Canad. Math. Bull. Vol. **53** (1), 2010 pp. 95–101
doi:10.4153/CMB-2010-020-7

Towards the Full Mordell–Lang Conjecture for Drinfeld Modules

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Abstract. Let ϕ be a Drinfeld module of generic characteristic, and let X be a sufficiently generic affine subvariety of $\mathbb{G}_a^g$. We show that the intersection of X with a finite rank ϕ-submodule of $\mathbb{G}_a^g$ is finite.

1 Introduction

McQuillan proved the Mordell–Lang conjecture in its most general form [19].

Theorem 1.1 **(The full Mordell–Lang theorem)** *Let G be a semi-abelian variety defined over a number field K. Let $X \subset G$ be a K^{alg}-subvariety, and let $\Gamma \subset G(K^{\mathrm{alg}})$ be a finite rank group,* i.e., *Γ lies in the divisible hull of a finitely generated subgroup of $G(K^{\mathrm{alg}})$. Then there exist algebraic subgroups $B_1, \dots, B_\ell$ of G and there exist $\gamma_1, \dots, \gamma_\ell \in \Gamma$ such that*

$$X(K^{\mathrm{alg}}) \cap \Gamma = \bigcup_{i=1}^{\ell} (\gamma_i + B_i(K^{\mathrm{alg}})) \cap \Gamma.$$

We note that in Theorem 1.1, if X does not contain any translate of a positive dimensional algebraic subgroup of G, then the full Mordell–Lang theorem says that $X(K^{\mathrm{alg}}) \cap \Gamma$ is finite. Also, a particular case of the full Mordell–Lang theorem (in the case Γ is the torsion subgroup G_{tor} of G) is the Manin–Mumford theorem, which was first proved by Raynaud [20].

Faltings [8] proved the Mordell–Lang conjecture for finitely generated subgroups Γ of abelian varieties G. His proof was extended by Vojta [25] to finitely generated subgroups of semi-abelian varieties G. Finally, McQuillan [19] extended Vojta's result to finite rank subgroups Γ of semi-abelian varieties G. Later, Rössler [21] provided a simplified proof of McQuillan's extension in which he used uniformities for the intersection of translates of a fixed subvariety $X \subset G$ with the torsion subgroup of the semi-abelian variety G. Essentially, Rössler showed that the full Mordell–Lang conjecture follows from the Mordell–Lang statement for finitely generated subgroups, combined with a uniform Manin–Mumford statement as proved by Hrushovski [18].

It is important to note that the exact translation of the Mordell–Lang conjecture to semi-abelian varieties in characteristic p is false due to the presence of isotrivial varieties. However, Hrushovski [17] saved the Mordell–Lang theorem for finitely generated subgroups of semi-abelian varieties in characteristic p by treating isotrivial varieties as *special*. The isotrivial case was treated by Rahim Moosa and the author

Received by the editors December 13, 2006.
Published electronically December 4, 2009.
AMS subject classification: **11G09**, 11G10.
Keywords: Drinfeld module, Mordell–Lang conjecture.

in [12] where there was obtained a full Mordell–Lang statement for isotrivial semi-abelian varieties in characteristic p. On the other hand, if we replace G by a power $\mathbb{G}_a^g$ of the additive group scheme, then the exact translation of the Mordell–Lang conjecture either fails (in characteristic 0) or it is trivially true (in characteristic p).

Inspired by the analogy between abelian varieties in characteristic 0 and Drinfeld modules of generic characteristic, Denis [6] proposed that analogs of the Manin–Mumford and Mordell–Lang theorems hold for such Drinfeld modules ϕ acting on $\mathbb{G}_a^g$ (in characteristic p). Denis conjectures describe the intersection of an affine subvariety $X \subset \mathbb{G}_a^g$ with a finite rank ϕ-submodule Γ of $\mathbb{G}_a^g$. Using methods of model theory, combined with some clever number theoretical arguments, Scanlon [22] proved the Denis–Manin–Mumford conjecture. In [9], the author proved the Denis–Mordell–Lang conjecture for finitely generated ϕ-modules Γ under two mild technical assumptions. In this paper, we extend our result from [9] to finite rank ϕ-submodules Γ.

