27 Dynamics of string theory at weak coupling

In previous chapters we have seen that string theory at the classical level shows promise of describing the Standard Model and can realize at least one scenario for the physics beyond: low-energy supersymmetry. But there are many puzzles, most importantly the existence of moduli and the related question of the cosmological constant. At tree level, in the Calabi–Yau solutions the cosmological constant vanishes. But whether this holds in perturbation theory and beyond requires an understanding of the quantum theory.

In studying string theory, we have certain tools:

1. weak coupling expansions;
2. long-wavelength (low-momentum, α') expansions.

We have exploited both these techniques already. In analyzing string spectra we worked in a weak coupling limit. There are corrections to the masses and couplings, for example; in string perturbation theory all but a few states that we have studied have finite lifetimes. At weak coupling these effects are small, but at strong coupling the theories will presumably look dramatically different.

In asserting that Calabi–Yau vacua are solutions of the string equations, we used both the above types of expansion. We wrote down the string equations both in lowest order in the string coupling and also with the fewest number of derivatives (two). Even at weak coupling and in the derivative expansion, we can ask whether Calabi–Yau spaces are actually solutions of the string equations, both classically and quantum mechanically. For example, we have seen that, at lowest order in both expansions, there are typically many massless particles. We might expect tadpoles to appear for these fields, both in the α' expansion and in loops. There is in general no guarantee that we can find a sensible solution by simply perturbing the original one.

Yet there are many cases where we can make exact statements. In both Type II and heterotic string theories, we can often show that Calabi–Yau vacua correspond to exact solutions of the classical string equations. We can also show that they are good vacua – there are no tadpoles for massless fields – to all orders of the string perturbation expansion. More dramatically, we can sometimes show that these vacua are good, non-perturbative, states of the theory. This is perhaps surprising since we lack a suitable non-perturbative formulation in which to address this question directly. The key to this magic is supersymmetry. In the framework of quantum field theory we have already seen that supersymmetry gives a great deal of control over the dynamics, both perturbative and non-perturbative. We were able to prove a variety of non-renormalization theorems from very simple starting points. The more that supersymmetry is involved, the stronger the results we could establish simply from symmetry considerations, without a detailed understanding

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of the dynamics. The same is true in string theory. We can easily prove a variety of non-renormalization theorems for string perturbation theory. We can show that with $N = 1$ supersymmetry in four dimensions the superpotential is not renormalized from its tree level form in perturbation theory; the gauge coupling functions are not renormalized beyond one loop. These same considerations indicate the sorts of non-perturbative corrections which can (and do) arise. In theories with more supersymmetries one can prove stronger statements: that the superpotential is not renormalized at all and that there are strong constraints on the kinetic terms. These sorts of results will be important when we try to understand weak–strong coupling dualities.

27.1 Non-renormalization theorems

In each superstring theory one can prove a variety of non-renormalization theorems. Consider, first, the case of ten dimensions. At the level of two derivative terms the actions with $N = 1$ or $N = 2$ supersymmetry (16 or 32 supercharges) are unique. So, *both perturbatively and non-perturbatively, there is no renormalization.* This is a variant of our discussion in four-dimensional field theories. If we dimensionally reduce the Type II theories on a six-dimensional torus, we obtain a four-dimensional theory with 32 supercharges ($N = 8$ in four dimensions); if we reduce the heterotic theory we obtain a theory with $N = 4$ supersymmetry in four dimensions (16 supercharges). In either case the supersymmetry is enough to prevent corrections to either the potential or the kinetic terms, not only perturbatively but non-perturbatively.

These are quite striking results. From this we learn that the question of whether the universe is four-dimensional or whether it has, say, four or eight supersymmetries, or none, is not simply a dynamical question (at least in the naive sense of comparing the energies of different states or their relative stability). Other issues, perhaps cosmological, must come into play. We will save speculations on these questions for later.

27.1.1 Non-renormalization theorems for world-sheet perturbation theory

Let us turn now to compactified theories. Consider first a Type II theory compactified on a Calabi–Yau space. In this case the low-energy theory has $N = 2$ supersymmetry. Again, this is enough to guarantee that there is no potential generated for the moduli, perturbatively or non-perturbatively. In other words, starting with a solution of the equations of the low-energy effective field theory, at lowest order in g_s and R^2/α', we are guaranteed that we have an exact solution to all orders – and non-perturbatively – in both parameters.

