

Earth rotation from a simultaneous reduction of
LLR and LAGEOS laser ranging data

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1. INTRODUCTION

As the techniques of lunar and artificial satellite laser ranging mature, emphasis is being placed upon the use of these observations to monitor the Earth's rotation. It is important to note, however, that at the present time neither technique alone can furnish all three components of this rotation to an accuracy which surpasses those results obtained from classical techniques. In the case of LAGEOS laser ranging, unmodeled secular orbital effects couple with axial Earth rotation in such a way that these effects are not separable in the analysis of those observations. In the case of lunar laser ranging, observations have been regularly available only from a single station for the past ten years or so with the result that a change in latitude along the McDonald Observatory meridian is not separable into the ordinary (x,y) components of polar motion. The main purpose of this paper is to present the first stages of an investigation to combine LAGEOS and lunar laser ranging observations. It is hoped that the proper implementation of such a process might eliminate the shortcomings inherent in each technique, while accentuating the advantages of each. This has the potential of producing all three components of the Earth's rotation to an accuracy and precision which is compatible with the present observational uncertainties.

II. DATA AND MODEL COMPATIBILITIES

As is the case in all investigations which seek to combine two different observation types, a great deal of ground work must first be laid before the data synthesis can be begun. Care must be taken to insure that the various data types to be combined are totally compatible and consistent with one another. Not only must such mundane matters as units and formats be unambiguously defined, but standards for such things as reference frames, theoretical and empirical models as well as fundamental and derived constants must be strictly and totally adhered to.

At the University of Texas at Austin, two parallel efforts are underway to obtain Earth rotation information from artificial satellite and lunar laser ranging observations. That within the Department of Aerospace Engineering and Engineering Mechanics has been concerned with the artificial satellite analysis; that within the Department of Astronomy and McDonald Observatory has been concerned with the lunar analysis. Although each has been performing its tasks completely independently of the other, each uses the extensive computing facilities of the University's main computer systems. This happy circumstance has alleviated many of the problems associated with the transfer of data and information between independent reduction and analysis systems. Also, the close proximity of the personnel of both groups assures precise communications and thereby has eased the reference model compatibility problems.

As might be expected, our initial efforts have been applied to testing algorithms and applying them to the LAGEOS and LLR data sets which were obtained during the short MERIT campaign which ran from August through October of 1980. The lunar data set consists of some 63 normal points which represents some 600-700 individual lunar laser ranging observations. Specific information about this data set can be found in the MERIT Campaign Report which should be generally available from the Bureau International de l'Heure in Paris. The LAGEOS data set contains in excess of 20,000 individual LAGEOS ranges and will be described elsewhere. In both cases, our analysis efforts are concerned with range residuals and partial derivatives which are supplied by the standard LLR and LAGEOS reduction packages which have been in regular use over the past few years at the University of Texas. Although, to the best of our knowledge, the current data sets are internally consistent and compatible, additional checks will be made continuously throughout the course of the total investigation to preserve and/or extend this integrity.

III. ANALYSIS TECHNIQUES

Because of the short term nature of the effects being sought by this investigation, our "observational equation" is a simple one and, for the LLR case, is similar to that presented by Stolz and Larden (1976), i.e.,

$$\begin{aligned} \rho_0 - \rho_C = & r [\sin \phi \cos \delta \cos(\lambda - H) - \cos \lambda \sin \delta \cos \phi] x \\ & - r [\sin \phi \cos \delta \sin(\lambda - H) - \sin \lambda \sin \delta \cos \phi] y \\ & + r \cos \phi \cos \delta \sin H \delta(UT1 - UTC) \end{aligned}$$

where $\rho_0 - \rho_C$ is the range residual (observed minus computed); r is the radius of the Earth and λ is its east longitude; H is the local hour angle of the retroreflector and δ is its declination; x , y , and $\delta(UT1 - UTC)$ are improvements to the nominal values of these Earth rotation parameters. Although the above expression is that which is

specifically used for LLR, a similar one has been used for the LAGEOS case.

