THE HUBBLE DIAGRAMS FOR QUASARS

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We discuss in this talk the optical and radio Hubble-diagrams for the brightest quasars. We shall infer from our results: (1) a strong correlation between redshift and received flux that is consistent with the cosmological interpretation of the emission-line redshifts; and (2) a quantitative upper limit on the permissible amount of pure luminosity evolution. We establish these conclusions twice, by examining the Hubble diagrams both with and without corrections for the selection effects introduced by the flux limits of the quasar catalogs considered.

The sample of 3C R and 4C quasars we use has been described by Schmidt (1976) in these preceedings (see also Schmidt 1968, 1972, 1974, Olsen 1970, Lynds and Wills 1972). We adopt the redshifts, optical and radio fluxes, and radio spectral indices given by Schmidt (1976) as well as the three sets of flux limits he has specified. We note that there are theoretical and observational advantages in defining quasars in terms of their absolute optical luminosity, including observational completeness, independence of observing conditions, and the possibility of isolating one of the most important physical variables (see Bahcall 1971). We eliminate therefore from the sample described by Schmidt those objects that are less than one magnitude brighter than the brightest ellipticals in rich clusters, i.e., we require $\log F_{2500} \geq 22.8$ (F_{2500} in watts per Hz, or $M_B \leq -23.2^m$, H_o = 55 km/s Mpc, Sandage(1973)). We are left with a sample of 112 objects. We assume $q_o = 1.0$ for specificity (and because selection effects are less critical for our purposes with $q_o = 1.0$ than, e.g., with $q_o = 0.0$).

The first step in conventional investigations of the Hubble diagram for galaxies is to chose as "standard candles" the brightest galaxies in rich clusters. We make an analogous choice. Following Bahcall and Hills (1973), we use the brightest quasars in equally-populated redshift bins, indicated by horizontal lines in Figure 1. In the quasar case, there are important selection effects that are associated with the flux limits of the radio and optical samples. We shall discuss these selection effects in detail later. A point by point comparison is made by Hills and Bahcall (1973) between the present treatment of the brightest

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quasars and what is usually done in forming the Hubble diagram of the brightest galaxies in rich clusters.

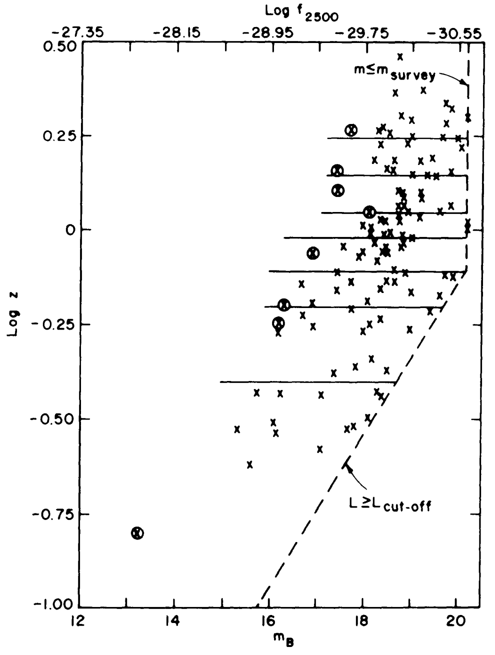

Figure 1. The redshift-magnitude diagram for the whole sample of 112 quasars

We consider first the optical Hubble diagram constructed from the raw data without any corrections. The results are shown in Figure 2a for the brightest quasars in each of eight redshift bins (the same objects are indicated by circles in Figure 1). The bins were chosen so that they all contain the same number of objects (14) in order to provide a sampling of the luminosity function that, except for flux limits, is unbiased with respect to redshift. There is obviously a strong correlation between redshift and apparent magnitude or flux. The best-fit straight line has the form

$$m_{uncorrected} = (4.43 \pm 0.44) \log Z + 17.3 \qquad , \qquad (1)$$

where we have replaced $\log f_{2500}$ given by Schmidt with an equivalent B magnitude, $m_B = -2.5 \log f_{2500} - 56.375$. The correlation coefficient between $\log Z$ and $\log f_{2500}$ is 0.956, i.e., the probability that this result would have arisen purely by chance is less than one in a thousand. Similar results have been obtained by Bahcall and Hills (1973), Burbidge and O'Dell (1973), Petrossian (1974) and others using inhomogeneous samples. The relation analogous to equation (1) for the brightest radio quasars is $\log f_{500}$ Mhz = $-24.9 - (1.1 \pm 0.35) \log Z$, with a correlation coefficient of -0.73 (less than 10 percent chance of having arisen by accident).

