FORUM

The Use of Bisectors

from Chief Officer Alan Davies

(British Tanker Company)

LIEUTENANT-COMMANDER BINI's interesting paper on the use of bisectors* shows the trap open to the believer in the 3-line fix and goes on to show the M.P.P. when four lines are used. However, as the ideal for the four lines is to have each at 90° apart, an error in either one or two lines provides the observer with an ellipse for the M.P.P. and is for practical purposes but a variation of the 3-line cocked hat circle of M.P.P. and little appears to have been gained by the working of the fourth line.

My own practice with stars is to observe two pairs, each star of a pair at 90° to the other but the pairs themselves at 45°, to give an ideal pattern as shown in Fig. 1. The advantage appears to lie in that, whereas three lines will usually

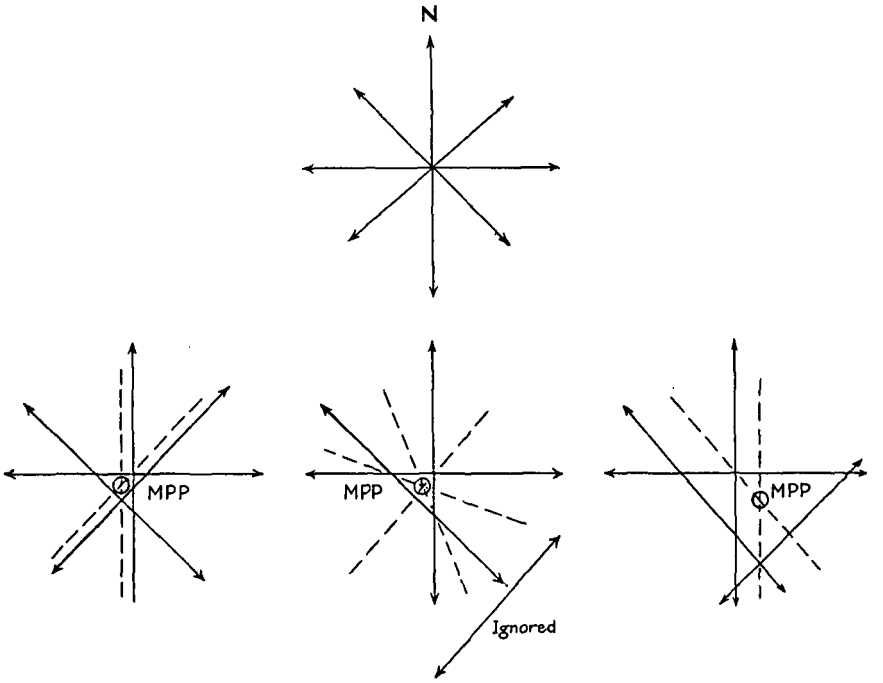

Fig. 1

provide a cocked hat, the fourth will indicate the likelihood of the M.P.P. lying within or without the triangle and if without, the direction in which it lies.

Where bisectors are used, the two bisectors intersecting at 45° will provide the M.P.P. unless one of the PL's is so far displaced as to have doubt thrown upon it, when the remaining three lines alone are used.

* M. Bini (1955). The use of bisectors in selecting the most probable position. This *Journal*, 8, 195.

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Lieutenant-Commander M. Bini writes:

There are several points that can be made about Mr. Davies's note.

1. The error affecting the bisector is given by:

$$\frac{E_{a1} - E_{a2}}{2 \sin(dA/2)}$$

where dA is the difference of azimuth and E_{a1} and E_{a2} the random errors. It appears, therefore, that when using pairs of stars 90° apart the algebraic difference of random errors is divided by 1·4, while if the stars are 180° apart the error is divided per 2. Thus a less reliable bisector is obtained by stars observed 90° apart instead of 180°.

2. The two bisectors cross at 45°, which is less accurate than if they cross at 90°.

3. The distribution of azimuths is less uniform because over a sector of the horizon of 135°—but in the worst case of 235°—no stars will be observed.

