Some topics in the theory of
ring structures on abelian groups

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In recent years, Fuchs [1] has described the absolute annihilator and
the absolute (Jacobson) radical of a torsion group, and Gardner [2] has
characterised the absolute annihilator of a completely decomposable
torsion-free group. In this thesis the problem of describing the absolute
annihilator and the absolute radical of certain abelian groups is
considered. This involves a discussion of the rings on these groups, and
the information so obtained allows us to answer several other questions
from the theory of ring structures on abelian groups.

Complete descriptions of the absolute annihilator are given for vector
groups, separable groups, certain mixed groups of torsion-free rank one,
reduced algebraically compact groups, cohesive groups, and reduced groups
whose quotients modulo torsion subgroups are divisible. Partial
characterisations are also provided for cotorsion groups, and torsion-free
groups of rank two. For the absolute radical of a group, complete
descriptions are provided for certain mixed groups of torsion-free rank one
and reduced algebraically compact groups and partial descriptions are given
for strongly indecomposable torsion-free groups of finite rank, completely
decomposable torsion-free groups, cotorsion groups, torsion-free groups of
rank two, and cohesive groups.

The properties of rings on some of the forementioned torsion-free
groups lead us to consider various aspects of nilpotence. Of particular
interest are the T-nilpotent rings on completely decomposable torsion-free
groups. A bound is also provided for the nil-degree, if it is finite, of

certain torsion-free groups. The mixed groups of torsion-free rank one discussed in this thesis motivate an investigation of the additive group of a regular ring. A question of Fuchs [1] concerning these groups is answered in the negative.

References

[1]   László Fuchs, *Infinite abelian groups*, Volume II (Pure and Applied Mathematics, **36-II**. Academic Press, New York and London, 1973).

[2]   B.J. Gardner, "Rings on completely decomposable torsion-free abelian groups", *Comment. Math. Univ. Carolinae* **15** (1974), 381-392.