This investigation is seeking values for the Earth rotation parameters averaged over 5 day intervals or less. In the case of LLR these short-term effects are well-separated from any unmodelled long term effects because it is believed that all short term (less than two weeks or so) lunar orbital and librational effects down to the few centimeter level are known. This is, of course, not yet the case for LAGEOS and it is certainly recognized that analysis efforts to extract orbital information from the LAGEOS data also extract axial Earth rotation information from that data, thus decreasing one's ability for obtaining accurate UT1-UTC information from this data type. It is believed that this study is the first attempt to obtain Earth rotation parameters by the simultaneous reduction of LLR and LAGEOS data at the observation level.

The LLR residuals which were used in this study are "post-fit", linearized residuals having been obtained after a normal global parameter improvement run on some 17 months of data centered approximately on the MERIT data set. Parameters in the global solution runs include linear, annual and lunar nodal period terms in UT. Linearly interpolated values of smoothed BIH Circular D x , y , and UT1-UTC which were modified by corrections given by Williams (1974) based on McClure (1973) have been used. Also used was the Woolard (1953, 1959) nutation series as modified by Melchior (1971). Simple checks have shown slight differences with the Wahr (1980) nutation series and the Yoder et al (1981) treatment of UT diurnal tidal terms, although we are presently upgrading our LLR reduction systems to these more recent treatments and the new IAU system of fundamental constants.

The LAGEOS residuals were computed with the model used to generate the LAGEOS long-arc trajectory designated LLA80.11. The gravity field used was LGM80.11.1; this geopotential is a preliminary LAGEOS-derived adjustment to the GEM10 field. The model includes the Wahr nutation series, the short period variations in UT from Yoder et al (1981), and BIH Circular D smoothed values for polar motion and UT1. The station positions were the LAGEOS-derived set designated LSC80.11. Orbit initial conditions were estimated from a sampled set of LAGEOS observations from 16 sites over the 124 day period from 30 June to 31 October 1980 (MJD = 44420-44543). The full set of data contained 508,000 observations while the sampled set, obtained by requiring that no two observations from any one site be less than one minute apart, contained 22,000 ranges. The unweighted RMS of the post-fit residuals was 0.42m. The estimated "single-shot" precision was 0.25m when averaged over all of the laser systems involved.

The remaining unmodelled long-period variations in the LAGEOS orbital elements were removed by smoothing the element residuals

from LLA80.11 with a Vondrak filter using $\epsilon=1.0E-06$ (half power at 50 days). Because of the high correlation of errors in UT and errors in the LAGEOS orbit node, this empirical adjustment to the LAGEOS orbital elements effectively filters a portion of any signal present in UT1-UTC. The small correlation of polar motion components x and y to the orbital elements implies that they are only slightly affected by the empirical adjustment. As the LAGEOS dynamical model matures the use of an empirically corrected orbit will be discontinued.

IV. NUMERICAL RESULTS

Using an observational equation of the type give in Section III, we have computed observational residuals and partial derivatives using standard lunar and LAGEOS data analysis packages. Several of the initial solution attempts are being reported here. To assess the solution algorithms of this package the first solution run was performed to obtain UT1-UTC and a constant bias from LLR observations alone. Since only single station LLR data is being used in this study the analysis is similar to that performed by Shelus et al (1976). The second solution run was performed to obtain x , y , and UT1-UTC estimates from LAGEOS observations alone. Only those LAGEOS observations which were close in time to LLR observations were used (a full analysis of the LAGEOS-only results is beyond the scope of this paper). Each of these two runs provided results which were similar to those obtained from analyses performed independently of this study. The results, which give deviations to BIH Circular D 5-day smoothed values, can be seen for UT1-UTC in Figures 1 for the LLR-only case and for x , y , and UT1-UTC in Figure 2 for the LAGEOS-only case.

