Canad. J. Math. Vol. 74 (2), 2022, pp. 603–605
http://dx.doi.org/10.4153/S0008414X20000838

Corrigendum: On an Enriques surface associated with a quartic Hessian surface

http://dx.doi.org/10.4153/CJM-2018-022-7

Ichiro Shimada

In the original article [1], we made a mistake in the calculation of the number of $\mathrm{Aut}(Y)$-equivalence classes of RDP-configurations on an Enriques surface Y covered by a $K3$ surface birational to a general quartic Hessian surface. Theorem 1.5 and Table 1.2 of the paper should be replaced by the following:

Theorem 1.5 *Up to the action of* $\mathrm{Aut}(Y)$, *the Enriques surface* Y *has exactly* 33 *nonempty* RDP*-configurations. Their* ADE*-types are given in* Table 1.2.

ADE-type	Number	ADE-type	Number
E_6	1	$A_3 + A_1$	1
$A_5 + A_1$	5	$2A_2$	1
$3A_2$	1	$A_2 + 2A_1$	1
D_5	1	$4A_1$	5
A_5	1	A_3	1
$A_4 + A_1$	1	$A_2 + A_1$	1
$A_3 + 2A_1$	5	$3A_1$	2
$2A_2 + A_1$	1	A_2	1
D_4	1	$2A_1$	1
A_4	1	A_1	1

Table 1.2: RDP-configurations on Y.

Received by the editors September 17, 2020.
Published online on Cambridge Core December 10, 2020.
AMS subject classification: 14J28.
Keyword: Enriques surface.

The corrected version of the theorem gives us the following:

Corollary *The automorphism group* $\mathrm{Aut}(Y)$ *acts on the set of smooth rational curves on Y transitively.* ■

The mistake in the original proof is located in the second paragraph of Section 7.6. Let $\mathcal{F}$ be the set of maximal nonideal faces of D_Y, and we consider the mapping $F \mapsto \mathcal{R}(F)$ on $\mathcal{F}$. Even when two faces $F \in \mathcal{F}$ and $F' \in \mathcal{F}$ are *not* $\mathrm{aut}(Y)$-equivalent, the RDP-configurations $\mathcal{R}(F)$ and $\mathcal{R}(F')$ can be in the same $\mathrm{aut}(Y)$-orbit. This happens when F^g and F' span a same linear subspace in $S_Y \otimes \mathbb{R}$ for some automorphism $g \in \mathrm{aut}(Y)$.

We briefly explain the algorithm to obtain the correct version of Theorem 1.6. The detail of this algorithm will be explained in the forthcoming paper by the author in a more general setting.

Let F be an element of $\mathcal{F}$, and we put $\Gamma := \mathcal{R}(F)$. Let $\langle\Gamma\rangle$ be the sublattice of S_Y generated by Γ, and $\langle\Gamma\rangle^\perp$ the orthogonal complement of $\langle\Gamma\rangle$ in S_Y. Then $\langle\Gamma\rangle^\perp$ is a hyperbolic lattice of rank $10 - \mu$, where $\mu := |\Gamma|$ is the total Milnor number of the surface $\overline{Y}$ corresponding to F. We consider the positive cone

$$\mathcal{P}_{\langle\Gamma\rangle^\perp} := \mathcal{P}_Y \cap (\langle\Gamma\rangle^\perp \otimes \mathbb{R})$$

of $\langle\Gamma\rangle^\perp$. A $\langle\Gamma\rangle^\perp$-*induced chamber* is a closed subset of the cone $\mathcal{P}_{\langle\Gamma\rangle^\perp}$ of the form $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D$, where D is an induced chamber in $\mathcal{P}_Y$ such that $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D$ contains a nonempty open subset of $\mathcal{P}_{\langle\Gamma\rangle^\perp}$. For example, the face F is a $\langle\Gamma\rangle^\perp$-induced chamber $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_Y$. Since $N(Y)$ is tessellated by induced chambers in $\mathcal{P}_Y$, the closed subset $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap N(Y)$ of $\mathcal{P}_{\langle\Gamma\rangle^\perp}$ is tessellated by $\langle\Gamma\rangle^\perp$-induced chambers. We denote by V_Γ the set of $\langle\Gamma\rangle^\perp$-induced chambers contained in $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap N(Y)$. Then, the group

$$\mathrm{aut}(Y,\Gamma) := \{g \in \mathrm{aut}(Y) \mid \Gamma^g = \Gamma\}$$

acts on V_Γ.

Let $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_0$ be an element of V_Γ, where D_0 is an induced chamber contained in $N(Y)$. Then,

- we can make the list of all induced chambers D_0' contained in $N(Y)$ such that $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_0 = \mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_0'$,
- we can make the list of all $\langle\Gamma\rangle^\perp$-induced chambers $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D'$ adjacent to $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_0$,
- for another $\langle\Gamma\rangle^\perp$-induced chamber $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_1$, where D_1 is an induced chamber contained in $N(Y)$, we can enumerate all the elements g of $\mathrm{aut}(Y)$ that maps D_0 to D_1, and
- combining the algorithms above, we can make the list of all $g \in \mathrm{aut}(Y)$ that maps $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_0$ to $\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D_1$.

Then, we can make a complete set of representatives

$$\mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D^{(0)}, \dots, \mathcal{P}_{\langle\Gamma\rangle^\perp} \cap D^{(m)} \tag{1.1}$$

of the orbits of the action of $\mathrm{aut}(Y,\Gamma)$ on V_Γ, where $D^{(0)}, \dots, D^{(m)}$ are induced chambers contained in $N(Y)$.

Let F' be another element of $\mathcal{F}$ such that $\Gamma' := \mathcal{R}(F')$ has the same ADE-type as Γ. Then Γ' belongs to the same $\mathrm{aut}(Y)$-orbit as Γ if and only if there exist a $\langle\Gamma\rangle^{\perp}$-induced chamber $\mathcal{P}_{\langle\Gamma\rangle^{\perp}} \cap D^{(k)}$ in the representatives (1.1) and an element g of $\mathrm{aut}(Y)$ that maps F' to $\mathcal{P}_{\langle\Gamma\rangle^{\perp}} \cap D^{(k)}$. Applying this criterion to all pairs of faces in $\mathcal{F}$, we obtain the orbit decomposition by $\mathrm{aut}(Y)$ of RDP-configurations of smooth rational curves on Y.

We apologize for possible confusion that this mistake may have caused.

Reference

[1] I. Shimada, *On an Enriques surface associated with a quartic Hessian surface*. Canad. J. Math. 71(2019), 231–246.

Department of Mathematics, Graduate School of Science, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima 739-8526, Japan
e-mail: ichiro-shimada@hiroshima-u.ac.jp