NEW RELATIVISTIC EFFECTS IN THE MOTION OF THE MOON

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ABSTRACT. A summary of the main relativistic effects in the motion of the Moon is presented. The results are based on the application of a novel approach to the restricted three-body problem in general relativity to the lunar motion. It is shown that the rotation of the Sun causes a secular acceleration in the relative Earth-Moon motion. This might appear to be due to a temporal "variation" of the gravitational constant.

I. The restricted three-body problem in general relativity has been investigated using an extension of the idea of the Fermi frame (Mashhoon 1977, 1984, 1985; Mashhoon and Theiss 1982). The new approach brings out clearly the fundamental role that local inertial frames play in general relativity. In contrast to the usual post-Newtonian approach to this problem (Brumberg 1958; Baierlein 1967; Lestrade, Chapront, and Chapront-Touzé 1982), the new method deals with manifestly coordinate-invariant quantities. This paper summarizes the results of the application of this method to the theory of the motion of the Moon.

The following results have been obtained assuming that the exterior field of the Sun is given by the Schwarzschild metric:

a) The dominant relativistic effects in the motion of the Moon's node and perigee consist of

(i) the geodetic precession effect (i.e., the precession of the Earth-Moon orbital angular momentum about the normal to the ecliptic), which results in the **advance** of the Moon's node and perigee by $\simeq 2$ arc seconds per century as already determined by de Sitter (1916).

(ii) The relativistic **tidal** influence of the Sun on the Earth-Moon orbit which causes a **retardation** in the lunar node and perigee by about $\simeq 0.2$ second of arc per century (Mashhoon 1985). Note that the advance of perigee due to the relativistic field of the Earth is $\simeq 0.06$ arc second per century.

b) The dominant relativistic influence of the Sun on the Earth-Moon distance is tidal in character and is given by a harmonic term of amplitude $\simeq 2$ cm and period of 1/2 synodic month (Mash-

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hoon and Theiss 1982; Mashhoon 1984, 1985). The expected relativistic influence of the planets on the Earth-Moon distance is negligibly small (Mashhoon 1985).

II. The angular momentum of the Sun is taken into account in a **post-Schwarzschild** approximation scheme. That is, if all multipole moments of the Sun are neglected except for the mass, the exterior gravitational field must be precisely the Schwarzschild field and **not** any approximate form of it (as usually assumed in the standard post-Newtonian approximation). It is found, in fact, that the standard post-Newtonian approach loses its validity when applied over times longer than the Fokker period of geodetic precession.

The exterior field of the Sun is thus taken to be the Kerr field linearized in angular momentum (i.e., the Schwarzschild field together with the Lense-Thirring term). On the basis of this approximation, a new gravitational effect of a rotating mass has been discovered (Mashhoon and Theiss 1982; Mashhoon 1985): A gyroscope in orbit about a rotating mass undergoes **relativistic nutation** as well as the well-known precessional effects due to Fokker (1920) and Schiff (1960). The amplitude of the nutation is proportional to $J_{\odot} \sin\alpha$, where α is the inclination of the ecliptic with respect to the equatorial plane of the Sun. Hence the nodding effect vanishes for an equatorial orbit. The period of nutation is the Fokker period of geodetic precession. Thus the proper angular momenta of the Earth and Moon, as well as the Earth-Moon orbital angular momentum, undergo relativistic nutation of amplitude 1/2 arc second and period of $\simeq$ 67 million years. The solar angular momentum is assumed to be $J_{\odot} = 1.7 \times 10^{48}$ gm cm^2 sec^{-1}.

The instantaneous value of the relativistic nodding of the Earth axis at an epoch t_o is essentially absorbed in the obliquity of the ecliptic, so that only its variation in time away from its value at t_o can be measured. Since the determination of the local **inertial** frame (defined by ideal gyroscopes) is influenced by relativistic nutation, the nodding effect is reflected in the tidal influence of the Sun on the Earth-Moon system. This is secular in character (i.e., it is proportional to $t-t_o$)over any time scale short compared to the Fokker period of geodetic precession. Therefore this effect, which is linear in $J_{\odot}$ and is basically caused by the gravitational "magnetic" field of the Sun (due to its mass current), is similar to a temporal "variation" of the gravitational constant.

