

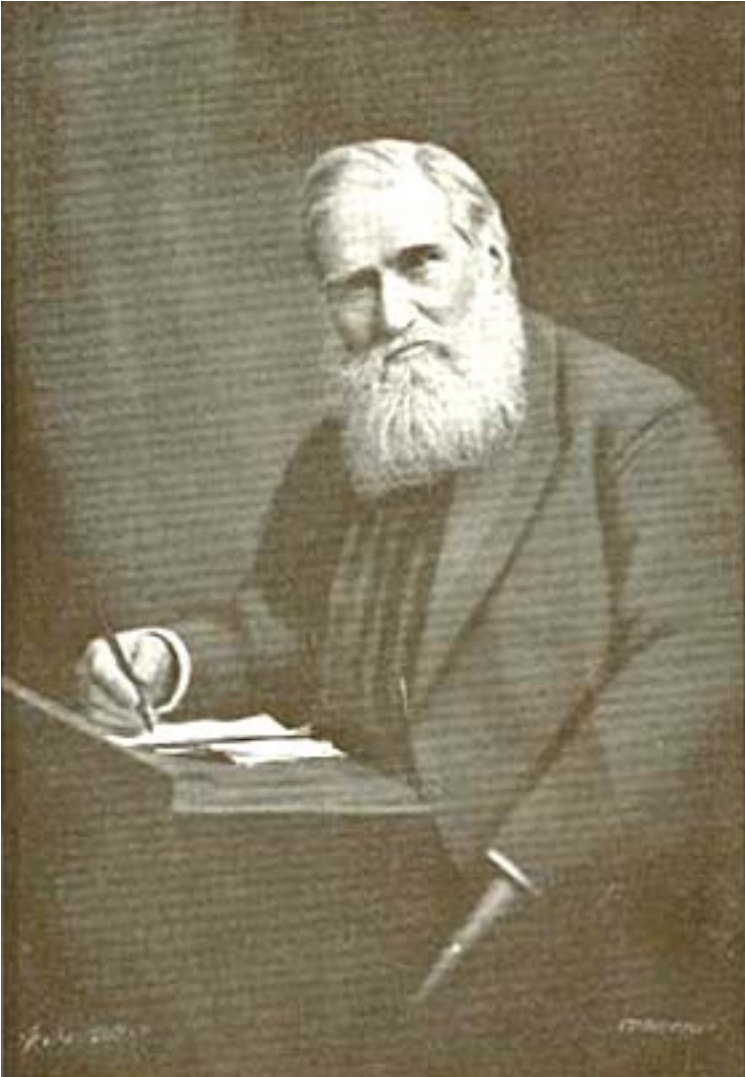
JOURNAL
OF THE
INSTITUTE OF ACTUARIES.

MEMOIR
OF
Thomas Bond Sprague, M.A., LL.D.,

*Fellow and Past President of the Institute of Actuaries;
Honorary Fellow and Past President of the Faculty of Actuaries in Scotland;
Fellow of the Royal Society of Edinburgh, &c., &c.
Born 29 March 1830. Died 29 November 1920.*

DR. SPRAGUE remained in complete retirement from all matters actuarial for fully twenty years before his death, and it is therefore difficult for the present generation to realize how unrivalled was his supremacy forty to sixty years ago. The young men of those distant days stood afar off and worshipped; and yet he was very approachable and considerate, and helpful to any who sought his assistance or advice. Once a friend he was always a friend, and generous in all his thoughts, and words, and deeds. In his private affairs it was the same, and for many years he was liberal with his means in giving assistance to deserving cases which came under his notice. There are not a few now in comfortable circumstances who have to thank him for setting them on their feet in times of financial stress, and thus enabling them to make a success instead of a failure of their lives.

Dr. Sprague was a man of action, whose actions were



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governed by theories formed after viewing the questions impartially, from various sides, and which were then tested practically, and proved to be sound. He possessed a great intellect, almost irresistible force of character, and unwearying industry, all combined with straightforward honesty of purpose, and candour in everything he said, or wrote, or did; and he had complete self-confidence, without self-conceit or undue self-esteem. He formed strong opinions, which he maintained with outspoken vigour against all opponents; but he was always willing to reconsider these opinions when necessity arose as a result of further investigations; and, when convinced that to change an opinion was the right thing to do, he frankly acknowledged the change, and was ready to state the reason why. It must be admitted, however, that he was sometimes rugged, and a little aggressive, and not always easy to get on with; and his earnestness led him on rare occasions to be perhaps over-vehement in his expressions. He had no thought of being personally discourteous, or of hurting the susceptibilities of those to whom he was for the moment in opposition. The opinions were attacked, and not the persons. These characteristics come out clearly in his voluminous writings, and more particularly in his Presidential Addresses.

