Drag effect

Brouwer and Hori (1) have discussed the motion of an artificial satellite through a medium causing drag on the motion by generalizing von Zeipel's method. The atmosphere is assumed to be spherically symmetric at least upward from the perigee height and stationary with respect to the Earth. The law of density is assumed to be isothermal. Hori is now revising this theory.

The theory of Cook *et al.* (2) is limited to nearly circular orbits with a slightly different law for the density.

Izsak (3) computed the periodic drag effect by the method of the variation of constants. Vinti (4) considered the effect of atmospheric drag on the secular variation of orbital inclination following the method of Garfinkel (5). The motion is separated into an initial elliptic stage, a quasi-steady spiral stage, and a final ballistic stage. The secular change is deduced separably for the spiral and the elliptic stages.

There are two representations for the effect of atmospheric drag which differ depending on the initial values of the orbital parameters. Other theories neglect the atmospheric rotation and hence commit errors of several percents. Westerman (6) presented a technique for the method which yields a unique expression for the secular change in each standard element, and computed (7) the life-time of an artificial satellite.

Jacchia (8) analyzed the observed drag effect for deducing the variable atmospheric density and especially the drag during the November 1960 events from the point of view of the solar-terrestrial relationship. Macé pointed out the effect of the atmospheric turbulence.

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Radiation pressure

Mello (1) has established an analytical theory of the motion of an artificial satellite under the action of the solar radiation pressure by taking into account the circumstance that the action is non-effective during the passage of the satellite in the shadow of the Earth. The effect is considered by multiplying the term in the perturbation function due to the radiation pressure by a factor called the shadow function. The expansions obtained are analogous to those in the satellite theory. Mello used Tchebychev's polynomials for the expansion. He concluded the non-existence of the secular terms in the major-axis, eccentricity and inclination, and computed the secular terms in the longitudes of the node and the perigee and the mean anomaly, and also the principal effect of long periods. He is planning to compute the observations of Echo and other satellites by modifying the theory for the case of small eccentricity.

Musen (2) and Kozai (3) independently worked out the effect of the solar radiation pressure on the motion of an artificial Earth satellite and computed the secular effect. Bryant (4) computed the effect by the method of Krylov-Bogoliubov. Sehnal (5) discussed the Poynting-Robertson

https://doi.org/10.1017/S0251107X00014383 Published online by Cambridge University Press

effect on the motion of an artificial satellite. Brouwer (6) discussed analytically the resonance caused by radiation pressure on the motion.

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Critical inclination

Brouwer's theory (1) on the motion of an artificial Earth satellite is based on von Zeipel's method of eliminating all short-period terms by a canonical transformation. Brouwer's elements are transformed by Cain (2) to the mean elements in the ordinary sense. Lyddane and Cohn (3) computed numerically the motion of an artificial Earth satellite by Cowell's integration method for verifying Brouwer's theory. The first-order terms in the semi-major axis are not sufficient. By taking the second-order terms in the semi-major axis they could verify Brouwer's theory satisfactorily. Lyddane (4) referred to Poincaré's canonical variables instead of Delaunay's in order to remedy Brouwer's theory on von Zeipel's method from the difficulty for $e = 0$, $I = 0$. The same difficulty has been dealt with by Smith (5) by ordinary co-ordinate transformation.

The expressions in Brouwer's theory show that the method fails when $1 - 5\cos^2 I = 0$. It corresponds to the inclination $I = 63° 26'$, which is called the critical inclination. There have been several hot discussions as regards to the critical inclination whether it is a real existence or it is just an illusion caused by the wrong treatment of the problem. As far as the present method of perturbation theory is concerned, that is, in separating the perturbation into the short-period, the long-period and the secular, the appearance of the critical inclination is essential, although it is tacitly assumed that the perturbation is small enough to be divisible into its parts, then integrated separately and finally summed over the separately integrated results, irrespective of the convergence of the solution.

Hagihara (6) referred to his general theory (7) of libration based on Poincaré-Andoyer's theory on the motion of the Hecuba group asteroids. After carrying out von Zeipel's transformation an integral $F = \text{constant}$ is obtained, where F is the new Hamiltonian. A pair of double points of the curve representing this integral corresponds to the critical inclination. It is shown that one of the double points corresponding to the critical inclination is a centre in Poincaré's terminology and is stable, while the other is a saddle point and is unstable. The orbits near the centre are libratory and those near the saddle point are of revolution. The width of the libration in inclination is very narrow. The solution is obtained in elliptic functions in both cases. It is noticed that there appears no critical inclination in Vinti's treatment with the assumption $J_2^2 + J_4 = 0$. Garfinkel (8) obtained similar results by a different method based on von Zeipel's transformation by making Vinti's parameter $J_2^2 + J_4$ explicitly.

Hagihara considered only the terms J_2 and J_4. Kozai (9) extended the discussion to include J_3 and J_5. Aoki (10) included all these terms from the start. By referring to the elliptic functions of Weierstrass he solved the problem completely in each of the different cases arising from the values of the constants in the problem.

Hori (11) expanded the solution in powers of the square root of J_2 in order to solve the problem near the critical point. He obtained the solution in the form of the elliptic integrals of the first and the second kinds.

Izsak (12) noticed that the expansion in powers of $\sqrt{J_2}$ fails when we include higher order

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