We also note that recently there has been significant progress in establishing additional links between classical diophantine results over number fields and similar statements for Drinfeld modules. The author [10] proved an equidistribution statement for torsion points of a Drinfeld module, which is similar to the equidistribution statement established by Szpiro–Ullmo–Zhang [24] (which was later extended by Zhang [28] to a full proof of the famous Bogomolov conjecture). Breuer [3] proved a special case of the André–Oort conjecture for Drinfeld modules, while special cases of this conjecture in the classical case of a number field were proved by Edixhoven–Yafaev [7] and Yafaev [27]. Bosser [2] proved a lower bound for linear forms in logarithms at an infinite place associated to a Drinfeld module (similar to the classical result obtained by Baker [1] for usual logarithms, or by David [4] for elliptic logarithms). Bosser's result was used by Thomas Tucker and the author [14] to establish certain equidistribution and integrality statements for Drinfeld modules. Moreover, Bosser's result is quite possibly also true for linear forms in logarithms at finite places for a Drinfeld module. Assuming this last statement, Thomas Tucker and the author proved [13] the analog of Siegel's theorem for finitely generated ϕ-submodules. We believe that our present paper provides an additional proof of the fact that the Drinfeld modules represent the right arithmetic analog in characteristic p for semi-abelian varieties in characteristic 0.

The plan for our paper is as follows: in Section 2 we provide the basic notation for our paper, while in Section 3 we prove our main result (Theorem 3.1).

2 The Mordell–Lang Theorem for Drinfeld Modules

First we note that all subvarieties appearing in this paper are considered to be *closed*. We define next the notion of a Drinfeld module.

Let p be a prime and let q be a power of p. Let C be a projective non-singular curve defined over $\mathbb{F}_q$. Let A be the ring of $\mathbb{F}_q$-valued functions defined on C, regular away from a fixed closed point $\infty \in C$. Let K be a finitely generated field extension of the fraction field $\mathrm{Frac}(A)$ of A. We let K^{alg} be a fixed algebraic closure of K, and let K^{sep} be the separable closure of K inside K^{alg}.

We define the operator τ as the Frobenius on $\mathbb{F}_q$, extended so that for every $x \in$

K^{alg} we have $\tau(x) = x^q$. Then for every subfield $L \subset K^{\text{alg}}$ we let $L\{\tau\}$ be the subring of endomorphisms of $\mathbb{G}_a$ defined over L (the addition is the usual addition, while the multiplication is given by the usual composition of functions).

Following Goss [16], *a Drinfeld module of generic characteristic* defined over K is a morphism $\phi\colon A \to K\{\tau\}$ for which the coefficient of τ^0 in ϕ_a is a for every $a \in A$ and there exists $a \in A$ such that $\phi_a \neq a\tau^0$. For the remainder of this paper, unless otherwise stated, $\phi\colon A \to K\{\tau\}$ is a Drinfeld module of generic characteristic.

A Drinfeld module $\psi\colon A \to K^{\text{alg}}\{\tau\}$ is isomorphic to ϕ (over K^{alg}) if there exists a nonzero $\gamma \in K^{\text{alg}}$ such that for every $a \in A$ we have $\psi_a = \gamma^{-1}\phi_a\gamma$.

For every field extension $K \subset L$, the Drinfeld module ϕ induces an action on $\mathbb{G}_a(L)$ by $a * x := \phi_a(x)$ for each $a \in A$. Let g be a fixed positive integer. We extend diagonally the action of ϕ on $\mathbb{G}_a^g$.

The subgroups of $\mathbb{G}_a^g(K^{\text{alg}})$ invariant under the action of ϕ are called *ϕ-submodules*. For a ϕ-submodule Γ its *full divisible hull* is

$$\Gamma' := \left\{x \in \mathbb{G}_a^g(K^{\text{alg}}) \mid \text{ there exists } 0 \neq a \in A \text{ such that } \phi_a(x) \in \Gamma\right\}.$$

We define the *rank* of a ϕ-submodule $\Gamma \subset \mathbb{G}_a^g(K^{\text{alg}})$ as $\dim_{\operatorname{Frac}(A)} \Gamma \otimes_A \operatorname{Frac}(A)$.