During his long connection with the Institute of Actuaries he was to a large extent its guiding spirit, and he did much to mould its destinies, and to bring it up to its present position of world-wide influence. In saying this, however, we cannot forget the services in the same direction of many other eminent men, whom we have always been proud to honour; and, if any one of these should be individually mentioned, he is Charles Jellicoe, the second President, and the first Editor of the *Journal*, of whom Dr. Sprague himself said in his first Presidential Address, that the Institute and the profession owed more to him than to any other single person.

In preparing this memoir, an effort has been made, not so much to review Dr. Sprague's writings, as to educe from them his character, and to make THE MAN stand forth as he was, so that even those may see him who never had the privilege of meeting him, and of associating with him: but the task has not been an easy one.

Thomas Bond Sprague was born on 29 March 1830, and was the eldest son of Thomas Sprague of London, wholesale stationer. He was educated at Tarvin Hall, a private school

near Chester, under the Head Mastership of Dr. John Brindley. While at school, he distinguished himself so much in mathematics that Dr. Brindley advised his father to send him to Cambridge, and he accordingly entered St. John's College as a Sizar. At the end of his first year he became a Proper Sizar, and in consequence of the excellent places he took in the College examinations, he became in due course a Scholar.

In 1853 he achieved the highest academic honours possible in mathematics, being Senior Wrangler and First Smith's Prizeman; and in the same year he was elected a Fellow of St. John's College, and appointed one of the College Lecturers. About this time he also qualified in law and was called to the bar, having in view the law as his future profession. But he became engaged to be married; and, progress in the law being slow, he sought a career which would be more immediately remunerative. He selected Life Assurance; and entered the Eagle Insurance Company in 1855 as a pupil of the late Charles Jellicoe, and thus became one of the distinguished group who, in after life, achieved great success.

After holding some minor appointments, Dr. Sprague was, for a short time, Actuary of the then Liverpool and London Assurance Company, until in 1861 he was appointed, at the age of 31, Actuary and Secretary of the Equity and Law Life Assurance Society, with which society he remained for twelve years. In 1873 the management of the Scottish Equitable Life Office became vacant, and the Directors sent a deputation to London to choose a manager; and, after careful consideration, they offered the post to Dr. Sprague, who accepted it. This involved his removal to Edinburgh, but it did not diminish his interest in the Institute of Actuaries, and he was very frequently in London to give his time and attention to Institute matters. In 1900, after being manager of the Scottish Equitable for twenty-seven years, he finally retired from active business life.

In 1893 the University of Aberdeen conferred on him the degree of LL.D., the diploma being dated the 6th of April of that year. Hence, in this memoir, the name and title "Dr. Sprague" are used throughout, for the sake of uniformity, and also because that designation is more familiar to the present generation of actuaries.

Dr. Sprague was elected Associate of the Institute of Actuaries on 25 February 1856, and was not long in giving evidence of his energies. His first paper, under date October

1856, is to be found in the *Institute Journal*, vol. vi, and is a short one on a certain method of distributing surplus. He was elected a Fellow of the Institute on 30 November 1857, and within a few months he contributed his second paper, dated July 1858, on the Terms upon which the Business of one Insurance Company may be equitably transferred to another. That paper appears in vol. vii of the *Journal*, and on subsequent occasions, when the failures of the Albert and of the European, in 1869 and 1872 respectively, had brought the question into prominence, he wrote other papers on analogous subjects.

Dr. Sprague was elected to the Council of the Institute for the first time in 1863, and he remained a member without break until his final retirement from business in 1900, a total period of thirty-eight years, which is a record. He became Editor of the *Journal* of the Institute in 1867, succeeding his former chief, Mr. Charles Jellicoe, who was the first Editor; and he retired from the editorship in 1883, having occupied the post for over sixteen years. The volumes of the *Journal* published during his editorship bear witness to his activity and zeal. Before retiring from the editorship, he prepared the index of the first twenty volumes of the *Journal*.

Dr. Sprague was elected President of the Institute, the eighth President, in 1882, and occupied the presidential chair for four eventful years, as, during his Presidency, the Institute received its Charter. In 1874 he was elected Fellow of the Faculty of Actuaries in Scotland, and was President from 1894 to 1896. No one else has occupied the Presidential chairs of both the Institute and the Faculty. He was also President of the Actuarial Society of Edinburgh on three occasions, 1874, 1882 and 1891. That Society was dissolved in 1901, its functions being taken over by the Faculty.

Dr. Sprague's unwearying devotion to the Institute of Actuaries, as embodying his own high ideals, comes out clearly in his first Presidential Address, delivered on 27 November 1882 (*J.I.A.*, vol. xxiv, p. 1). There is nothing selfish in it, but, on the contrary, an earnest striving for the raising of the status of the profession, and, through the profession, for the good of the world at large. The address consists, for the most part, of a brief but comprehensive history of the Institute from the preliminary discussions and meetings, which took place early in 1848, and its establishment by formal resolution adopted on 8 July 1848, until the time that Dr. Sprague was speaking, on

the eve of the granting of the Charter. That history is well worthy of study even at the present day.