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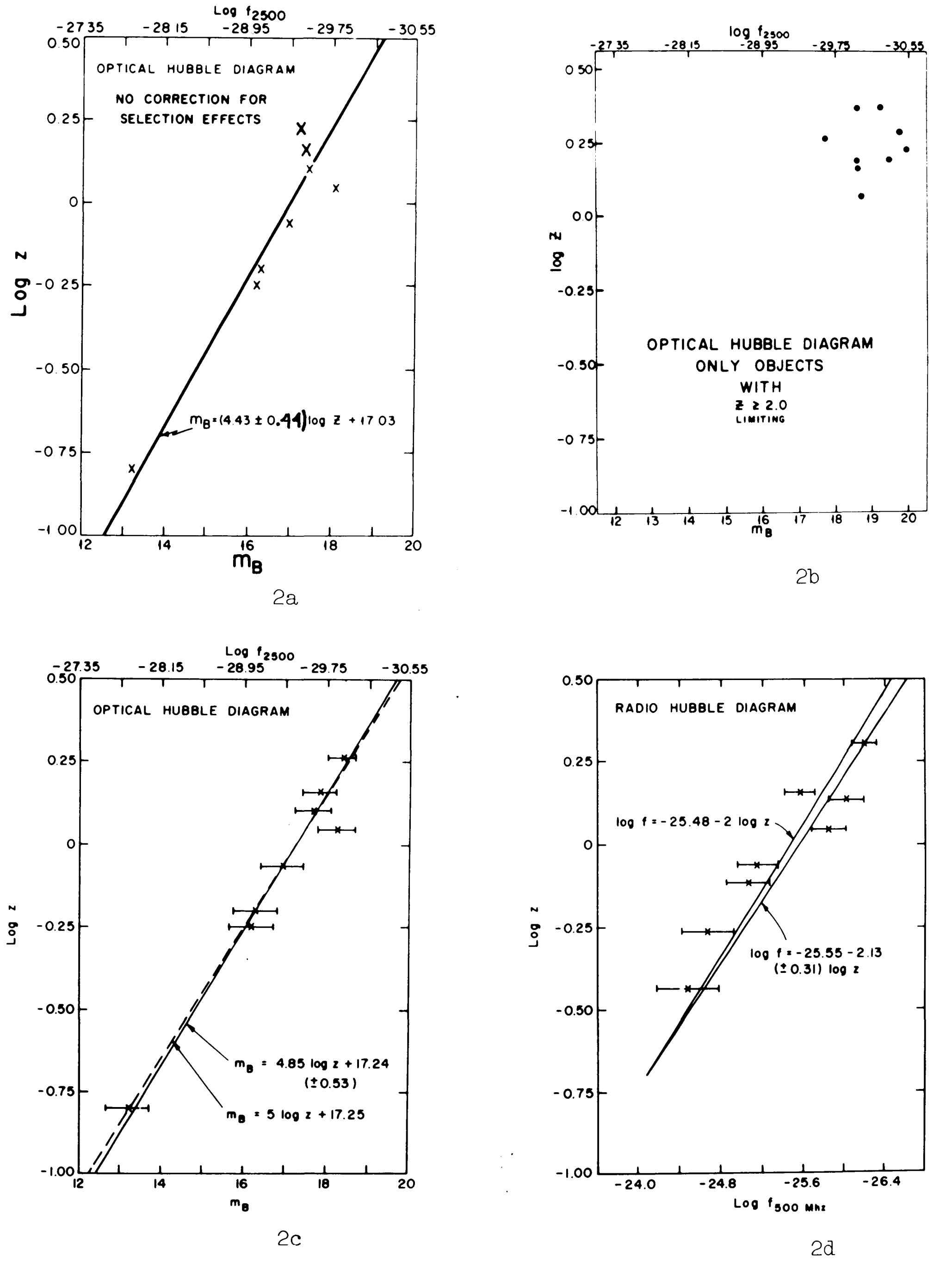

Figure 2. The Hubble Diagrams for the Brightest Quasars

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The strong correlation observed in the raw data between the apparent magnitudes of the brightest objects and their redshifts is consistent with a cosmological origin for the redshifts. No local theory that we are familiar with can explain this correlation. We assume in what follows that the emission-line redshifts of quasars are caused by the expansion of the universe.

