. Cartan lectured on exterior differential systems, and Chern visited him at his home every other week for discussions. The Julia seminar that year was also devoted to Cartan's work, and Chern there met André Weil and other young French mathematicians.

In the summer of 1937, he crossed the Atlantic, the United States and the Pacific to take up the position of Professor at Tsinghua, only to be faced with the outbreak of the Sino-Japanese war. His university had moved, together with Peking and Nankai University, to Kunming. There, despite the deprivations of war and the lack of communication with the outside world, he found the time to pore over Cartan's work and form his own vision of where geometry should be going. He also taught many students who later made substantial contributions in mathematics and physics — one such was C. N. Yang of Yang–Mills fame. He was already known internationally when in 1943 he was able, via a chain of military flights through India, Africa, Brazil and Central America, to make his way to the Institute for Advanced Study in Princeton.

In Princeton, Weyl and Veblen were well aware of his papers. It was a quiet place at the time because of the absences due to war work, but he made contact with Chevalley and Lefschetz and also with Weil in nearby Lehigh University. They had a common background in having studied Cartan and Kähler and, in Weil's words ⟨**5**⟩, "...we seemed to share a common attitude towards such subjects, or towards mathematics in general; we were both striving to strike at the root of each question while freeing our minds from preconceived notions about what others might have regarded as the right or the wrong way of dealing with it." Discussions with Weil revealed the properties of characteristic classes, all expressed in terms of sphere bundles, as vector bundles were as yet unheard of. Weil explained to Chern the Todd–Eger classes, derived in the spirit of Italian geometry. These discussions provided the foundation of Chern's most famous work on characteristic classes, and at the time they emerged in his new intrinsic proof [**25**] of the general Gauss–Bonnet theorem — by his own account, one of his favourite theorems ⟨**4**⟩.

When the war ended in 1945, Chern began his return to China but only reached Shanghai in March 1946. There, he was given the task of setting up an Institute of Mathematics as part of the Academia Sinica. He did this very successfully, nurturing several outstanding mathematicians, but Nanjing, where the Institute was located, was getting more and more dangerous in the turmoil of the civil war. Weil, now in Chicago, and Veblen and Weyl in Princeton, were getting concerned about his fate and both Chicago and the Institute offered him visiting positions, and then a full professorship at Chicago. So in 1949 he returned to the USA, this time with his family, to spend most of his working life there.

Chern's work in Nanjing and Chicago became quite topological, there being several papers on the topology of fibre bundles, some with Spanier, as well as differential geometric applications. His talk at the 1950 International Congress [**50**] shows how far the interaction of differential geometry and topology had come by this time. This is a thoroughly modern statement, a million miles from the work of fifteen years earlier. Chern's students in Chicago included K. Nomizu, L. Auslander and J. Wolf.

In 1960, Chern took up a Professorship in Berkeley — an expanding department and a milder climate made the move attractive. He immediately started a differential geometry seminar (which of course continues to this day), and he attracted visitors both young and old. P. A. Griffiths started his collaborations as a graduate student 'sent to learn from Chern', as did J. Simons, M. do Carmo and many others. His own PhD students included S.-T. Yau, A. Weinstein, P. Li, J. Millson and many more.

Although approaching retirement, in 1978 Chern, together with I. M. Singer and C. Moore, prepared a response to the NSF's request for proposals for a Mathematical Institute to reflect the "need for continued stimulation of mathematical research" in an environment that regarded American mathematics to be in a 'golden age'. Approval came in 1981, and Chern became the first Director of MSRI from 1982 until 1985. It was of course a huge success, but Chern continuously supported it in many ways, not least from the proceeds of his 2004 Shaw prize. The new building at MSRI is, naturally, named Chern Hall.

Chern's interest in Chinese mathematicians continued throughout his years in the USA. He had an aim: "Chinese mathematics must be on the same level as its Western counterpart, though not necessarily bending its efforts in the same direction." During the 1980s, he initiated three developments in China: an International Conference on Differential Geometry and Differential Equations, the Summer Education Centre for Postgraduates in Mathematics, and the Chern Programme, aimed at organizing Chinese postgraduates in mathematics for further study in the United States. In 1984, he was invited by China's Ministry of Education to return to his alma mater, Nankai University, to create the Nankai Research Institute of Mathematics. A residence, 'The serene garden', was built by the University for Chern, and he and his wife lived there every time they returned to China. While he was the Director, he invited many overseas mathematicians to visit; he donated more than 10,000 books to the Institute, and his $50,000 Wolf Prize to Nankai University.

In 1999 he finally returned to China for good, and the MSRI held a farewell party for him, at which Chern said: "The study of mathematics should be an undertaking of youngsters. There's nobody else my age that's still working on frontier research in mathematics around the world. I have a simple belief: it's that I still want to do something for the development of mathematics during the remainder of my life." In fact, he did continue to do mathematics, and just before his death was grappling with an old problem about the existence or otherwise of a complex structure on the 6-sphere, but perhaps the best testament to his achievement in his final years was sitting next to President Jiang Zemin in the Great Hall of the People, at the opening of the 2002 International Congress of Mathematicians.