Definition 2.1 An algebraic ϕ-submodule of $\mathbb{G}_a^g$ is an irreducible algebraic subgroup of $\mathbb{G}_a^g$ invariant under ϕ.

Denis proposed the following problem [6, Conjecture 2].

Conjecture 2.2 **(The full Denis-Mordell–Lang conjecture)** *Let $X \subset \mathbb{G}_a^g$ be an affine variety defined over K^{alg}. Let Γ be a finite rank ϕ-submodule of $\mathbb{G}_a^g(K^{\text{alg}})$. Then there exist algebraic ϕ-submodules $B_1, \ldots, B_\ell$ of $\mathbb{G}_a^g$ and there exist $\gamma_1, \ldots, \gamma_\ell \in \Gamma$ such that*

$$X(K^{\text{alg}}) \cap \Gamma = \bigcup_{i=1}^{\ell} (\gamma_i + B_i(K^{\text{alg}})) \cap \Gamma.$$

Conjecture 2.2 was proved by Thomas Tucker and the author [15] in the case $\operatorname{trdeg}_{\mathbb{F}_q} K = 1$ and Γ is a finitely generated subgroup of rank 1. In this paper we will deal with the "function field" case of Conjecture 2.2, *i.e.,* $\operatorname{trdeg}_{\mathbb{F}_q} K > 1$. Before stating our result, we need to introduce the following notion.

Definition 2.3 We call the modular transcendence degree of ϕ the smallest integer $d \geq 1$ such that a Drinfeld module isomorphic to ϕ is defined over a field of transcendence degree d over $\mathbb{F}_q$.

In [9, Theorem 4.11] the author proved the following result towards Conjecture 2.2.

Theorem 2.4 *With the above notation, assume in addition that the modular transcendence degree of ϕ is at least 2. Let $X \subset \mathbb{G}_a^g$ be an affine subvariety defined over K^{alg} such that there is no positive dimensional algebraic subgroup of $\mathbb{G}_a^g$ whose translate lies inside X. Let Γ be a finitely generated ϕ-submodule of $\mathbb{G}_a^g(K^{\text{alg}})$. Then $X(K^{\text{alg}}) \cap \Gamma$ is finite.*

In Theorem 3.1 we extend the previous result to all finite rank ϕ-submodules Γ. We also note that our proof immediately extends to provide a stronger statement than Theorem 3.1, provided the result from Theorem 2.4 is strengthened.

Remark 2.5 We have two technical conditions in Theorem 2.4 that we will also keep in our extension from Theorem 3.1. The condition that ϕ has modular transcendence degree at least equal to 2 is a mild technical condition, however necessary due to the methods employed in [9]. The condition that X does not contain any translate of a positive dimensional algebraic subgroup of $\mathbb{G}_a^g$ is satisfied by all sufficiently generic affine subvarieties X.

3 Proof of Our Main Result

We continue with the notation from Section 2. We define the *torsion submodule* of ϕ as

$$\phi_{\mathrm{tor}} = \{x \in \mathbb{G}_a^g(K^{\mathrm{alg}}) \mid \text{ there exists } a \in A \setminus \{0\} \text{ such that } \phi_a(x) = 0\}.$$

Next we state our main result.

Theorem 3.1 *Let K be a finitely generated field of characteristic p and let g be a positive integer. Let $\phi\colon A \to K\{\tau\}$ be a Drinfeld module of generic characteristic. Assume the modular transcendence degree of ϕ is at least 2. Let $X \subset \mathbb{G}_a^g$ be an affine subvariety defined over K^{alg} such that there is no positive dimensional algebraic subgroup of $\mathbb{G}_a^g$ whose translate lies inside X. Let Γ be a finitely generated ϕ-submodule of $\mathbb{G}_a^g(K)$, and let Γ' be its full divisible hull. Then $X(K^{\mathrm{alg}}) \cap \Gamma'$ is finite.*

In our proof of Theorem 3.1 we need a uniform version of Scanlon's result [22]. He proved the Manin–Mumford theorem (or equivalently, the Denis–Mordell–Lang conjecture in the case $\Gamma = \phi_{\mathrm{tor}}$) for Drinfeld modules (see his Theorem 1).