From an examination of these figures we see that our initial expectation that UT1-UTC "power" has been lost from the LAGEOS observations is confirmed since the deviations from BIH values for UT1-UTC are much smaller from the LAGEOS-only results than from the LLR-only results. This assumes, of course, that the LLR-only results are "correct". Having confirmed our expectations, we next proceed to the next step whereby we may "tie" the short-term signature from the LAGEOS data type to the long term signature from the LLR data type by attempting simultaneous solutions. Figure 3 shows the results for our first such attempt. In this case we have opted to only consider the x and y partial derivatives (not UT1-UTC) from the LAGEOS data set simultaneously with all three partial derivatives (x , y , and UT1-UTC) from the LLR set. All observations going into the solutions are given equal weight in spite of the overwhelming amount of LAGEOS data with respect to the LLR data. The signature for the UT1-UTC results are similar to the LLR-only results, and the signatures for the x and y results are similar to the LAGEOS-only results, as would be expected.

A very important sidelight of this investigation surfaces from our

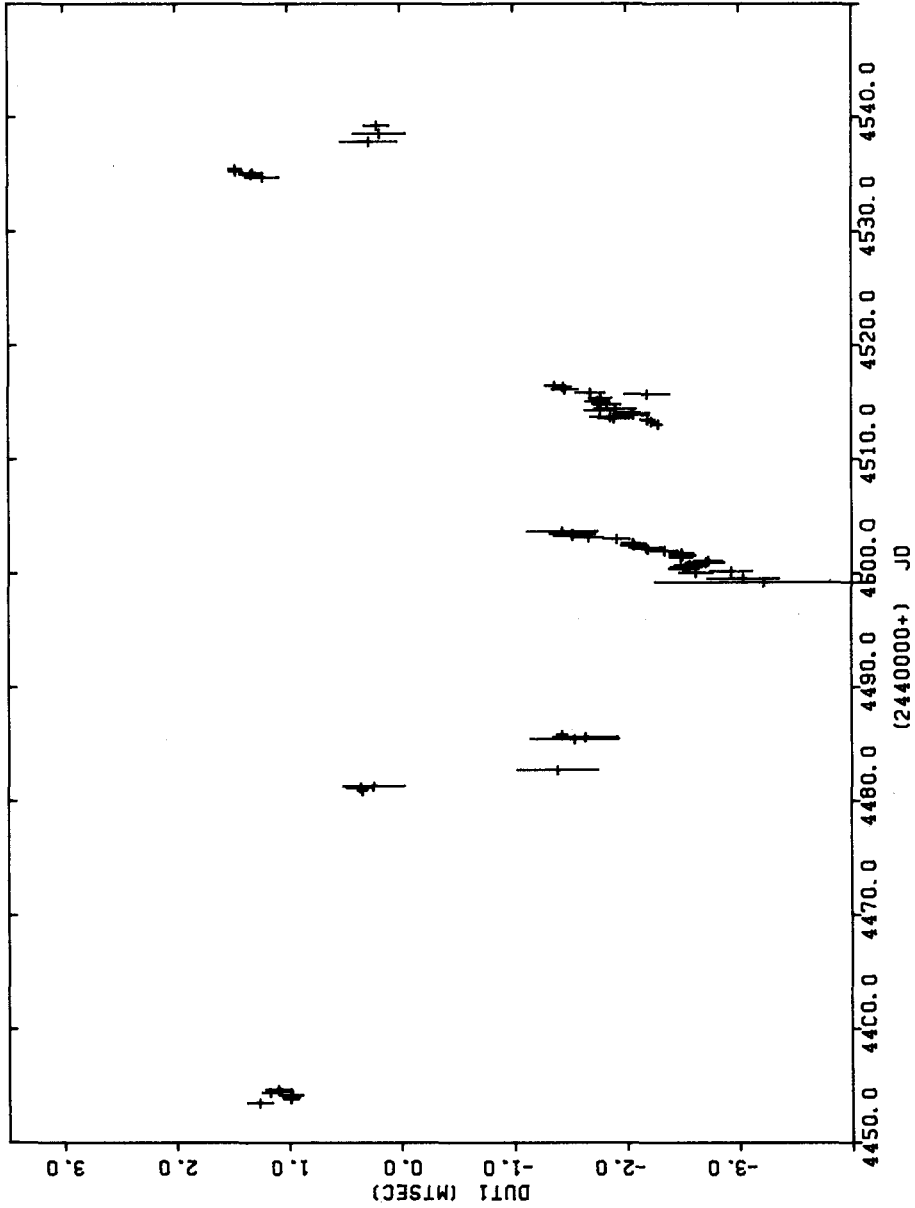