Now consider the compactification of the heterotic string theory on the same Calabi–Yau space, with spin connection equal to the gauge connection. Then the world-sheet theory, as we saw, has two left-moving and two right-moving supersymmetries. It is identical to the theory which describes the corresponding Type II background. But we have just established that the Calabi–Yau space is a solution of the classical string equations, which means that there is a corresponding superconformal field theory with central charge

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$c = 9$. This is an exact statement, so the background corresponds to an exact solution of the classical string equations. This does *not* establish that the Calabi–Yau space corresponds quantum mechanically to an exact vacuum, as it does in the Type II case. For example, the intermediate states in quantum loops in the two theories are different.

We can establish this result in a different way. Consider the $h_{1,1}$ (1, 1)-forms $b^{(a)}_{i\bar{i}}$; one of these is the Kahler form, where $b_{i,\bar{i}} = g_{i,\bar{i}}$. In world-sheet perturbation theory we have seen that these fields decouple at zero momentum. The fact that all scattering amplitudes involving external b particles vanish at zero momentum has consequences for the structure of the low-energy effective Lagrangian: only derivatives of b appear in the Lagrangian. This is reminiscent of the couplings of Goldstone bosons; the Lagrangian, in world-sheet perturbation theory, is symmetric under

$$b(x) \to b(x) + \alpha \tag{27.1}$$

for constant α. We will refer to fields exhibiting such perturbative shift symmetries quite generally as *axions*.

This result implies a non-renormalization theorem for sigma-model perturbation theory; b lies in a supermultiplet with r^2, the modulus which describes the size of the Calabi–Yau space. This is apparent from the fact that they are both Kaluza–Klein modes associated with the metric $g_{i,\bar{i}}$; r^2 is the symmetric part and b is the antisymmetric part. So this is similar to the situation in which we could prove non-renormalization theorems in field theory. Different orders of sigma model perturbation theory are associated with different powers of r^{-2}. But, in holomorphic quantities such as the superpotential and gauge coupling function, additional powers of r^{-2} are accompanied by powers of b. So only terms which are independent of r^{-2} are permitted by the shift symmetry. As a result, the superpotential computed at lowest order is not corrected in sigma model perturbation theory. This means that particles which are moduli at the leading order in α' are moduli to all orders of sigma model perturbation theory.

This non-renormalization theorem does not quite establish that these are good solutions of the classical string theory; there is still the possibility that non-perturbative effects in the sigma model will give rise to potentials for the lowest-order moduli. Indeed, our argument for the vanishing of the b couplings is not complete. At zero momentum the vertex operator for b, V_b, is topological; while it is the integral of a total divergence, it does not necessarily vanish. There generally exist classical Euclidean solutions of the two-dimensional field theory – instantons – for which the vertex operator is non-zero. These world-sheet instantons raise the possibility that non-perturbative effects on the world sheet will lift some of or all the vacuum degeneracy. For the (2, 2) theories, however, we already know that this does not occur. Earlier we argued, by considering the compactification of the related Type II theories, that the corresponding sigma models are exactly conformally invariant. It is possible (and not terribly difficult), by examining the structure of the two-dimensional instanton calculation (i.e. for the "world-sheet instanton") to show that no superpotential is generated. While we will not review this analysis here, the techniques involved are familiar from our discussion of four-dimensional instantons. One wants to determine whether instantons can generate a superpotential. It is necessary, as in four dimensions, to count the fermion zero modes and see whether they can lead to a non-vanishing correlation function at zero momentum for an appropriate set of fields. In the $(2, 2)$ case one finds that they cannot. One can then ask whether quantum corrections (i.e. due to small fluctuations) to the instanton result can yield such a correction. Here one notes that, as in perturbation theory, holomorphy fixes uniquely the dependence on the coupling. So if the lowest-order contribution vanishes, higher orders will vanish as well.

In the case of $(2, 0)$ compactifications of the heterotic string the situation is more complicated. Perturbatively, we can argue, as before, that solutions of the string equations at lowest order are solutions to all orders in the α' expansion. Non-perturbatively, however, the situation is less clear. For such compactifications there is no corresponding Type II compactification, so we can not rely on the magic of $N = 2$ supersymmetry; it is necessary to examine in detail the effects of world-sheet instantons. In general, if one does the sort of zero-mode counting described above then one finds that it is possible to generate a superpotential. But in many cases one can argue that there are cancelations, and the superpotential vanishes.

