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Long-term integration error of KS regularized orbital motion

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Abstract. We show the effectiveness of the KS regularization and the method of variation of parameter for integrating of the perturbed two body problems.

The numerical integrations are the strong tools for celestial mechanics. Developing the highly accurate integration schemes[1] has mainly devoted to an reduction of integration error. However we showed it is effective to transform equation of motion (EOM) by KS (Kustaanheimo-Stiefel) regularization and it reduce the integration error, because the EOM is rewritten in the form of perturbed harmonic oscillator. In the KS form, the independent variable is not t but the fictitious time s.

We found that the error growth of the position is proportional to s, not depending on the types of perturbation, the integrator, nor the nominal eccentricity. The physical time must be calculated by the integration because of the time transformation. The error of physical time grows linearly when the harmonic oscillator parts of EOM are integrated by the time symmetric integrators[2], while to s^2 integrating the same parts by the traditional ones.

Further we discovered that the KS regularization evades the stepsize resonance/instability of symmetric multistep method for the special second order ordinary differential equations (ODEs) when integrating the Kepler problem, and we indicated that the harmonic oscillator potential can only avoid the that resonance/instability.

Although we can perform the fast and highly accurate integration in the condition, $\Delta x \propto t$, the KS regularization have limits; the symmetric multistep method for special second order ODEs cannot deal with the acceleration depending on velocity v, and that for general first order ODEs often faces the numerical instability.

We considered the application of the method of variation of parameter (MVP) to the KS regularization[3]. KS element is defined as the amplitudes and

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[1] Symplectic integrator and symmetric multistep method

[2] Leapfrog and symmetric multistep method

[3] Stiefel *et al* (1967), and Stiefel &Scheifele(1971)

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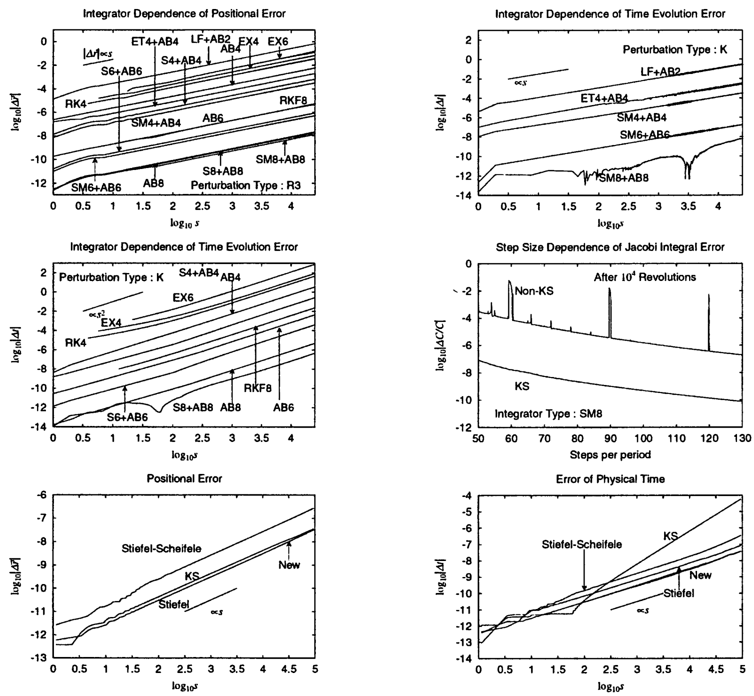

phases of harmonic oscillator. We confirmed that the MVP reduces the integration error of both the position and physical time and it grows only linearly with respect to s for the any kind of perturbation. Since Stiefel's KS element is not complete in the sense the element of physical time is not given, we introduced a time element and determined the third set of KS element. Unfortunately, this good property of KS regularization and its MVP fail in the general N-body problem because the fictitious time is proper to each body. Therefore the KS regularization and its MVP are effective to study the long term behavior of perturbed two body problems.

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