Figure 1. Differences in UT1-UTC determined by LLR with respect to BIH Circular D 5-day smoothed values.

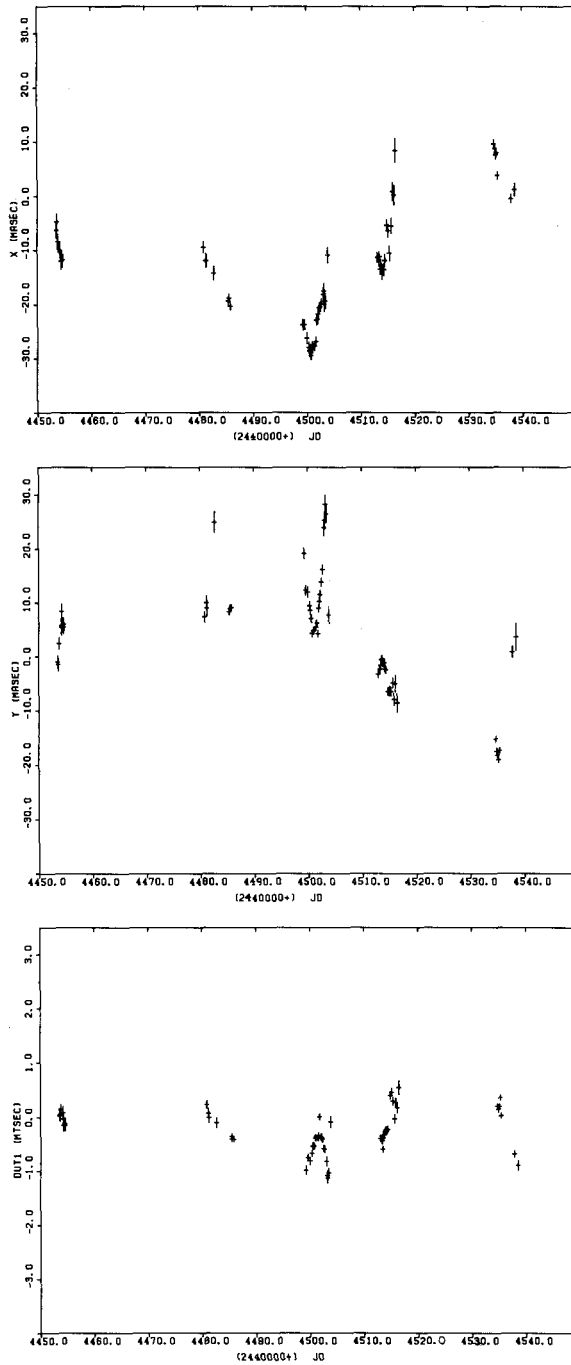


Figure 2. Differences in x, y, and UT1-UTC determined by LAGEOS with respect to BIH Circular D 5-day smoothed values.

processing of the observations in a manner different from most. Although a "window width" is selected similar to most other investigations (for instance, when one is computing two day averages, one chooses a window width of two days; all observations which fall through that window are allowed to enter into that solution), we do not necessarily move our window one full window width before performing the next solution. We feel that this technique can give a more complete representation of the information content of each data set. However it does have the drawback that each individual solution run is not completely independent from neighboring solutions. In all of the results which are presented here we have used a "window width" of two days and have "slid" this window by 0.04 days after each solution. A solution is performed only if there are at least three LLR observations in a particular data set. After sliding the window, a new solution is performed only if the LLR data set has changed.

