CORRESPONDENCE

Multiple Decrement Tables

The Editor, *7 October* 1947
The Journal of the Institute of Actuaries Students' Society

Sir,

As your reviewer of our booklet on *Multiple Decrement Tables* seems to be very concerned with the students' welfare he will not object if we break with the usual formalities and reply to some of his criticisms, since we believe that his remarks, rather than ours, will confuse the average student.

If we had been writing a paper on the subject we should have devoted space to an examination of other peoples work, but as the booklet was for students it was necessary for us to give only a straightforward presentation.

When preparing the booklet we gave careful consideration to Karup's work and we do not think that we have misunderstood it. Karup commenced with the postulate of a set of *independent* forces, the word 'independent' having the meaning which it has in probability theory. We have defined such a force in our booklet as one which operates so that the subpopulation of decrements is a random sample of the original population relative to the other forces operating. That is, the meaning of 'independent' is opposite to the meaning of our word 'selective'. Karup then showed that if all the independent forces operate together their values remain unchanged. His mathematical proof demonstrates that the value of one force at age x does not depend on the incidence of the other forces in the neighbourhood of age x, a theorem which is, of course, an implication of the definition of 'independence'. In other words, as your reviewer says, $(a\mu)^{\alpha}$ and μ^{α} are indistinguishable.

It is therefore confusing to term a set of independent forces operating together as a set of 'dependent' forces. There is nothing whatever to be gained from this terminology since the independent forces remain independent whether they operate singly or together.

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It is clear then that we have no quarrel with Karup's position. From our point of view, however, it is too limited. We required a model which would serve when the forces were 'dependent' in the profounder sense, that is, when decrements were selective. Our basic idea is a population subject to diminution by several causes of decrement. We then give this idea a mathematical representation, and the forces are so defined that the addition rule for forces and the product rule for survival factors follow logically. In this demonstration nothing whatever is asserted about 'independent' or 'dependent' forces. The model applies whether the decrements are selective or not, and the main purpose of pp. 18–22 was to show students how to make the proper interpretation when decrements were selective.

The notation $(a\mu)^{\alpha}$ and μ^{α} was used merely to indicate whether we were discussing a multiple decrement table or a single decrement table, and we are at pains to point out that in the case of a multiple decrement table and its family of single decrement tables $(a\mu)_x^{\alpha} \equiv \mu_x^{\alpha}$, etc.

Even if the actuary obtains estimates of, say, μ_x^{α} and μ_x^{β} from different sources and then combines them to form a double-decrement table to be applied to other data, he does not necessarily assume independence in Karup's sense. He merely assumes that the values of $(a\mu)_x^{\alpha}$ and $(a\mu)_x^{\beta}$ in his data approximate to the values of μ_x^{α} and μ_x^{β} used in the table. In making this judgement he must take into account all the evidence including any concerned with whether the decrements are selective or not. Thus our view is that independence in Karup's sense must be considered only at the application stage and not in the development of the mathematics. It is particularly important in considering the meaning of a single decrement table which is actually one of a family.

We would also stress that the meaning to be given to 'dependent' and 'independent' when attached to the word 'rate' is very simple. A dependent rate is a function of all the forces operating, whilst an independent rate is a function of one force only. It would be an improvement to use another terminology.

Finally there are two minor points on which we should like to comment. Your reviewer used the words 'original' and 'novel' in connexion with a formula. These words are the reviewer's and not the authors'. We made no claim to originality in this respect. It is clear, however, from your reviewer's misunderstanding that the work as a whole must have some original aspects.

We are curious about the meaning which your reviewer attaches to the word 'real' (line 13, p. 114). Since the logical consistency of our set-up rests on the product rule being satisfied (we stress this more than once), any relationships which do not satisfy this rule are 'logically indefensible'. All that your reviewer does at the bottom of p. 114 is to say that in a double-decrement table if ${}_tp_x^\alpha = 1 - tk$ and ${}_tp_x^\beta = 1 - tl$, where k and l are constants, then it is logically impossible for ${}_t(ap)_x = 1 - tm$, where m is a constant. This is obvious, since according to the product rule

$${}_t(ap)_x = {}_tp_x^\alpha\, {}_tp_x^\beta = (1 - tk)(1 - tl),$$

which is a quadratic expression and not a linear expression in t.

Yours faithfully,

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[*We fear that our reviewer did not make himself clear in his strictures on our correspondents' interpretation of Karup's theorem. It would, perhaps, have been better to point out that the footnote on p. 13 of our correspondents' booklet is a precise negation of this theorem. Karup, in fact, proved that if*

$${}_t(ap)_x = {}_tp_x^\alpha\, {}_tp_x^\beta\, {}_tp_x^\gamma \cdots,$$

then, necessarily, $(a\mu)_x^\alpha \equiv \mu_x^\alpha, \quad (a\mu)_x^\beta \equiv \mu_x^\beta, \ldots,$

if the μ's are to be continuous 'forces'. The problem is not that of a discrimination between 'independent', 'dependent' and 'selective' forces but of the clear understanding of a mathematical concept, continuity.

Our reviewer's incautious use of the word 'real' deserves criticism. However, in the passage cited by our correspondents, he was attempting *to draw a distinction between the unnecessary rigidity of the requirement that approximate expressions should satisfy the mathematical relation connecting their exact counterparts, and the fundamental postulate that a pair of approximate formulae to be used together should not be derived from their true mathematical values by means of mutually contradictory hypotheses.* ED.]

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