# EVALUATION OF DETERMINANT OF 11TH ORDER IN THE THEORY OF PARTITIONS

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Determinants [1] of 5th and 7th orders have already been discussed in connection with the Theorems 4 and 5 of Atkin and Swinnerton-Dyer [2]. The determinant under consideration occurs in the investigation of Dyson's rank function for $q = 11$ given by Atkin and Hussain [3]. As regards the notation it may be mentioned that the author has adopted the same as that of Atkin and Hussain [3].

Let

$$\begin{aligned} a &= -x^{-4}P(2)/P(1), & b &= x^{-5}P(4)/P(2) \\ c &= x^{2}P(3)/P(4), & d &= -x^{-3}P(5)/P(3). \\ e &= -x^{10}P(1)/P(5), \end{aligned} \tag{1.1}$$

Let $\Delta_{11}/y^5$ be equivalent to

$$\begin{vmatrix} b & 0 & 0 & 0 & cx^4 & 0 & x^6 & ex^7 & 0 & dx^9 & ax^{10} \\ ax^{-1} & b & 0 & 0 & 0 & cx^4 & 0 & x^6 & ex^7 & 0 & dx^9 \\ dx^{-2} & ax^{-1} & b & 0 & 0 & 0 & cx^4 & 0 & x^6 & ex^7 & 0 \\ 0 & dx^{-2} & ax^{-1} & b & 0 & 0 & 0 & cx^4 & 0 & x^6 & ex^7 \\ ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 & 0 & 0 & cx^4 & 0 & x^6 \\ x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 & 0 & 0 & cx^4 & 0 \\ 0 & x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 & 0 & 0 & cx^4 \\ cx^{-7} & 0 & x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 & 0 & 0 \\ 0 & cx^{-7} & 0 & x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 & 0 \\ 0 & 0 & cx^{-7} & 0 & x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b & 0 \\ 0 & 0 & 0 & cx^{-7} & 0 & x^{-5} & ex^{-4} & 0 & dx^{-2} & ax^{-1} & b \end{vmatrix}$$

Then

$$\Delta_{11}/y^5 = \lambda\mu - 17\lambda^2 - 108\mu + 346\lambda - 131. \tag{1.2}$$

Multiplying the $(i+1)$th row by $x^i w_r^i$ where $i = 0$ to 10 and adding it is found that $(aw_r + b + cw_r^7 + dw_r^2 + ew_r^4 + w_r^5)$ is a common factor where $w_r$ is an 11th root of unity.

Thus

$$\begin{aligned}\Delta_{11}/y^5 &= \prod_{r=1}^{11} (aw_r + b + cw_r^7 + dw_r^2 + ew_r^{15} + w_r^5),\\ &= \prod_{r=1}^{11} (aw_r^{-4} + bw_r^{-5} + cw_r^2 + dw_r^{-3} + ew_r^{10} + 1);\\ &= y^{-5} f^{12}(y)/f^{12}(y^{11});\\ &= \lambda\mu - 17\lambda^2 - 108\mu + 364\lambda - 131;\end{aligned}$$

Vide the results (11.4) to (11.7) of Atkin and Hussain (3).

## References

[1] Proceedings of the Pakistan Statistical Association 6 (1957) 27—28
[2] A.O.L. Atkin and P. Swinnerton-Dyer, Some properties of Partition, Proc. London Math. Society (3) 4 (1953) pp. 84—106.
[3] A.O.L. Atkin and S. M. Hussain, some properties of Partition (2) Transactions of the American Math, Society, 89, (1958) 184—200.

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