The results presented in Figure 3 are not satisfying from several points of view. First, there were no attempts made to normalize the effects of each observation type through proper weighting parameters. As has already been mentioned, the LLR data are normal points while the LAGEOS data are shot-by-shot data. A far more serious objection arises from the fact that only a very weak tie is established between the two data types because the UT information from the LAGEOS data has been ignored and only single station LLR data exists. A crude attempt at normalization was made for the fourth solution run (Figure 4) wherein the third solution was performed again except that the LLR normal points were given a weight 5.0 with respect to unity for the LAGEOS shot-by-shot points. As might be expected, Figures 4 and 5 are quite similar.

V. DISCUSSION

Although the results presented here are preliminary, they are indicative of the great progress which has been realized recently at the observation by observation level in the combination of LAGEOS and LLR results for Earth rotation. Each technique is certainly mature enough that consistency and compatibility between such different data types has been accomplished. The presence of such a two-pronged analysis effort opens the door to a more proper and satisfying data synthesis. Our next steps will progress to more realistic ties between the two data types. This will entail using the LLR results to help separate the unmodelled orbital effects of LAGEOS from axial Earth rotation instead of merely ignoring the effects of UT1-UTC in the LAGEOS data. Simultaneously the x and y results from LAGEOS will be used to improve the LLR results. Once separation is obtained the short term LAGEOS results will be "anchored" by the long-term LLR results, thereby giving the UT1-UTC parameter the same significance and resolution as the x and y parameters.

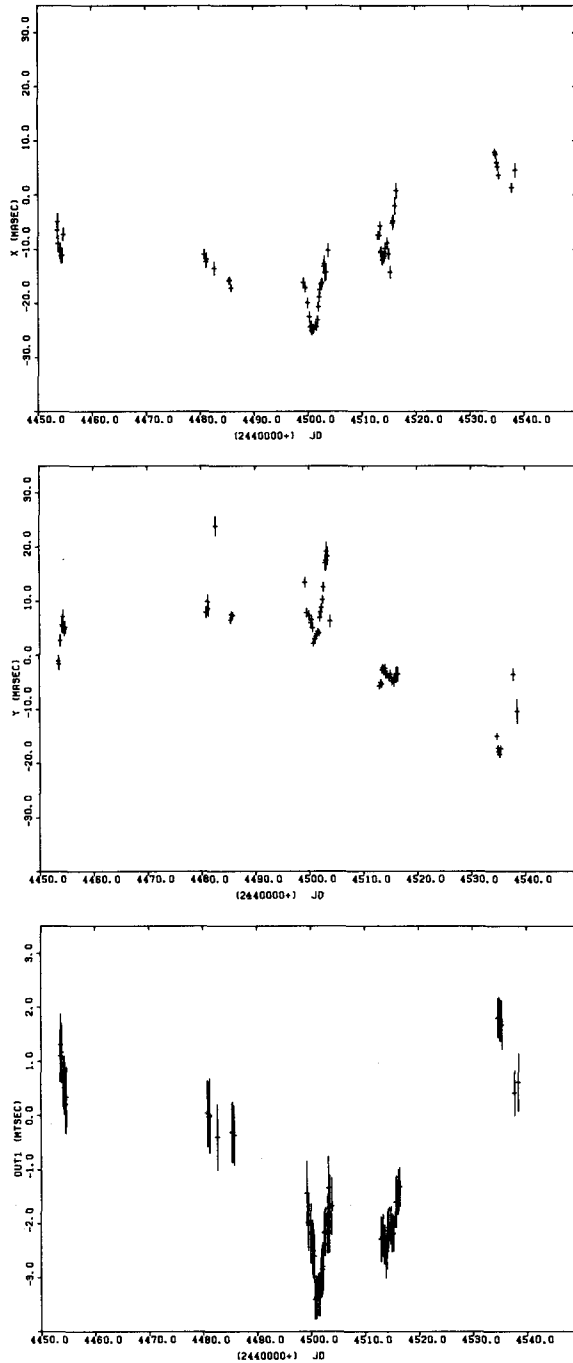


Figure 3. Differences in x, y, and UT1-UTC determined by LLR and LAGEOS with respect to BIH Circular D 5-day smoothed values (LAGEOS sampled shot-by-shot data and LLR normal point data equally weighted).