There were great difficulties in starting the Institute on account of the opposition of a small minority of influential and respected men, who seem to have thought that professional matters should be kept private, and that publicity would be injurious to the members of the profession, by stimulating undue competition from outside quarters. It was through these men that the long delay arose in the granting of the Charter, but on this point Dr. Sprague says that, on the whole, in his opinion the delay had been beneficial, because the Institute had thus been left for a period of thirty-four years to develop itself by its own resources, without any external aid or encouragement, and had thus attained to a high and influential position.

Dr. Sprague looked upon the educational facilities provided by the Institute for its young members as one of its most important functions. He thought that all the young men connected with life assurance should be trained in the scientific department, although they could not all secure high appointments. Even without such high appointments they might be very useful members of the profession ; and, should they drift away from it to obtain posts elsewhere, their actuarial training would always be a great asset. It was, however, to the encouragement of publicity that Dr. Sprague attached the most importance, and he had no fear of competition. On this point he said :

“ Very much in consequence, I believe, of the teaching and example of the leading members of the Institute, it has become much more the practice for actuaries to work in the light of day, and to explain fully their processes, and the reasons which influence them. In the olden time I believe it was the custom for actuaries to conceal their methods, or only to reveal them to their most favoured pupils ; and, if an actuary had constructed a private mortality table, this was perhaps the most valued part of his stock-in-trade. These ideas, I think, may be fairly said to be now quite out of date.”

He adopted with high approval the opinions given by others in the very early days of the Institute. For instance, he quoted the late Mr. Peter Hardy, the father of the late Mr. R. P. Hardy who was a friend of so many of us, as saying :

“ The Institute was not originated in any selfish feeling of professional vanity. Its founders took no counsel together how to narrow the access to our profession, or how to exclude other competitors than ourselves from a fair share of its honours

or emoluments. It was originated with a far higher, a far nobler motive, in the honest hope of educating the future actuary, of improving among ourselves our present theories, and of enlarging the bounds of that science on which the safety of so many millions sterling depends, and in which the social happiness of so many thousand families is involved."

Also, he welcomed appreciation from actuaries of other countries, and quoted, as follows, the late Mons. E. Maas, Manager of the "Union" Company of Paris, a Corresponding Member :

"A few years ago men engaged in the same pursuits were envious of each other, and endeavoured to conceal all that could afford any improvement to their competitors. The English people, moved by a nobler spirit, did not fear to exhibit to all mankind the treasures of their industry, so that everyone might profit by their example, and take advantage in imitating them. The same feeling induced the Institute of Actuaries to convene the actuaries of all nations on the native ground of insurance, to participate in the discoveries of their science and the results of their experience."

Dr. Sprague himself followed the practice which he inculcated, and many volumes of the *Journal* contain the papers in which, whenever he discovered or worked out anything that he thought would be useful to others, he gave full details, and kept nothing back for his own private advantage.

The first great work of the Institute in which Dr. Sprague took part was the preparation of what are now known as the Institute of Actuaries' Life Tables. On 14 January 1862, the Council of the Institute of Actuaries appointed the following six gentlemen a Committee to carry through the investigations: Messrs. A. H. Bailey, Archibald Day, W. B. Hodge, Charles Jellicoe, T. B. Sprague, and Robert Tucker; and, after correspondence, the Managers and Actuaries in Scotland appointed six to co-operate with the London Committee. They were Messrs. David Chisholm, John M. McCandlish, James Meikle, Samuel Raleigh, Alexander G. Ramsay, and William Smith. To the name of each of these twelve gentlemen we must now prefix the words "the late", Dr. Sprague being the last of them to pass away.

The experience was that of twenty Offices, and was published in 1869, and at the Annual Meeting of the Institute on 5 June of that year, Mr. Samuel Brown, the then President, submitted an early copy of the volume of Experience, and, in doing so, specially referred to Dr. Sprague, and to Dr. Sprague only, for

his efficient help in preparing the experience. At the same time he announced that a scheme of notation was being prepared by the Council for general use, and again he mentioned Dr. Sprague, and Dr. Sprague only, as having given exceptionally valuable help. That scheme of notation was published in the volume of monetary tables based upon the experience, and has since, with extensions, been adopted by resolutions of International Actuarial Congresses as the notation to be used throughout the world. The volume of subsidiary and monetary tables was published in 1872, the life tables having been graduated by Mr. Woolhouse, and the subsidiary and monetary tables having been calculated by Mr. Peter Gray, whose name at that time was a household word among Actuaries.