We can also calculate an upper limit for optical luminosity-evolution by using the uncorrected luminosity of the brightest object in each of the eight redshift bins. Assumming $L_{brightest}(t)=L_{brightest}(t_o) \exp -\beta t$, we find

$$\beta_{upper\ limit} = 1.46 \pm 0.91 \tag{2}$$

where t is the age of the universe in the units of the present age. It should be stressed that the value of β given by equation (2) is an upper limit since we have ignored so far the selection effect in the sample arising from a fixed cut-off in optical flux. This cut-off naturally gives rise to an apparent brightening of objects at small t (large Z) i.e., a positive β. In fact, we shall see below that a value of β equal to zero is consistent with the present data and we shall assume, in what follows, pure density evolution ($\beta \equiv 0$).

Equation (1) implies an important physical result: the optical luminosity of the brightest quasars has not changed appreciably over many generations (possibly hundreds of quasar lifetimes). This suggests that at least the brightest quasars were not very different at early times ($t \simeq 0.15$) than they are today.

The limits on luminosity evolution for the brightest radio quasars are less informative. One finds, analogous to equation (2), $\beta(\text{radio})_{\text{upper limit}} = 3.6 \pm 1.4$. The relatively large values for both the variance and the absolute value of β for the radio-brightnesses could be primarily a result of the importance of the radio flux limits in selecting the present sample.

We must now consider what to do about the selection effects introduced by the limiting optical and radio fluxes. The simplest procedure would be generalize a suggestion by Burbidge and O'Dell (1973) and use only objects that are bright enough to be unaffected by either flux limit. They also suggested using the second and third brightest objects in each redshift bin as additional tests. However, Burbidge et al. (1973) ignored all radio selection effects. This certainly cannot be done with the present sample; for 85 out of the 112 quasars, or 75 percent, the maximum distance to which they could be seen and remain in the sample is determined by their radio, not optical, flux. If we require quasars to be intrinsically bright enough to be visible at $Z \geq 2.0$ in both radio and optical wavelengths, we find that only 4 objects out of the 3C R sample qualify. There are 9 objects in the 4C Olsen-sample that are bright enough and 7 in the Lynds-Wills 4C sample. With the present approach, we cannot combine objects from the different samples since the optical luminosity function depends upon the radio selection procedure. We also cannot use quasars with $Z > 2.0$ since they would correspond to a greater intrinsic luminosity cut-off and hence require us to eliminate some of the smaller Z objects. We therefore choose the sample with the largest number (9) of intrinsically bright objects, the Olsen 4C sample. The results are shown in Figure 2b. It is obvious that the 9 bright quasars selected in this way are all at such large redshifts that one cannot determine meaningfully a slope in the red-shift-magnitude diagram even if we ignore statistical fluctuations in this small sample. The brightest objects are extremely rare and are naturally found only at the largest redshifts. The total number of objects remaining in the sample of 112 quasars treated as described above is less than were in each of the eight independent redshift bins of Figure 1 and 2a. A formal least squares solution yields $m_b = (0.92 \pm 2.34) \log Z + 18.74$. The bunching effect at Z(limiting) shown in Figure 2b cannot be avoided by using a different value for Z(limiting). For example, if Z(limiting) = 1.0, then 8 out of the allowed 9 objects have log Z within ± 0.07 of each other. We conclude that further progress requires an explicit calculation of the selection effects so that quasars at smaller redshifts may be used.

Bahcall and Hills (1973) described a method by which a given luminosity function can be used to evaluate both the selection effects associated with the fixed limiting fluxes and the variances expected because of the finite sample size in each bin (14 objects in the present case). The later feature is important because it allows us to make a self-consistent test for goodness-of-fit. In what follows, we use the Bahcall-Hills procedure to construct both the optical and radio Hubble diagrams. The basic assumptions are: (1) the emission-line redshifts are of cosmological origin and (2) the shape of the luminosity function is independent of time (redshift), i.e., there is no luminosity evolution.