It is important to understand that the non-renormalization theorems do not imply that the Calabi–Yau manifold is itself an exact solution to the classical string equations; rather, the point is that a solution is guaranteed to exist nearby. There can be – and are – tadpoles for massive particles in sigma model perturbation theory. A tadpole corresponds to a correction of the equations of motion as follows:

$$\nabla^2 h + m^2 h = \Gamma. \tag{27.2}$$

This is solved by a perturbatively small shift in, the h field;

$$h = -\frac{\Gamma}{m^2}. \tag{27.3}$$

For the massless fields, however, one cannot find a solution in this way, and in general, if there is a tadpole, there is no nearby (static) solution of the equations. This is why the low-energy effective action is such a useful tool in addressing such questions: it is precisely the tadpoles for the massless fields which are important.

27.1.2 Non-renormalization theorems for string perturbation theory

In field theory we proved non-renormalization theorems by treating couplings as background chiral fields and exploring the consequences of the holomorphy of the effective action as a function of these fields. In string theory we have no coupling constants, but the moduli determine the effective couplings and, since they are themselves fields, they are restricted by the symmetries of the theory. We exploited this connection in the previous subsection to prove non-renormalization theorems for sigma model perturbation theory. In this subsection we prove similar statements for string perturbation theory.

We begin with the heterotic string theory, on a Calabi–Yau manifold or an orbifold. In this case we have seen that there is a field S which we called the dilaton (it is sometimes called the *four-dimensional dilaton*). The vertex operator for the imaginary part of this field, $a(x)$, at $k = 0$ is simply

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$$V_a = \int d^2\sigma \, \epsilon^{ab} \, \partial_a X^\mu \partial_b X^\nu b_{\mu\nu}. \tag{27.4}$$

This is, again, a total derivative on the world sheet. So this particle, which we saw earlier is an axion, decouples at zero momentum. Again there is a shift symmetry – this is just the axion shift symmetry. Again, this means that the superpotential must be independent of S. But, since powers of perturbation theory come with powers of S, this establishes that the superpotential is not renormalized to all orders of perturbation theory!

As in the world-sheet case there can be non-perturbative corrections to the superpotential, and this raises the possibility that potentials will be generated for the moduli. We will see shortly that gluino condensation, as in supersymmetric field theories, is one such effect.

First, we consider other string theories. In the case of Type II compactified on a Calabi–Yau space, the $N=2$ supersymmetry is enough to ensure that no superpotential is generated perturbatively or non-perturbatively: Calabi–Yau spaces correspond to exact ground states of the theory, and the degeneracies are exact as well. As in field theories with $N = 2$ supersymmetry, corrections to the metric (the Kahler potential (26.44)) are possible. Theories with more supersymmetry (heterotic on tori, or Type II theories on $K3$ with $N = 4$ supersymmetry or Type II theories on tori with eight supersymmetries) are even more restricted.

27.2 Fayet–Iliopoulos *D* terms

In deriving non-renormalization theorems for string perturbation theory, we established that there is no renormalization of the superpotential or of the gauge coupling function beyond one loop. But this is not quite enough to establish that there is no renormalization of the *potential*. We must also check whether Fayet–Iliopoulos terms are generated. From field-theoretic reasoning we might guess that any renormalization would occur only at one loop. In globally supersymmetric theories in superspace, a Fayet–Iliopoulos term has the form

$$\zeta^2 D = \int d^4\theta \, V. \tag{27.5}$$

This term is just barely gauge invariant: under $V \to V + \Lambda + \Lambda^\dagger$ it is invariant because $\int d^4\theta \, \Lambda = 0$ since Λ is chiral. If we treat the gauge coupling (or any other couplings) as a background field, any would-be corrections to D would have the form

$$\int d^4\theta \, g(S, S^\dagger) V, \tag{27.6}$$

which is only invariant if g is a constant. Thus any D term is independent of the coupling, in the normalization where $1/g^2$ appears in front of the gauge terms. So at most there is a one-loop correction.

Before going on to string theory, it is interesting to look at the structure of any one-loop term. Call the associated $U(1)$ generator Y. If the supersymmetry is unbroken then massive

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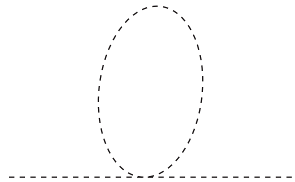

Fig. 27.1 The Feynmann diagram which contributes to the D term.

fields come in pairs with opposite values of Y, so only massless fields contribute. The Feynman diagram which contributes to the D term is shown in Fig. 27.1. It is given by:

$$\zeta^2 = \text{Tr}\, Y \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}. \tag{27.7}$$