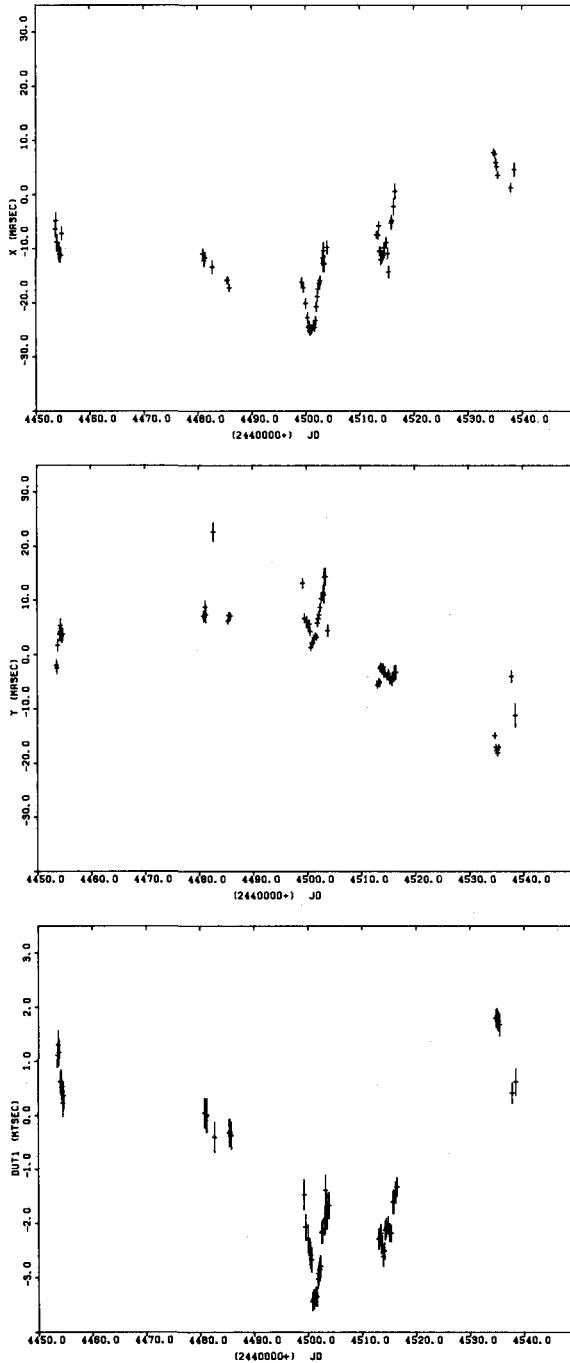


Figure 4. Differences in x,y, and UT1-UTC determined by LLR and LAGEOS with respect to BIH Circular D 5-day smoothed values (LLR normal point data weighted by a factor 5 with respect to LAGEOS shot-by-shot data).

Further progress will be also accomplished by a further investigation of the relative weighting schemes for LAGEOS versus LLR data to more reasonably combine normal point and shot-by-shot data. It may be also attempted to work with LLR shot-by-shot data and/or LAGEOS normal point data to obtain this next level of compatibility.

VI. ACKNOWLEDGEMENTS

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DISCUSSION

Langley : How often was a value for UT1 estimated ?

Shelus : According to our current research, we are considering 1, 2 and 5-day averages where our "window" slides about 10 % of its width. A new solution is performed each time that the data set changes. Therefore, our individual results are not all linearly independent within any given run of data.