The Newtonian constant of gravitation is assumed to be a constant in general relativity as well. Nevertheless, the question has been raised whether this constant could in fact vary over cosmological time scales. Mach's ideas on the origin of inertia may be invoked to argue that due to the universal expansion, the recession of distant masses may have a weakening influence on the strength of gravitation. Let $t_o=0$ be the epoch at which observations begin, and consider a temporal variation of G according to the formula

$$G = G_o(1+\eta t) \tag{1}$$

over time scales short compared to the Hubble time. The hypothesis of the temporal variation of G implies that $\eta \sim -H_o$, where H_o is the Hubble

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constant (10^{-10}–10^{-11}/yr). It is important to point out that there is no firm observational basis for this hypothesis at the present time. If G does vary according to equation (1), the relative variation of the Earth-Moon separation will be given by $-\eta t$.

Secular variations of the Earth-Moon distance are also caused by the secular acceleration of the Moon due to the solar gravitational "magnetic" field. It might appear that these variations depend on the fact that the relative motion of the Earth-Moon system is referred to a parallel-propagated frame. Astronomical observations are usually referred to a fixed frame determined at a certain epoch of observation. However, at any given time the local frame is related to the fixed frame by a rotation. If the length of time (as measured from the fixed epoch) is short compared to the Fokker period, the frequency of the rotation is determined by the superposition of the Fokker and Schiff effects. It is clear from the operational definition of the Earth-Moon separation that this distance is independent of the choice of the local frame.

The secular tidal acceleration due to solar rotation induces harmonic, secular, and mixed variations in the Earth-Moon distance. The secular term is given by $At \sin\phi$, where ϕ is the angle formed by the lines of the ascending nodes which are the intersections of the ecliptic with the equatorial plane of the Sun and with the Earth-Moon orbital plane, and A is defined by

$$A = \frac{3}{2}\left(\frac{\omega_o}{\Omega_o}\right)^2 \alpha\beta\omega_F\,\xi \quad . \tag{2}$$

Here ω_o is the Keplerian frequency of the Earth around the Sun, Ω_o is the Keplerian frequency of the Moon around the Earth, β ($\simeq 5°$) is the inclination of the Earth-Moon orbital plane with respect to the ecliptic, ω_F is the Fokker frequency and ξ is given by

$$\xi = \frac{J_\odot}{M_\odot r_o^2 \omega_o} \quad , \tag{3}$$

where r_o is the Earth-Sun distance. The mixed terms are of the form $At \sin(at+b)$ with dominant periods of 1/2 yr, 1/2 synodic month, and 1/2 month. Thus the secular variation of the Earth-Moon distance (with $A \sim 10^{-16}$/yr) is expected to correspond to a temporal "variation" of G at the level of $\sim 10^{-16}$/yr.

The relativistic tidal influence of the solar quadrupole moment (quadratic in $J_\odot$) on the Earth-Moon system is expected to produce a similar secular effect. Preliminary calculations (Theiss 1984) indicate that this effect may be larger than the effect of the solar gravitational "magnetic" field. Can a temporal variation of G, if "proven" observationally, be really due to the rotation of the Sun? This may in fact provide a further confirmation of Einstein's theory of gravitation. The complete determination of the effect of solar quadrupole moment on the motion of the moon is presently under investigation.

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REFERENCES

Baierlein, R.: Phys. Rev. **162**, 1275 (1967).
Brumberg, V.A.: Bull. Inst. Theoret. Astron. (USSR), **6**, 733 (1958).
de Sitter, W.: Mon. Not. Roy. Astron. Soc. **77**, 155 (1916).
Fokker, A.D.: Proc. Roy. Acad. Amsterdam, **23**,729 (1920).
Lestrade, J.-F., Chapront, J., and Chapront-Touzé, M.: in "High-Precision Earth Rotation and Earth-Moon Dynamics", O. Calame, ed. (Reidel, Dordrecht), p. 217 (1982).
Mashhoon, B.: Astrophys. J. **216**, 591 (1977).
Mashhoon, B.: Gen. Rel. Grav. **16**, 311 (1984).
Mashhoon, B.: Found. Phys. **15**, 497 (1985).
Mashhoon, B., and Theiss, D.S.: University of Cologne report (1982).
Schiff, L.I.: Phys. Rev. Lett. **4**, 215 (1960).
Theiss, D.S.: Ph.D. thesis, University of Cologne (1984).

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