Theorem 3.2 *Let $\phi\colon A \to K\{\tau\}$ be a Drinfeld module and let $X \subset \mathbb{G}_a^g$ be an affine variety defined over K^{alg}. Then there exist algebraic ϕ-submodules $B_1, \dots, B_\ell$ of $\mathbb{G}_a^g$ and there exist $\gamma_1, \dots, \gamma_\ell \in \phi_{\mathrm{tor}}$ such that*

$$X(K^{\mathrm{alg}}) \cap \phi_{\mathrm{tor}} = \bigcup_{i=1}^{\ell} (\gamma_i + B_i(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}.$$

In [22, Remark 19], Scanlon notes that his proof of the Denis–Manin–Mumford conjecture yields a uniform bound on the degree of the Zariski closure of $X(K^{\mathrm{alg}}) \cap \phi_{\mathrm{tor}}$, depending only on ϕ, g and the degree of X (see also [23]). In particular, we obtain the following uniform statement for translates of X.

Corollary 3.3 *Let $X \subset \mathbb{G}_a^g$ be a subvariety which contains no translate of a positive dimensional algebraic subgroup of $\mathbb{G}_a^g$. Then there exists a positive integer N such that for every $x \in \mathbb{G}_a^g(K^{\mathrm{alg}})$, the set $(x + X(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}$ has at most N elements.*

Proof Because X contains no translate of a positive-dimensional algebraic subgroup, for every $x \in \mathbb{G}_a^g(K^{\mathrm{alg}})$ the algebraic ϕ-modules B_i appearing in the intersection

$(x + X(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}$ are trivial. In particular, the set $(x + X(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}$ is finite. Thus, using the uniformity obtained by Scanlon for his Manin–Mumford theorem, we conclude that the cardinality of $(x + X(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}$ is uniformly bounded above by some positive integer N. ■

We will also use the following fact in the proof of our Theorem 3.1.

Fact 3.4 *Let $\phi\colon A \to K\{\tau\}$ be a Drinfeld module. Then for every positive integer d, there exist finitely many torsion points x of ϕ such that $[K(x){:}K] \leq d$.*

Proof If $x \in \phi_{\mathrm{tor}}$, then the canonical height $\widehat{\mathrm{h}}(x)$ of x (as defined in [5, 26]) equals 0. Also, as shown in [5], the difference between the canonical height and the usual Weil height is uniformly bounded on K^{alg}. Actually, Denis [5] proved this last statement under the hypothesis that $\mathrm{trdeg}_{\mathbb{F}_q} K = 1$. However, his proof easily generalizes to fields K of arbitrarily finite transcendence degree. For this we need the construction of a *coherent good set of valuations* on K as done in [11] (see also the similar construction of heights from [26]). Essentially, a coherent good set U_K of valuations on K is a set of defectless valuations satisfying a product formula on K (for more details, we refer the reader to [11, §§2, 3]). Then Fact 3.4 follows by noting that there are finitely many points of bounded Weil height and bounded degree over the field K (using Northcott's theorem applied to the global function field K).

Moreover, [11, Corollary 4.22] provides an effective upper bound on the size of the torsion of ϕ over any finite extension L of K in terms of ϕ and the number of places of L lying above places in U_K of bad reduction for ϕ. Because for each field L such that $[L{:}K] \leq d$, and for each place $v \in U_K$, there are at most d places w of L lying above v, we conclude that there exists an upper bound for the size of torsion of ϕ over all field extensions of degree at most d over K in terms of ϕ, d and the number of places in U_K of bad reduction for ϕ. ■

We are ready to prove Theorem 3.1.

Proof of Theorem 3.1. First we note that at the expense of replacing K by a finite extension, we may assume X is defined over K.