The Albert failed on 13 August 1869, and the European on 12 January 1872; but, for some time before even the failure of the Albert, there had been ominous premonitions that the catastrophes were impending, and bills were introduced into Parliament whereby the promoters hoped to make such disasters impossible for the future. The Council of the Institute took a very active interest, and put forth strenuous efforts to guide the legislation in the right direction. The first bill was crude, and would have done more harm than good. The watchword of the Institute was "Liberty with Publicity": no interference by the State, but, instead, the publication in respect of each Company of summaries of its business, of the particulars for valuation, and of the principles followed, sufficient to enable outside actuaries to judge as to its real position. The result was the Act, passed on 9 August 1870, which, by general consent, is held to have worked well, and which has now been superseded by the Act of 1909 on very similar but somewhat extended lines as regards Life Insurance, but which also provides for Insurance business of all other classes.

Dr. Sprague was one of the foremost advocates of Liberty with Publicity, and on 25 April 1870, when the Bill was before Parliament, he read a paper (*J.I.A.*, vol. xv, p. 411) on "The Proper Method of Estimating the Liability of a Life Insurance Company." At that time there were influential advocates of legislation to impose on the Companies a strict net premium valuation, and to set up a standard of solvency; but against both of these suggestions Dr. Sprague took a firm stand, and in the paper above referred to he said regarding valuations:

“ In valuing by the net-premium method, the supposition is tacitly made that the expenses chargeable to a policy are equally spread over its existence. But this is very far indeed from the truth, as the expenses incurred in the first year of the policy's existence far exceed those of subsequent years. Indeed, by far the greatest part of the expenses of every Life Insurance Company are chargeable to the new business—in particular, all advertising, inspection of agencies, and medical fees, and a considerable proportion of the postage, directors' fees, salaries, and office expenses. For if a company decided, as some have done, on working out the existing business, on taking no new business, and closed its doors with the intention of working out the existing business as economically as possible, it is obvious that the expenses, exclusive of the commission, might be reduced to a very trifling sum. This reduced sum, or at all events such a sum as an established company would undertake to conduct the business for, is all that should be charged to the old business.

“ If we keep these considerations steadily in view, we shall see that the only case in which the net-premium method of valuation can be expected to lead to satisfactory results is when, from the magnitude of the business transacted, or other causes, the total expense of conducting the business, irrespective of commission, amounts only to a small percentage of the premiums. In all other cases we must seek for some other rule.”

He returned to this point, after fifteen years' experience of the Act, in his fourth Presidential Address, delivered on 30 November 1885 (*J.I.A.*, vol. xxv, p. 293). There he said :

“ The theoretical expression for the value of a policy takes no account of expenses of management ; and in adopting this formula as the basis of the valuation of a company, it is assumed that the expenses are in every case less than the uniform loading added to the uniform annual net premium. This is an assumption, however, which is not justified by the facts, both as regards the total business of most young companies, and as regards the new business of most old companies. It is therefore necessary, in order to get an appropriate formula to introduce into our calculations the amount of expense at which the new business is obtained ; and thus, for old and young companies alike, we conclude that the proper reserve to be made for the liabilities is less than that given by the ordinary net-premium method of valuation.”

There is nothing in the Act of 1870 about a standard of solvency, but Dr. Sprague took up this question in a paper on Legislation as to Life Insurance and Life Insurance Companies, read on 28 November 1870, just after the Act had been passed (*J.I.A.*, vol. xvi, p. 77). He there maintained :

“Firstly: That no legislative enactments whatever can succeed in absolutely securing the solvency of Life Insurance Companies.

“Secondly: That even if this could be done, it would be highly undesirable that it should be attempted.”

He supported these two propositions by lengthy and cogent reasoning, and summed up by saying:

“It is impossible to say that one particular estimate of the value of the liabilities is right, and all others wrong. It is of course easy to find limits above and below which difference of opinion ceases, so that if a Company with known liabilities possesses a certain amount of assets, all actuaries will agree in saying that it is solvent, while on the other hand if it possesses less than a certain other amount of assets, all actuaries will agree in saying that it is insolvent. But, these limits would be very wide; and if the Company possesses an intermediate amount of assets, probably some actuaries will say that in their opinion it is solvent, while others will say that in their opinion it is insolvent. The conclusion we arrive at therefore is, that the solvency of Life Insurance Companies being so much a matter of opinion, the Government cannot with propriety attempt to decide the question by laying down a legal standard of solvency.”

This may be thought to be ancient history, but even ancient history has sometimes a tendency to repeat itself: and we can see from what Dr. Sprague did in 1870, and from what he wrote on the subject in the *Journal* at the time, that he would have ever been on the alert, and that he would at all times have put forth all his powers to oppose legislation involving government control of Insurance Companies, any standard of valuation, and more particularly any standard of solvency.

