LETTER TO THE EDITOR

Dear Editor,

The threshold behaviour of epidemic models

My attention has kindly been drawn to the paper by Metz (1978). I regret that I did not see this paper prior to writing my own paper (Ball (1983)).

Metz's paper considers a wide class of essentially homogeneously mixing 'general' epidemic models. It falls naturally into two parts. In the first he uses an integral equation approach to provide a deterministic analysis, whilst in the second he considers branching process approximations to his epidemic models. This latter part contains some overlap with my paper, as I outline below, though there are important distinctions.

Metz showed weak convergence of a sequence of general epidemics to a branching process. Like myself, he used a construction given in Mollison (1977) to obtain a sample path of an epidemic process from that of an appropriate branching process. However, epidemics for different initial susceptible population sizes were not coupled together and thus he could only show weak convergence, rather than the strong convergence proved in my paper. The emphasis in Metz's paper is on clarifying results of direct biological relevance, for which weak convergence is quite adequate. However, I believe that the simpler proof of the stronger result given in my paper is mathematically more pleasing.

Metz also indicates how the deterministic and branching process approximations to the stochastic epidemic can be pieced together. His argument runs roughly as follows. The branching process approximation displays two modes of behaviour, namely minor and major outbreaks. If a minor outbreak occurs then the branching process approximation is valid throughout the time course of the epidemic, whilst in the event of a major outbreak the branching process approximation breaks down when the time t is sufficiently large. In such circumstances the stochastic epidemic can be approximated by a random translate of a suitably defined deterministic epidemic, the random translate being determined by the asymptotic size of the branching process.

Finally, I regret not having included the paper by Whittle (1955) in the bibliography of my paper. This paper contains the first statement of a stochastic epidemic threshold theorem. The approach was to sandwich the epidemic process between two birth-and-death processes, the 'faster' of which is identical to the limiting birth-and-death process of my paper.

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Yours sincerely,
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