4. Mr. Davies says that three lines will usually provide a cocked hat and the fourth will indicate whether the M.P.P. is more likely to lie within or without the triangle. Here, however, two comments can be made:

(*a*) Four lines of position will generally provide four cocked hats (Fig. 2). Which of them is the good one and why?; and which is the 'fourth' line?

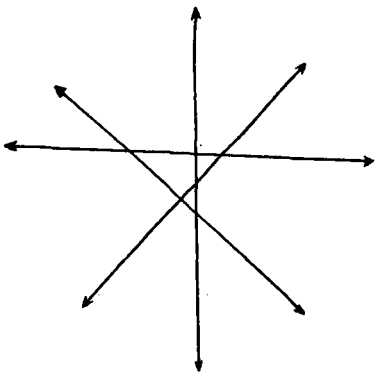

Fig. 2

(*b*) If the fourth line has to indicate the likelihood of the M.P.P. being inside or outside the cocked hat one would suppose this fourth line to be more reliable than the other three. As a principle this cannot be accepted as all the lines must be given the same weight and therefore used together in order to get out of them the best M.P.P. they provide.

Let us draw again the plot of Fig. 2 in the way proposed, i.e. adding to the lines the little arrows pointing in the direction of the observed bodies (Fig. 3): it now appears that all the lines are shifted the same amount in the same direction. No random error exists and the fix is quite good, apart from the systematic error responsible for the shifting of the lines—it could be an error in the index correction, or the effect of an abnormal dip, &c.

The correct use of bisectors shows us this, whereas choosing any one of the four cocked hats would have been misleading.

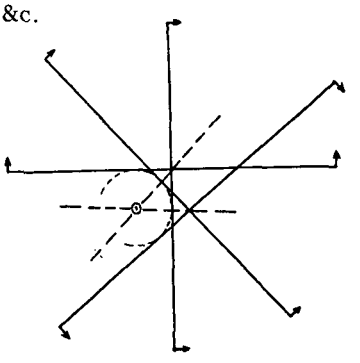

Fig. 3

5. It is quite true that when using four stars 90° apart, an error in one of the lines provides the observer with an ellipse of error for the M.P.P.; but it must be remembered that the real advantage of using four stars is that it enables the navigator to judge straightaway whether there is a random error and what its probable order of magnitude is.

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This is a property of the four-line plot, which the three-line plot cannot have; it affords the possibility of a useful evaluation of errors following the general principles pointed out in paragraph 6 of my article.

Mr. J. B. Parker comments:

A brief examination of Mr. Davies' note suggests the following comment:

1. *Accuracy.* If the errors in the four position lines are wholly random (i.e. no common personal error and no blunders) Mr. Davies' method is just as accurate as symmetrical shooting at 90° intervals.

2. *Bias.* If a personal error affects all four sights, the true position may be offset from the apparent position as judged by eye from the diagram. This would be the case if the azimuths are 000°, 045°, 090° and 135°. A use of the method of bisectors would, however, give the right point.

The more orthodox round-the-clock shooting (azimuths 000°, 090°, 180° and 270°) would give a less misleading plot (Bini's Fig. 5); so would Mr. Davies' methods if the second and fourth azimuths had been 225° and 315°.

3. *Blunders.* It is true that if, for the orthodox method, the 000° and 180° sights (say) appear inconsistent, it is not, in general, possible to say which is right without further observations. But this is also true for Mr. Davies' method, as is shown by the following diagram (Fig. 4).

Which is the blunder, Position Line 2 or Position Line 4? According to Mr. Davies' second paragraph, the fourth position line indicates the likelihood of the M.P.P. lying within the triangle ABC: but by the same token, the second position line indicates the likelihood of the M.P.P. lying within the triangle DEC. Where do we go from here?