He would have maintained that vigilance is imperative, but he would not have resisted all legislation. From his writings we know that he thought the Act of 1870 might have been carried further with advantage, and at the present day it is very likely he would have said, for instance, that it is quite proper to give wider powers to the Board of Trade than it possesses, but that the powers to be granted must be minutely scrutinized.

It has already been mentioned that, on account of the opposition of influential men, who had formed themselves into the then Actuaries' Club, there had been interminable delays in securing a Charter for the Institute. The question was, however, taken up again seriously, and at the Annual Meeting held on 3 June 1882, the meeting at which Dr. Sprague was elected to

his first Presidency, a resolution was passed on the recommendation of the Council to apply for a Charter, and thereupon an application was presented to the Privy Council on 15 September 1882. The application was signed by Dr. Sprague as President, and by the then four Vice-Presidents, Messrs. M. N. Adler, G. W. Berridge, C. J. Bunyan, and R. P. Hardy, who have all now passed away. It was found that the Actuaries' Club was still in opposition, and had lodged an objection to the granting of the Charter; but negotiations took place in a friendly spirit between the Council of the Institute and the Actuaries' Club, and the objections of the Actuaries' Club were overcome, and all the members of the Club, fourteen in number, were brought into the Institute as Fellows under the Charter. Not one of these fourteen is now with us, all having passed away. The Charter was granted on 29 July 1884, and bye-laws had to be prepared, and under the Charter the then Council was to remain in office until the bye-laws had been approved by the Privy Council. Hence the then Council remained in office unchanged for four years, 1882 to 1885 inclusive. The bye-laws were allowed by the Privy Council on 26 February 1886, and this was reported at the Annual Meeting held on 5 June 1886, under the Presidency for the last time of Dr. Sprague, who then retired from the chair.

A great deal is due to Dr. Sprague for the success of this application for the Charter, but he himself at the Annual Meeting in 1886 offered a very generous and just acknowledgment to the late Mr. William Sutton, who was himself subsequently President of the Institute. Dr. Sprague said that he was sure all would pay a tribute to the services of Mr. Sutton, who had done most to bring about the change in the constitution of the Institute. Mr. Sutton had drafted the Charter and bye-laws, which had then been carefully considered by the Council; and also his influence as Actuary to two Government Departments had been invaluable. Mr. Sutton at that time was Actuary to the Registrar of Friendly Societies, and also Actuarial Adviser to the Board of Trade; and he had found, under the Friendly Societies Act more especially, that it was very important that the position of actuaries should be defined and settled, so that unauthorized persons might not be permitted to practise. We may therefore say that the final success in obtaining the Charter was due in great part to Dr. Sprague and Mr. Sutton.

The reconciliation between the members of the actuarial

profession was complete, and later on the old Actuaries' Club and the Institute of Actuaries' Club, which had been formed from the leading members of the Institute on much the same lines as the Actuaries' Club, were fused, and continued under the old name of The Actuaries' Club.

Dr. Sprague was a great mathematician, and yet in comparatively few of his actuarial writings does much of the higher mathematics appear. He kept that in reserve, and preferred, whenever possible, to restrict himself to more simple ordinary algebra. Nevertheless he had an essentially mathematical mind, which it was his delight to apply to practical work. He possessed in rare degree what may be called mathematical instinct. He thought mathematically, and he spoke mathematically, and he wrote mathematically, even when there was but little appearance of mathematics on the surface. All this comes out clearly in his Presidential Address, delivered to the Actuarial Society of Edinburgh on 13 November 1874 (*J.I.A.*, vol. xviii, p. 403). The address was on the Usefulness of Mathematical Studies to the Actuary; and, on a first perusal, it seems to be almost absurdly elementary; but, on further acquaintance, it is found to contain deep wisdom. He says that the mathematician takes nothing, not even the simplest thing, for granted; and that by him language is used in a strict and precise manner. Words are, as it were, symbols, and sentences formulas; and hence the mathematician thinks clearly and accurately, and speaks and writes without ambiguity. All this presupposes what has been called above the mathematical instinct, which Dr. Sprague combined with profound mathematical knowledge; but here it may be remarked that, with mathematical instinct, combined with even only moderate attainments in the higher mathematics, a man will not be liable to fall into error. He will think for himself, and will avoid pitfalls by which the unwary may be entrapped. He will travel safely; whereas a man who has read a great deal of mathematical literature, and who has learned and remembered a great deal, but who is not endowed with the mathematical instinct, and who has not thought for himself, will be in danger; and may try, as it were, to force a round formula, very good in itself, into a square hole. He will sometimes fail to perceive when the abstruse methods which are at his finger ends are not really applicable to the case in hand; and he will be apt to use them recklessly, and under conditions where they cannot produce satisfactory results.

This is more particularly so in the case of actuaries, who have, as Dr. Sprague says, “to deal even more with figures than with formulas”, and who “must submit every formula to the test of actual numerical calculation”; and on this point he adds that :

“In many branches of applied mathematics, it has been too much the custom in treatises designed for the English student—especially, perhaps, for the student who is reading for mathematical honours at the universities—to disregard the numerical application of the formulas proved.”