So a vanishing D term requires that the trace of the $U(1)$ generator vanish. The one-loop diagram is quadratically divergent, but let us rewrite Eq. (27.7) in a way which resembles expressions we have seen in string theory. We can introduce a "Schwinger parameter," which we will call τ_2. Then

$$\begin{aligned}\zeta^2 &= 2\pi\, \text{Tr}\, Y \int_0^\infty d\tau_1 \int \frac{d^4k}{(2\pi)^4} e^{-2\pi\tau_2 k^2} \\ &= \frac{1}{32\pi^3} \text{Tr}\, Y \int_0^\infty \frac{d\tau_2}{\tau_2^2} \int_{-1/2}^{1/2} d\tau_1.\end{aligned} \tag{27.8}$$

We have written the expression in this way because we want to consider it as an integral over the modular parameter of the torus. At this stage the integral is still quadratically divergent. But, under modular transformations, the complex τ plane is mapped into itself an infinite number of times. We can define a *fundamental domain*,

$$-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}, \quad |\tau| \geq 1. \tag{27.9}$$

If we restrict the integration to the fundamental domain, the result is finite. In string theories, the correct answer terms out to be

$$\zeta^2 = \frac{1}{192\pi^2} \text{Tr}\, Y. \tag{27.10}$$

This result can be derived by a straightforward string computation. However, in string models where $\text{Tr}\, Y$ is non-zero we can give a low-energy field theory argument which completely fixes the coefficient of the D term and also sheds light on possible perturbative corrections. If $\text{Tr}\, Y \neq 0$, the low-energy theory has a gravitational anomaly. This anomaly is rather similar to the gauge anomalies we discussed in the context of field theory. It arises from a diagram with one external gauge boson and an external graviton. String models with such anomalies typically have gauge anomalies as well, which we can readily evaluate. As an example, consider the compactification of the $O(32)$ heterotic string on a Calabi–Yau space, with spin connection equal to the gauge connection. In this case the low-energy gauge group is $SO(26) \times U(1)$. There are $h_{1,1}$ 26s with $U(1)$ charge 1, and $h_{2,1}$ 26s with $U(1)$ charge -1. There are also corresponding singlets, with charges $+2$ and -2

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respectively. These are in precise correspondence with the fields we found in E_6; the 26s arise in parallel to the $O(10)$ 10s and the singlets arise in parallel to the $O(10)$ singlets. But now it is clear that there are anomalies in the gauge symmetries. For example, there is a $U(1) \times O(26)^2$ anomaly proportional to

$$A = h_{2,1} - h_{1,1} \tag{27.11}$$

and a $U(1)^3$ anomaly given by

$$A' = (h_{2,1} - h_{1,1})(26 - 8). \tag{27.12}$$

This is, however, a modular invariant configuration of string theory, so there should not be any inconsistency, at least in perturbation theory. Therefore something must cancel the anomaly. The cancelation is actually a variant of the mechanism discussed originally by Green and Schwarz in ten dimensions, now specialized to four dimensions. We know that there is a coupling:

$$\int d^2\theta \, S W_\alpha^2. \tag{27.13}$$

This gives rise to a coupling of the axion to the $F\tilde{F}$ terms of each group. The anomaly calculation in the low-energy theory implies a variation of the action proportional to the anomaly coefficient and $F\tilde{F}$. So, if the axion transforms under the gauge symmetry as

$$a(x) \to a(x) + c\omega(x) \tag{27.14}$$

then this can cancel the anomaly. It is crucial that the anomaly coefficients are the same for each group.

We can check whether this hypothesis is correct. If $a(x)$ transforms as above then it must couple to the gauge field. The required covariant derivative is

$$D_\mu a = \partial_\mu a - \frac{1}{c} A_\mu. \tag{27.15}$$

So, from the kinetic term in the action there is a coupling of A_μ to a. One can compute this coupling without great difficulty and verify that it has the required magnitude.