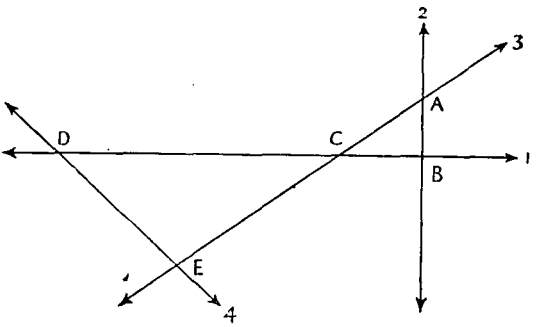

Fig. 4

Summing up, it is fair to say that the principle of taking sights 'all round the clock' is a sound one, though even if this is not done a good position can be obtained by the method of bisectors. There is no great gain in choosing the stars to be roughly 90° apart, and Mr. Davies' methods appear to be quite as good as those described by Commander Bini.

Lieutenant-Commander Bini writes:

I quite agree with Mr. Parker's comment, especially with what he says in the last paragraph: '. . . the principle of taking sights "all round the clock" is a sound one, though even if this is not done a good position can be obtained by the method of bisectors'.

In fact, it quite often happens at sea that part of the horizon is not suitable for observations (especially in the west at dawn and in the east at sunset) and, therefore, that one is unable to take sights 'all round the clock'. In these cases also the method of bisectors shows its usefulness; see for instance how in

two identical looking fixes the M.P.P. changes according to the azimuths of the observations and what different reliability the two fixes can be given (Figs. 5, 6).

Apart from the actual distance between the two M.P.P.'s—which could be some miles—the method of bisectors shows that the case in Fig. 5 is quite a reliable fix as there is only a systematic error, whereas the case in Fig. 6 is less reliable because there is a random error, or a blunder: two of the lines—as seen from the M.P.P.—are pointing outwards and the other two inwards.

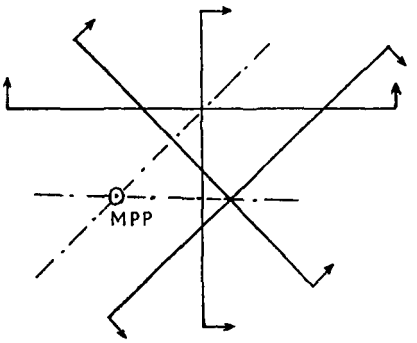

Fig. 5. Az 0°, 90°, 45°, 135°.

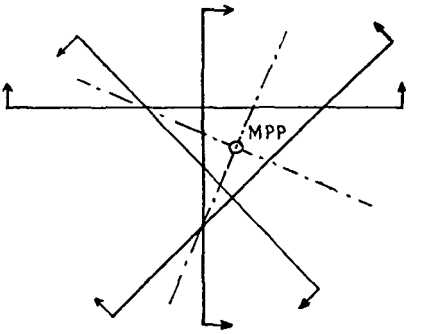

Fig. 6. Az 0°, 90°, 225°, 315°.

Mr. Parker's comment ends by saying: '. . . there is no great gain in choosing the stars to be roughly 90° apart, and Mr. Davies' method appears to be quite as good as those described by Commander Bini'.

Now if we speak of *methods* I cannot agree because what Mr. Davies proposes, i.e. using three stars to get a cocked hat and a fourth one to indicate the likelihood of the M.P.P. being inside or outside it cannot be called a *method* for the reasons already pointed out in my comment on Mr. Davies' note (and principally because it will never be able to establish which *is* the fourth line).

However, if, as I believe, Mr. Parker means that it is not necessary to use stars 90° apart and that also if they are observed in a different way a good M.P.P. can be obtained *by using the bisectors*, then I again agree with Mr. Parker. In fact the method of bisectors indicates the practice of taking stars 90° apart as the best way of sight taking (with four stars), but it can be applied in any case.

Dead Reckoning Error over the North Atlantic

Mr. P. G. Powell, Chief Navigating Officer of Trans-Canada Airlines, has allowed publication of the following note, originally presented at an I.A.T.A. meeting.

1. INTRODUCTION. Air traffic controllers separate aircraft on the North Atlantic with reference to dead reckoning (D.R.) positions of aircraft. Two such D.R. positions are transmitted to a.t.c. regularly from each aircraft, one usually 10 to 30 minutes following a fix, the other one hour's run farther ahead. The accuracy of these positions is important, since it has a direct bearing on horizontal separation standards and optimum time between position reports.

D.R. accuracy has been investigated previously on a theoretical basis. How-