Chern received many awards for his work, including the US National Medal of Science in 1975, the Wolf Prize in Mathematics in 1983, and the Shaw Prize in 2004. He was elected a Foreign Member of the Royal Society in 1985.

Shiing-Shen Chern died in Tianjin, China on December 3rd 2004, aged 93. His wife of 61 years, Shih-ning, had died four years earlier. He is survived by a son, Paul, and a daughter, May Chu.

2. *Mathematical work*

2.1. *Characteristic classes*

Chern's proof of the general Gauss–Bonnet theorem [**25**] was a pivotal event in the history of differential geometry, not just for the theorem itself but for what it led to — in particular, the *Chern classes*. Recall the classical theorem of the same name for a closed surface in $\mathbb{R}^3$: it states that the integral of the Gaussian curvature is 2π times the Euler characteristic. This link between curvature and topology has several features: one is Gauss's *theorema eregium*, which says that the Gauss curvature, while ostensibly defined by the second fundamental form, which measures the way in which the surface sits in Euclidean space, is in fact intrinsic and determined by the first fundamental form, or metric, and its derivatives. So, clearly, whatever its integral is depends only on the intrinsic geometry. On the other hand, there is a very natural and useful extrinsic interpretation of this integral as the degree of the Gauss map: the unit normal to the surface at each point defines a map to the 2-sphere, and its topological degree is the invariant. The problem was to extend this result to (even-dimensional) manifolds in higher dimension. In 1926 H. Hopf had generalized the Gauss map approach to hypersurfaces in $\mathbb{R}^n$, but to put the intrinsic problem in context, one should recall that even the definition of a manifold was only formulated correctly by Whitney in 1936, and Cartan even in 1946 considered that "the general notion of manifold is quite difficult to define with precision". Basic properties such as embedding in Euclidean space, or existence of triangulations, had not been established. Allendoerfer and Weil ⟨**1**, **2**⟩ had in 1943 given a proof for manifolds embedded in Euclidean space with higher codimension: this used Weyl's formula for the volume of tubes: in modern language this can be interpreted as a generalization of the Gauss map from the unit normal bundle to a sphere. It was still an extrinsic proof, but the integrand was recognized to be intrinsic.

Chern's proof was entirely intrinsic — he used the unit tangent sphere bundle of the manifold and identified a natural differential form α on it. Its exterior derivative $d\alpha$ has a number of terms, one of which is the correct curvature integrand on the manifold itself. He then applies Hopf's theorem relating the index of a vector field to the Euler characteristic — the vector field in modern language gives a section of the sphere bundle outside the singular points, and Chern shows that the extra terms in $d\alpha$ do not contribute in the integral.

The novel content came from studying the intrinsic tangent sphere bundle, and using the exterior differential calculus that Chern had learned at the hands of Cartan. It provided a link between topology and differential geometry at a time when the very basics of the topology of manifolds were being laid down. In fact, he wrote several papers on the geometry and topology of fibre bundles [**32**, **38**, **42**–**44**], at the same time and often independently of topologists such as Steenrod, but he was always interested in geometrical interpretations. Given the focus over the past 25 years in four-dimensional Riemannian geometry, it is interesting to see Chern introducing 'a new topological invariant' for four-manifolds [**30**], without knowing that it is the signature, and in [**53**] discussing the formula relating this invariant and the Euler characteristic in what is now called the 'self-dual case'.

The successful attack on the Gauss–Bonnet theorem led him to study the other invariants of sphere bundles, to see whether curvature could represent them. He started with Stiefel–Whitney classes, but their mod 2 property "seemed to be a mystery" ⟨**5**⟩, and Pontryagin classes were not known then, so he moved into Hermitian geometry and discovered the famous Chern classes [**32**], whose importance in algebraic geometry, topology and index theory cannot be overestimated.

Chern consistently wrote about connections and curvature, absorbing the Weil approach through invariant polynomials on the Lie algebra to generalize to principal bundles, and — once vector bundles had replaced sphere bundles — he gave in [**82**] perhaps the cleanest description of covariant derivatives and characteristic classes, at the same time as solving problems in higher-dimensional complex geometry.

Throughout his work on characteristic classes and curvature, Chern was always concerned with the geometry of forms living on fibre bundles. Perhaps this came from the recognition that his early work with Cartan was really concerned with this, though the language was not then available. In any event, he recognized that there was more than just the topological characteristic class to be obtained, and this emerged in a strong form in his work with J. Simons on Chern–Simons invariants. Nowadays, the Chern–Simons functional is an everyday tool for theoretical physicists, but in the first papers [**102**, **108**] it was related to problems of obstructions to conformal immersion.