More interesting, however, is to consider the implications of supersymmetry. We can generalize the coupling above to superspace. The transformation law for a now becomes a transformation law for S:

$$S \to S + \Lambda + \Lambda^\dagger, \tag{27.16}$$

where Λ is the chiral gauge transformation parameter. The gauge-invariant action for S is

$$-\int d^4\theta \, \ln\left(S + S^\dagger - \frac{1}{c}V\right). \tag{27.17}$$

If we expand this Lagrangian in a Taylor series, we see that, in addition to the $A_\mu \partial^\mu a$ coupling, we generate a Fayet–Iliopoulos D term,

$$\int d^4\theta \frac{1}{c(S + S^\dagger)} V. \tag{27.18}$$

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One can verify that this term – and the other terms implied by this analysis – are present. First, we can ask: at what order in perturbation theory should each of these terms appear? To establish this we need to remember that the standard supergravity Lagrangian is written in a frame where M_p^2 appears in front of the Einstein term in the effective action. In the string frame, it is the dilaton – essentially S – which appears in front. If we rescale the four-dimensional metric according to

$$g_{\mu\nu} \to S g_{\mu\nu} \tag{27.19}$$

then S appears in front of the Lagrangian. With this same rescaling, the "kinetic" term, which had an S in front, has S^3. The Fayet–Iliopoulos D term, originally had a $1/S$ in front. Correspondingly, the resulting scalar mass term would be proportional to $1/S^2$. After the metric rescaling this would be independent of S; that is, in the heterotic string theory the D term should appear at one loop, in accord with our field theory intuition. Similarly, the coupling $A_\mu \partial^\mu a$ should appear at one loop, while there should be a contribution to the cosmological constant at two loops. All these results can be found by straightforward string computations (some of them are described in the suggested reading).

In essentially all the known examples this one-loop D term does not lead to supersymmetry breaking. There always seem to be fields which cancel the D term. Consider, again, the $O(32)$ theory. Here we can try to cancel the D term by giving an expectation value to one of the singlets, 1_{-2}. The question is whether this gives a non-zero contribution to the potential when we consider the superpotential. The most dangerous coupling is a term $1_{-2}1_{+2}$ involving some other singlet. But such terms are absent at lowest order, and their absence to higher orders is guaranteed by the non-renormalization theorems. Charge conservation forbids terms of the form 1^n_{-2}; there are no other dangerous terms. So this corresponds to an exact "F-flat" direction of the theory, in which all F vevs vanish. So, in perturbation theory there exists a good vacuum. While a general argument is not known, empirically this possibility for cancelation appears to arise in every known example.

What does the theory look like in this new vacuum?

1. Supersymmetry is restored and the vacuum energy vanishes.
2. The $U(1)$ gauge boson has a mass-squared of order g_s^2 times the string scale.
3. The longitudinal mode of the gauge boson is principally the imaginary part of the charged scalar field whose vev canceled the D term. There is still a light axion.

From the perspective of a very low energy observer, the D term is not a dramatic development. It plays some role in determining the physics at a very high energy scale (albeit not quite as high as the string scale). What is perhaps most impressive is the utility of effective-field-theory arguments in sorting out a microscopic string problem. Prior to the discovery of the D term, for example, there had been many papers "proving" a strict non-renormalization theorem for the *potential*; this, we see, is not correct (it is not hard to determine, in retrospect, what went wrong in the original proofs). The effective-field-theory arguments make clear when the potential is renormalized in perturbation theory and when it is not. They also permit one to easily find the "new vacuum" in cases where a Fayet–Iliopoulos term appears. It is possible, in principle, to find this vacuum by string methods, but this is distinctly more difficult. Finally, these arguments give insight into the non-perturbative fate of the non-renormalization theorems.

27.3 Gaugino condensation: breakdown of axion shift symmetries beyond perturbation theory

We have seen that, in string theory, if supersymmetry is unbroken at tree level in some particular constructions then it is unbroken to all orders of perturbation theory. The argument, as in field theory, allows exponential dependence on the coupling. In the case of a heterotic string compactified on a Calabi–Yau space, gaugino condensation, as in supersymmetric field theories, generates a superpotential on the moduli space.

Consider the $E_8 \times E_8$ theory compactified on a Calabi–Yau space, with spin connection equal to the gauge potential and without Wilson lines. In this case there is an $E_6 \times E_8$ gauge symmetry. There are typically several fields in the 27 of E_6, but there are no chiral fields transforming in the E_8. One has a pure E_8 supersymmetric gauge theory. The couplings of the E_6 and E_8 are equal at the high scale, so the E_8 coupling becomes strong first. This leads, as we have seen, to gaugino condensation. We have also seen that at tree level there is a coupling

$$SW_\alpha^2. \tag{27.20}$$

Just as before, this leads to a superpotential for S,

$$W(S) = Ae^{-3S/b_0}. \tag{27.21}$$

One often hears this described as a “field theory analysis,” as if it is not necessarily a feature of the string theory. But string theory obeys all the principles of quantum field theory. If we correctly integrate out the high-energy string effects then *the low-energy analysis is necessarily reliable.* So the only question is: are there terms in the low-energy effective action that lead to larger effects? One might worry that, since we understand so little about non-perturbative string theory, it would be hard to address this. But, with some very mild assumptions, we can establish that *the low-energy effects are parametrically larger than any high-energy string effects.*