At the time of the Address in 1874, there was not much in the way of higher mathematics required of actuarial students for their examinations; but Dr. Sprague looked forward to a change in that respect, and he wrote :

“It can scarcely be said that the extraction of the facts (for a mortality experience) is a process for which much mathematical knowledge is required; but when the facts have been extracted, the process of adjusting them so as to deduce a satisfactory mortality table, that shall proceed without any abrupt changes, and shall yet not deviate too far from the original facts, is a problem requiring for its complete solution the very highest mathematical attainments. Not only is an intimate knowledge of the Differential and Integral Calculus useful for the purpose, but a knowledge of the Method of Least Squares may also be extremely serviceable. Until we have come to this point, we have been able only to speak of a knowledge of the differential calculus as serviceable in a general way to the actuarial student; but when we come to the present part of our subject, and examine the methods that have been applied with the most success to the graduation of mortality tables, we are compelled to say that no actuary can be considered to be fully qualified for the discharge of all the duties he may be called upon to undertake, who has not a competent knowledge of the differential and integral calculus. It is not to be forgotten, moreover, that an extensive use has of late years been made of the calculus, to obtain more exact values of annuities payable by half-yearly, quarterly, or other instalments; also of complete annuities, and in a variety of other questions, especially in the Theory of Complete Annuities. It is not yet essential for an actuary to be familiar with these investigations, but it is impossible to say how soon it may become so; and in the meantime a study of them cannot fail to give the actuarial student a greater command over the simpler parts of his subject.”

Only two papers, illustrative of Dr. Sprague’s mathematical genius, can be referred to here: the first on “A new Formula for Interpolation” (*J.I.A.*, vol. xxii, p. 270, July 1880), and the

second, on "The Graphic Method of Adjusting Mortality Tables" (*J.I.A.*, vol. xxvi, p. 77, October 1886). They differ much from each other, but each possesses its own well marked character.

Dr. Sprague's method of interpolation is that which is now known as the Osculatory method, a name first given by Dr. Karup. Dr. Sprague foreshadowed it in an earlier paper in 1867 (*J.I.A.*, vol. xiii, p. 305) on the Value of Annuities payable half-yearly, &c., on page 322 of which he indicated its principles, without developing them. The problem is, from equidistant, usually quinquennial, pivotal values of a function, $u_0, u_1, \&c.$, to u_5 , to interpolate the values intervening between u_2 and u_3 in such a way that when interpolation is similarly effected between the values u_1 and u_2 , and the values u_3 and u_4 , there shall be no break at the points of meeting of the adjacent partial curves at u_2 and at u_3 .

Dr. Sprague secured the necessary smoothness by arranging that, at the point of meeting of each pair of adjacent partial curves, these curves shall have the same tangent, and the same radius of curvature, that is, that they shall have the same first differential coefficient, and the same second differential coefficient. The values of the differential coefficients at the points u_2 and u_3 have to be determined by means of the given pivotal values of the function; and that is done by supposing parabolic curves of the fourth order to be passed through the points u_0 to u_4 , and through the points u_1 to u_5 respectively. Starting with the equations to these curves, and working by an abstruse mathematical process, the five leading sub-divided differences of u_2 are determined, and the terms intervening between u_2 and u_3 are then inserted by addition.

The process followed by Dr. Sprague in deducing the formula may be simplified and shortened a little, but it must always be abstruse. Nevertheless, the arithmetical application of the formula is easy, and the resulting completed table is free from all roughness. Dr. Sprague used the formula for the five select columns of his Select Mortality Tables, the pivotal values having been calculated by a graphic method; and the columns are very smooth.

In the earlier paper of 1867 Dr. Sprague mentions a much shorter formula of the same kind, with only three differences. At the point of meeting he gives to the two partial curves only the same tangent, and not necessarily the same radius of curvature; that is, he gives them the same first differential

coefficient, and ignores the second. It is found from experience that, for the purpose of constructing or of graduating mortality and other tables used in life contingencies, and for pension funds, &c., the shorter formula is all that is required, and that it gives excellent results.

This, the first of the papers chosen as illustrative of Dr. Sprague's methods, bristles with the higher mathematics. It is on interpolation; but that also necessarily includes graduation; and Dr. Sprague himself used the formula for the purpose of graduating his Select Tables.

The second illustrative paper is on graphic graduation; and in it Dr. Sprague condemns in language without restraint all mathematical formulas for graduation. He does not, however, condemn mathematics, but, on the contrary, he shows how there must be a sound knowledge of the mathematical properties of curves if a graphic graduation is to be really scientifically successful. On this point he says, "In order to understand and successfully apply the graphic method of graduation, it is necessary to study carefully the relations that exist between the progression of the numbers and their differences, and the form of the curve." In fact, according to Dr. Sprague, when graphically drawing a curve we must not, as some have said, be guided solely by the eye, but we must allow mathematical instinct to have full play.