Because each polynomial ϕ_a is separable we conclude that $\Gamma' \subset \mathbb{G}_a^g(K^{\mathrm{sep}})$. Let $z \in X(K^{\mathrm{sep}}) \cap \Gamma'$. For each field automorphism $\sigma\colon K^{\mathrm{sep}} \to K^{\mathrm{sep}}$ which restricts to the identity on K we have $z^\sigma \in X(K^{\mathrm{sep}})$ (because X is defined over K). By the definition of Γ', there exists a nonzero polynomial $P \in A$ such that $\phi_P(z) \in \Gamma$. Because ϕ_P has coefficients in K, we obtain that $\phi_P(z^\sigma) = (\phi_P(z))^\sigma = \phi_P(z)$. The last equality follows from the fact that $\phi_P(z) \in \Gamma \subset \mathbb{G}_a^g(K)$. We conclude that $\phi_P(z^\sigma - z) = 0$, and so $T_{z,\sigma} := z^\sigma - z \in \phi_{\mathrm{tor}}$. Moreover, $T_{z,\sigma} \in (-z + X(K^{\mathrm{alg}})) \cap \phi_{\mathrm{tor}}$ (because $z^\sigma \in X$). Using Corollary 3.3 we conclude that for each fixed $z \in X(K^{\mathrm{alg}}) \cap \Gamma'$, the set $\{T_{z,\sigma}\}_\sigma$ has cardinality bounded above by some number N (independent of z). In particular, we obtain that z has finitely many Galois conjugates over K, and so,

$$[K(z){:}K] \leq N. \tag{3.1}$$

Similarly we get $[K(z^\sigma){:}K] \leq N$, and so (using also (3.1)), we conclude

$$[K(T_{z,\sigma}){:}K] \leq [K(z, z^\sigma){:}K] \leq N^2.$$

As shown by Fact 3.4, there exists a finite set of torsion points w for which $[K(w):K] \leq N^2$. Hence, recalling that N is independent of z, we conclude that the set

$$H := \{T_{z,\sigma} \mid z \in X(K^{\mathrm{alg}}) \cap \Gamma' \text{ and } \sigma\colon K^{\mathrm{sep}} \to K^{\mathrm{sep}}\}$$

is finite. Because H is a finite set of torsion points, there exists a nonzero polynomial $Q \in A$ such that $\phi_Q(H) = \{0\}$. Therefore, $\phi_Q(z^\sigma - z) = 0$ for each $z \in X(K^{\mathrm{alg}}) \cap \Gamma'$ and for each σ. Hence $\phi_Q(z)^\sigma = \phi_Q(z)$ for each σ. We conclude that

$$\phi_Q(z) \in \mathbb{G}_a^g(K) \text{ for every } z \in X(K^{\mathrm{alg}}) \cap \Gamma'. \tag{3.2}$$

Let $\Gamma_1 := \Gamma' \cap \mathbb{G}_a^g(K)$. Because Γ' is a finite rank ϕ-module and $\mathbb{G}_a^g(K)$ is a *tame* module (*i.e.,* every finite rank submodule is finitely generated; see [26] for a proof of this result), we conclude that Γ_1 is finitely generated. Let Γ_2 be the finitely generated ϕ-submodule of Γ' generated by all points $z \in \Gamma'$ such that $\phi_Q(z) \in \Gamma_1$. More precisely, if $w_1, \dots, w_\ell$ generate the ϕ-submodule Γ_1, then for each $i \in \{1, \dots, \ell\}$, we find all the finitely many z_i such that $\phi_Q(z_i) = w_i$. Then the finite set of all z_i generates the ϕ-submodule Γ_2. Thus Γ_2 is a finitely generated ϕ-submodule and moreover, using equation (3.2), we conclude that

$$X(K^{\mathrm{alg}}) \cap \Gamma' = X(K^{\mathrm{alg}}) \cap \Gamma_2.$$

Because Γ_2 is a finitely generated ϕ-submodule, Theorem 2.4 finishes the proof of Theorem 3.1. ∎

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