The basic assumption is that, as in field theory, non-perturbatively the theory obeys a discrete shift symmetry (for a suitable normalization of a):

$$a(x) \to a(x) + 2\pi. \tag{27.22}$$

When we discuss non-perturbative string theory, we will give some evidence for this assumption; it will turn out to be one of the milder assertions on the subject of string duality. For now, we note that if we accept this assumption then, any superpotential for S arising from high-energy string effects will be of the form

$$W_{\text{np}} = C_n e^{-nS} \tag{27.23}$$

for integer n. So, such effects are exponentially smaller than gaugino condensation.

What does the low-energy theory look like? The dilaton potential goes rapidly to zero for large S, i.e. in the weak coupling limit. We might have hoped that somehow we would find that supersymmetry is broken and the moduli fixed. But, instead, gaugino condensation

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leads to a runaway potential. At large S we have just argued that no additional string effects can stabilize this behavior.

We can imagine more elaborate versions of this phenomenon, involving matter fields as well, in some sort of hidden sector. But it is difficult to construct models where the moduli are stabilized in any controlled fashion along these lines.

27.4 Obstacles to a weakly coupled string phenomenology

We have seen that string theory is a theory without dimensionless parameters. This is an exciting prospect, but it also raises the question: how are the parameters of low-energy physics then determined? We have argued that the answer to this question lies in the dynamics of the moduli: the expectation values of these fields determine the couplings in the low-energy Lagrangian.

In non-supersymmetric string configurations, perturbative effects already lift the degeneracy among different vacua, giving rise to a potential for the moduli. In the previous section we have learned that in supersymmetric compactifications non-perturbative effects generically lift the flat directions of the potential. In other words, the moduli are not truly moduli at the quantum level. At best, we can speak of approximate moduli in regions of the field space where the couplings are weak. The potentials, both perturbative and non-perturbative, all tend to zero at zero coupling. This is not surprising; with a little thought it becomes clear that this behavior is not specific to perturbation theory or some particular non-perturbative phenomenon such as gaugino condensation; at very weak coupling, we expect that the potential always tends rapidly to zero. This means that if the potential has a minimum, this occurs when the coupling is not small. This is troubling, for it means that it is likely to be hard – if possible at all – to do computations which will reveal detailed features of the state of string theory which describes the world we see around us.

In the next chapter we will see that much is known about non-perturbative string physics. Most striking is a set of dualities which relate regimes of very strong coupling in one string theory to weak coupling in another. While impressive, these by themselves do not help with the strong coupling problem we have elucidated above; if, at very strong coupling, the theory is equivalent to a weakly coupled theory then the potential will again tend to zero. In other words, it is likely that stable ground states of string theory exist only in regions where no approximation scheme is available.

Perhaps just as troubling is the problem of the cosmological constant. Neither perturbative nor non-perturbative string theory seems to have much to say. The potentials are more or less of the size one would guess from dimensional analysis (and the expected dependence on the coupling). Perhaps most importantly they are, up to powers of the coupling, as large as the scale set by supersymmetry breaking.

There are, however, some reasons for optimism. Perhaps the most important is provided by nature itself: the gauge and Yukawa couplings of the Standard Model are small. Another is provided by string theory. As we will discuss later, there are ways in which large pure numbers can arise dynamically in the theory. These might provide mechanisms

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to understand the smallness of couplings, even in situations where asymptotically the potential vanishes. Finally, we will see that there is, at present, only one proposal to understand the smallness of the cosmological constant, and string theory may provide a realization of this suggestion.

Suggested reading

The result that there are no continuous global symmetries in string theory is fundamental. For the heterotic theory, it appears in Banks and Dixon (1988). Non-renormalization theorems for world-sheet perturbation theory and issues in the construction of $(0, 2)$ models were described by Witten (1986) and by Green *et al.* (1987). The non-renormalization theorem for string perturbation theory is described in Dine and Seiberg (1986). The space–time argument for the Fayet–Iliopoulos D term appears in Dine *et al.* (1987c); world-sheet computations appear in Atick *et al.* (1987) and Dine *et al.* (1987a). World-sheet instantons are discussed in Dine *et al.* (1986, 1987b); cancelations of instanton effects relevant to $(0, 2)$ theories were studied by Silverstein and Witten (1995).