Dr. Sprague, with various diagrams, gives a beautiful analysis of the nature of curves, and then supplies copious numerical examples of graphic graduation. He furnishes the figures all complete; and one cannot but be struck by his conscientious industry. He spares no labour to make his final results perfect.

Dr. Sprague's paper on graphic graduation led to a heated controversy. In it he had examined the results of several graduations by mathematical formulas, including that of Mr. Woolhouse, and compared them with his own graphic graduation; and he summed up by saying "We must include in one general condemnation all such graduation formulas as Mr. Woolhouse's, Mr. Higham's and Mr. Ansell's."

Naturally, Mr. Woolhouse replied, and Dr. Sprague replied again, and much dust was raised; but, in the end, something like peace was restored. Dr. Sprague made his "final remarks" in *J.I.A.*, vol. xxix, p. 232. To test the matter thoroughly, he made a graduation by Woolhouse's formula of the Text Book table and said: "Taking a general view of the figures, we infer

“that, although Mr. Woolhouse’s method slightly disturbs the law of the series (l_x) to which it is applied, the practical effect of this disturbance on the probability of dying is so slight as to be of no practical importance except at extreme old ages. I feel therefore bound to admit that my objection to the method, on the ground of its tendency to disturb the law of the series, although theoretically well founded, may be disregarded in practice.”

Besides being an actuary of unusual erudition, Dr. Sprague was a first-class business man; and of this the prosperity during his management, extending over thirty-nine years, of the two great Insurance Societies of which he was in succession Chief Officer, is sufficient evidence. The business of life assurance, using the words “life assurance” in their broadest sense, was the business of his life and in all his investigations he kept in view their immediate practical application, and refrained from acting on any theories he might have formed until he had put them to practical test. His writings all bear witness to this, his prevailing characteristic; and one would like to refer to many of them in some detail; but, in such a memoir as this, to do so is impossible. They are too voluminous. The mere list of his communications to the *Journal* of the Institute alone occupies nearly five and a half pages of the index. Then, there are actuarial publications of his to be found elsewhere. In April 1874 he brought out a volume entitled “Life Insurance Accounts”, in which he analysed minutely all the returns to be made by companies under the Act of 1870, and gave careful explanations of their meaning, and of how the schedules should be filled in—all of which was very useful at the time, when the new form of the accounts was but little understood except by the few.

In 1876 he contributed to the 9th edition of the Encyclopædia Britannica an exhaustive article on “Annuities”, in some parts highly mathematical, which superseded the article by Joshua Milne written many years before, and which was reviewed by the late Mr. Sutton in the *Journal*, vol. xx, p. 112. There are also to be found in the Transactions of the Actuarial Society of Edinburgh sundry contributions from Dr. Sprague, including some of his Presidential Addresses to that Society and his too little-known paper “On Probability and Chance, and their connection with the Business of Insurance.” All of these we must pass over, and confine ourselves to touching lightly on only two or three of the most salient points.

Dr. Sprague's monumental Select Tables have already been mentioned. In two elaborate papers in the *Journal*, vol. xxi, p. 229, dated November 1878, and vol. xxii, p. 391, dated January 1881, their construction and use were fully explained, and monetary tables at 4 per-cent interest were given; and in 1896 they were published separately in book form, with extensive monetary tables at four rates of interest.

Early in his managerial career, Dr. Sprague took up with his usual earnestness the question of investments in reversionary securities of all kinds, and he investigated them from every point of view, and examined them in all their phases. The result is a long series of papers, developing the mathematical theory of, and the principles to be followed when carrying out transactions in, reversions, reversionary interests, &c., whether of the more simple descriptions, or whether complicated by probabilities of marriage, of the birth of issue, &c., &c. The titles of a few of these papers may be quoted, just to show how comprehensive were Dr. Sprague's researches:

J.I.A., vol. xiv, p. 417, On the Valuation of Reversionary Life Interests.

J.I.A., vol. xviii, p. 77, On the Apportionment by mutual consent of a Fund between the Life Tenant and the Reversioner.

J.I.A., vol. xxiv, p. 327, On the Calculation of the values of Benefits that depend on the Death without Issue of One or More Persons.

J.I.A., vol. xxv, p. 160, On the Probability that a Marriage entered into at any age will be fruitful; and that, if a Marriage has been childless for several years, it will afterwards become fruitful.

Perhaps the most important paper on this class of subjects is that in *J.I.A.*, vol. xxi, p. 406, which was read before the Institute on 31 March 1879. It has the comprehensive title, "On the Construction of a Combined Marriage and Mortality Table, from the Rates of Marriage and Mortality among any body of Men; and on the Calculation of the Values of Annuities and Assurances that depend on the Contingency of Marriage, as well as Death; and their Application to determine the Rate of Premium for an Assurance against the Contingency of a Bachelor of a given age leaving Issue."

