On a paper by Copson and Ferrar

By A. Erdélyi.

[*Extract from a letter to W. L. Ferrar.*]

Concerning your paper[1] written in collaboration with Professor Copson, I found the expansion of

$$F(\lambda) = \frac{1}{2\pi}\int_0^\infty e^{i\lambda \cosh t} \frac{\sin\theta}{\cosh t + \cos\theta} dt$$

for small values of λ in the following manner: The elementary substitution $\tan\frac{1}{2}\phi = \sin\theta/(e^t + \cos\theta)$ yields

$$F(0) = \frac{1}{2\pi}\int_0^\infty \frac{\sin\theta}{\cosh t + \cos\theta}\, dt = \frac{1}{2\pi}\int_0^\theta d\phi = \frac{\theta}{2\pi},$$

i.e. your equation (5.11). By reason of the differential equation

$$\frac{dF}{d\lambda} + i\cos\theta \, . \, F = -\tfrac{1}{4}\sin\theta \, . \, H_0^{(1)}(\lambda), \quad F(0) = \frac{\theta}{2\pi},$$

it follows that

$$F(\lambda) = e^{-i\lambda\cos\theta}\left\{\frac{\theta}{2\pi} - \tfrac{1}{4}\sin\theta \, . \int_0^\lambda H_0^{(1)}(\nu)\, e^{i\nu\cos\theta}\, d\nu\right\}.$$

Thus I have expressed $F(\lambda)$ as a *finite* integral. Expansion of the integrand yields at once the expansion of $F(\lambda)$ for small values of λ.

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[1] *Proc. Edinburgh Math. Soc.* (2), **5** (1938), 159-168.

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