2.2. *Geometrical structures, connections and differential equations*

Chern's early work was influenced by Blaschke and Cartan, and involves the consideration of differential geometries more general than Riemannian geometry, often associated with distinguished families of submanifolds. Some of this was motivated by attempts to extend general relativity — for example, Weyl geometry and path geometry. The latter considers a manifold which has a distinguished family of curves on it which behave qualitatively like geodesics: given a point and a direction, there is a unique curve of the family passing through the point and tangent to the direction. Veblen and his school in Princeton had worked on this, and it was through this work that they probably first heard of Chern. There is a close relation with projective structures, and there are curvature-type local invariants. Cartan studied the differential equation $y'' = F(x, y, y')$, where F is a polynomial in y' of degree three, by these methods, using a natural projective connection over a two-dimensional space, but the general case, and a higher-order equation studied by Chern in [**6**, **13**], involved a connection over a fibre bundle. Chern's papers follow Cartan very closely, with the then terminology of 'infinitesimal displacement of elements', which makes the subject difficult to understand nowadays.

Families of submanifolds of higher dimension were considered, particularly 'webs', which was Blaschke's subject when he originally visited China. In modern language, a web is a family of foliations in general position, but there are again local diffeomorphism invariants of curvature type. Later in his career, Chern collaborated with Griffiths [**115**, **116**, **118**] on a web-related problem in algebraic geometry — an algebraic curve of degree d in projective n-space meets a generic hyperplane in d points. By duality this gives a d-web of hyperplanes in the dual space.

Curvature invariants in another aspect of holomorphic geometry came up in his work with J. Moser [**110**] on the geometry of real hypersurfaces of $\mathbb{C}^n$, picking up on a problem once considered by Cartan.

These higher-order connections are only gradually being understood nowadays, but they were undoubtedly formative in Chern's mathematical development, in particular in formalizing geometrical objects as connections on intrinsically defined bundles far more general than the tangent bundle.

The study of these connections, especially when formulated in the language of exterior differential systems, leads continuously into the Cartan–Kähler theory, and Chern has written a number of papers on this [**133**, **147**, **156**]. When, in the mid-1970s, soliton equations such as the KdV equation, together with its Bäcklund transformations, began to be studied in this way, he was well-prepared to apply these methods [**119**, **124**, **132**, **144**]. His knowledge of classical differential geometry, not unexpectedly, also enabled him to recognize the geometrical origins of the sinh–Gordon equation [**128**].

2.3. *Euclidean geometry*

The classical differential geometry of surfaces in Euclidean space still carries unsolved problems, and most differential geometers are attracted to some aspect of this — Chern was no exception. His main interest was in global properties, and in particular the use of holomorphic methods. It is well known that any metric in two dimensions can, by choosing *isothermal coordinates* (x, y), be written in the form $h(dx^2 + dy^2)$, and then the complex parameter $z = x + iy$ gives a surface in $\mathbb{R}^n$, the structure of a Riemann surface. The proof of this, with appropriate regularity conditions, was somewhat obscure until Chern gave an elementary proof using the Cauchy kernel [**55**, **57**] and then put the method to use in globally characterizing the sphere among surfaces where there is a functional relationship between the mean curvature and the Gaussian curvature [**58**].

The holomorphic aspect came to the fore also in his proof in [**81**] that the Gauss map of a minimal surface in $\mathbb{R}^n$ (which goes into the Grassmannian $G(2, n)$, a complex manifold) is antiholomorphic. Using this, he generalized to higher dimensions the Bernstein theorem that says that a minimal graph $z = f(x, y)$ defined on the whole of $\mathbb{R}^2$ must be a plane.

Another Euclidean area of research linking curvature and topology is the generalization by Chern and Lashof [**62**, **66**] of Fenchel's theorem of 1929, that the integral of the curvature of a closed curve is at least 2π. They use the same Gauss map as Allendoerfer and Weil for a manifold in $\mathbb{R}^n$, and pull back the *absolute value* of the volume form — for Gauss–Bonnet in $\mathbb{R}^3$ the integrand changes sign. In this case they obtained a lower bound for the integral in terms of the sum of the

Betti numbers, and they discuss the cases where equality holds. This generated a whole area of study of *taut* and *tight* submanifolds.

2.4. *Other contributions*

In a life as long and full as Chern's, there are many more, highly significant, contributions — on holomorphic mappings, minimal submanifolds, G-structures and Hodge theory. He also returned to some favourite themes over the decades. One was Blaschke's use of integral geometry and generalizations of the attractive Crofton's formula which measures the length of a curve by the average number of intersections with a line [**14**, **16**, **18**].

Another was the subject of Finsler metrics. In a retrospective millennial paper 'Back to Riemann' [**185**], he pointed out: "In 1948 I published a paper solving the problem of equivalence of Finsler manifolds . . . the paper was summarized in Rund's book and has been otherwise completely ignored." Riemann originally suggested an arbitrary norm on the tangent space, but decided to consider only one coming from an inner product because the calculations were simpler. Chern's exposure to other types of geometries and connections other than the Levi–Civita connection led to a number of papers exploring this geometrical structure, and (working together with with Bao and Shen) to the book [**184**]. There is indeed now a certain resurgence in the area, and it demonstrates once again the breadth of Chern's view of geometry, and his ability to isolate new and interesting developments.

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Mathematical Institute
24–29 St Giles
Oxford, OX1 3LB
United Kingdom

N. J. Hitchin