This difference equation has solution

$$y_n = A(\sqrt{(s^2+1)} - s)^n + B(-1)^n(\sqrt{(s^2+1)} + s)^n. \qquad (2)$$

Since we know (see [3], for example) that

$$y_0 = 1/\sqrt{(s^2+1)}, \quad y_1 = 1 - 1/\sqrt{(s^2+1)}$$

it follows from (2) that $A = 1/\sqrt{(s^2+1)}$, $B = 0$ which proves (1).

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To the Editor, *The Mathematical Gazette*

WHERE ARE ALL THE NUMBERS?

DEAR SIR.—I much enjoyed Mr. Davies' entertaining article " Where are all the numbers? " (*Gazette* LV, No. 394 (December 1971), pp. 379–382). Like him I have been very impressed by the countability of not merely the rational but also the algebraic numbers, and their consequent total failure to make any substantial contribution to " filling the line ". And like him I am far happier believing that those other reals, the transcendentals, exist, that there are lots of them, enough to fill up the gaps, and that they are there on the real line somewhere—although I have heard that there is a school of thought which holds this to be an unjustified act of faith, and who would doubtless point to Mr. Davies' difficulties as a vindication of their position.

However, ignoring such doubts, for an excited moment I thought I could provide Mr. Davies with a genuine little block of fully paid-up, certified transcendentals, with which to make a real start on filling in his line. I refer to the set

$$\{x\colon\ x = \sum_{i=1}^{\infty} a_i/10^{i!}, \quad \text{with} \quad 1 \leqslant a_i \leqslant 9 \quad \text{and} \quad a_i \text{ integral}\}$$

of which, for example, $0\cdot110001000000000000000001000\ldots$ is a member, and to Courant and Robbins' well known book *What is Mathematics?*, pp. 104–107, for Liouville's Theorem, which establishes their transcendental pedigree.

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But a few moments' thought showed me that, after all, I had not achieved the impossible, that the measure of this set is still zero, and there is as much gap on the line as before. But at least some of the burden is lifted from e, π and γ, and Mr. Davies has here a *non-countable* collection with which to be able to say " numbers such as ... fill up the gap " left by the algebraists; although of course he will need the (denumerable) axiom of choice to take full advantage of its non-countability.

Also it is encouraging to find an accessible and reasonably elementary proof that a particular number is transcendental.

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To the Editor, *The Mathematical Gazette*

PROGRESS IN CONGRESS?

DEAR SIR,—Rather astonishingly, John Cameron's article on " Establishing a Pecking Order " (*Gazette* LV, No. 394 (December 1971), pp. 391–5) seems, in spite of the mass of new terminology, to have missed the point. His rule of thumb " make the Congress of lower order as quickly and sparingly as possible and work on from there " is natural and tempting; but it doesn't work!

As Mr. Cameron himself points out, an " unexpected " bonus can be obtained in Congress (5). Using his rule of thumb, you would first get a Progress (4) (5 weighings) and insert the last gress, which takes 3 more weighings, with bad luck. But Mr. Cameron gives a correct alternative which reduces the number of weighings to 7.

However, contrary to what Mr. Cameron says, this sort of thing *does* happen again. An extra " unexpected " bonus can be gained in Congress (9), which therefore takes only 19 weighings, and yet another in Congress (10) which takes only 22. (Mr. Cameron's formula gives 20 and 24, respectively.) This is what gives the puzzle its zest!

Since there is a reasonably obvious lower bound to the number of weighings of $[\log_2 (n!)]+1$ (for $n > 2$), it is known that there are no further bonuses for $n = 11$; but there is room for one with $n = 12$, which is why the number of weighings needed for Congress (12) is not known (so far as I have heard).

Now for the disappointment. I do not know the method which gets Congress (9) in 19 weighings! My figures above are taken from Roland Sprague's *Recreations in Mathematics* and details are not given.

But since Mr. Cameron doesn't consider the possibility of " unexpected " bonuses, except to say boldly that they can't happen, I think I'll stick with Roland Sprague for the time being.

Yours sincerely,

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P.S. I've just found a way of getting Congress (10) in 23 weighings, which disproves Mr. Cameron's formula, anyway.