This paper was illustrated mainly with statistics derived from the British Peerage families, but also with some derived from other sources; and it gave the graduated tables of Marriage

and Mortality, which follow the usual Mortality Table form ; and it gave also the monetary values of Marriage and Mortality benefits.

These tables of Dr. Sprague are really the precursors of the Service Tables now prepared for Pension Funds, which are used for measuring the values of benefits falling in on death, or on withdrawal, or on retirement on pension, and sometimes even on marriage, but which also include varying salaries as an additional complication.

At the present day the statistics used by Dr. Sprague are, for the most part, obsolete on account of changed social conditions ; but the principles remain for our guidance and help.

The only other portion of Dr. Sprague's work which can be referred to here is his papers on legal subjects, on which, as barrister, he was an authority. Besides law being involved in many of his other papers, he wrote on the Policies of Assurance Act 1867, on the Sales of Reversions Act 1867, and on the Married Women's Property Acts 1870 and 1882 ; and his very last contribution on assurance matters was that on Lost Policies, and was read before the Institute on 26 April 1897, being followed by a most interesting discussion. It appears in the *Journal*, vol. xxxiii, p. 373, and is valuable at the present day, although naturally circumstances are not all the same.

Dr. Sprague's first paper appeared in the number of the Institute *Journal* for October 1856, and his last in the number for October 1897, there being thus exactly forty-one years between them.

One feature of many of Dr. Sprague's contributions to the *Journal*, which must be mentioned, although not without regret, is the peculiar spelling which he adopted. He was an ardent advocate of spelling reform, and thought he was doing good service to the cause by using, in season and out of season, spelling of a phonetic character. The Council objected, holding that, although some of them were in favour of reformed spelling, yet a scientific journal was not the place to introduce it. But Dr. Sprague was obdurate, and refused to contribute more papers unless his spelling were admitted ; and the Council gave way. In later years, however, Dr. Sprague relented, and, for some time before he retired, he allowed his papers to appear in orthodox garb. The papers in the phonetic spelling are irksome to read, and thus lose some of their value.

Dr. Sprague did not confine his energies to matters actuarial, but took an active interest in other branches of science. He

was Fellow of the Royal Society of Edinburgh, and of the Edinburgh Mathematical Society, and he was a member of the Edinburgh Field Naturalists and Microscopical Society, to all of which Societies he made communications; and on one occasion at any rate he also contributed to the proceedings of the British Association.

In the publications of the Royal Society of Edinburgh there are papers by him :

On the Nature of the Curves whose Intersections give the Imaginary Roots of an Algebraic Equation. (1883.)

On a New Algebra, by means of which Permutations can be Transformed in a variety of ways, and their Properties investigated. (1893.)

He also contributed to that Society papers on actuarial subjects, but these have been either reprinted in the *Journal* of the Institute of Actuaries, or their subject matter repeated by Dr. Sprague in other papers in that *Journal*.

To the Edinburgh Mathematical Society he contributed the following papers :

Note on the evaluation of Functions of the Form 0^0 . (1884-85.)

On the Different Possible non-linear arrangements of Eight Men on a Chess Board. (1889-90.)

On the Transformation and Classification of Permutations. (1890-91.)

On the Geometrical Interpretation of i^i . (1893-94.)

On the Eight Queens' Problem. (1898-99.)

On the Singular Points of Plane Curves. (1902-03.)

His outdoor recreations supplied him with material for communications to the Edinburgh Field Naturalists and Microscopical Society, of which the following is a list :

Bones and shells taken from a kitchen midden on Inchkeith during 1881.

The " Green Balls " of Loch Kildonan.

On the Growth of Leaves.

Fibre Balls.

Ripple Marks on Sand.

On the occurrence of the Fresh Water Mussel.

Notes on the Bournemouth Cliffs.

Notes on the Entomostraca (Water-fleas) of Midlothian.

The last of these papers was prepared conjointly with his daughter Miss B. Sprague, and it gained a prize for a beautifully mounted collection of the minute crustaceans treated of.

Dr. Sprague was twice married, first in 1859, to Miss Margaret Vaughan Steains, younger daughter of Mr. James Steains of Liverpool, by whom he had eleven children, eight of whom survive ; and, second, in 1908, to Miss Jean Elizabeth Stuart, third daughter of Mr. Morris Forsyth Stuart, Navy and Army Contractor of Edinburgh, who also survives him.

It is worthy of note that his eldest surviving son, Dr. A. E. Sprague, is, at the time of writing, President of the Faculty of Actuaries in Scotland. This is the first occasion of both father and son having achieved that great distinction.

GEORGE KING.