The ideas behind the procedure may be most clearly visualized by supposing for a moment that the luminosity function (radio or optical) is a uniform distribution between L(cut-off) and a maximum quasar luminosity, L(MAX). Then for redshifts so small that luminosities less than L(cut-off) could have appeared in the Schmidt-catalog, there is no correction because we are sampling the entire luminosity function equally in each bin. However, for redshifts above some critical value ($Z_c \simeq 0.78$, $\log Z_c = -0.11$ for optical luminosities in the Schmidt sample) only objects with luminosities L(limiting,Z) > L(cut-off) could appear in the quasar catalog. By randomly sampling (14 times in our case) the luminosity function between L(limiting,Z) and L(MAX) we obtain progressively brighter objects as Z increases [L(limiting,Z) approaches L(MAX)]. Also the standard deviation of the brightest object in each bin will decrease as L(limiting,Z) approaches L(MAX). This simple example gives the correct physical picture of the correlations to be calculated; the rest is details. We mention, incidentally, that of the two processes described above, (1) binning into equally populated samples and (2) calculating luminosity corrections, binning is more important and calculating the corrections is more complicated.

We obtain luminosity functions by requiring that (Schmidt 1968) the average value of V/V_{MAX} be equal to one-half when density evolution is included. This does not uniquely or optimally define a luminosity function, but the corrections we calculate are not sensitively affected by small differences in luminosity functions. Some satisfactory evolutionary laws are of the form $\rho(t) = t^{\alpha} \exp \gamma(1-t)$, with $\alpha = 0$, $\gamma = 10^{+4}_{-3}$, $\alpha = 1$, $\gamma = 15$, $\alpha = 2$, $\gamma = 17$, $\alpha = 3$, $\gamma = 20$, and $(1 + Z)^n$ with $n = 5.5^{+2.5}_{-1}$. We prefer evolutionary laws that are expressed in terms of cosmic time since these are simplest to interpret theoretically. Moreover, laws of the form $e^{+\alpha}e^{\gamma t}$ are suggested by quasar theories in which stellar or galactic evolution is important and, for $\alpha > 1$, these laws predict very few quasars with $Z > 2.5$.

We have calculated luminosity functions using several of the derived evolutionaly laws; the results do not depend much on the particular evolutionary law that was chosen. For $q_o = 1.0$ and $\rho(t) = \exp 9.5(1-t)$, we find with

$$\Phi(L)dL = \Phi^*(L/L_*)^{\alpha} \exp{-L/L_*}\, dL/L_* \qquad (3)$$

that for optical wavelengths $\alpha = -1 \pm 1$, $\Phi^* = 0.2 \pm 0.1$ Gpc^{-3}, and $F^*(2500) = (4.5 \pm 0.9) \times 10^{23}$ watts HZ^{-1} ($M^* = -25.3 \pm 0.2$). For the radio luminosity function, $\alpha = -2.4 \pm 0.3$, $\Phi^* = (3 \pm 2) \times 10^{-3}$Gpc^{-3} and $F^*(2500) = 2.1 \pm 0.8) \times 10^{28}$ watts HZ^{-1}. The errors quoted are equivalent to 1-σ errors. Functions of the general form given by equation (3) have been used by Schechter (1976) and others to describe galaxy luminosity functions (Schechter obtains $\alpha \cong -1.25$ and M^*_B galaxy $\simeq -20.85$). We used functions of the form given in equation (3) since they provide convenient empirical fits to the average luminosity functions defined by the present sample. The most important feature expressed by equation (3) is that there is a sharp steepening of the slope of the empirical luminosity functions near a characteristic luminosity L_*.

The corrections for the selection effects can be calculated from the empirical luminosity functions if we assume that the N-quasars in each redshift bin are Poisson-distributed over the accessible (at the given Z) part of the luminosity function. We have, up to a common factor,

$$\int_{L_{limiting}(Z)}^{\infty} \Phi(L)dL = N \quad , \qquad (4a)$$

$$\int_{L_{brightest}(Z)}^{\infty} \Phi(L)dL = \ell n2 \quad , \qquad (4b)$$

with (in our case) N=14. A slightly more complicated equation determines the variance σ^2 (log L(brightest,Z). For the radio corrections, each of the three samples (3C R, Olsen, and Lynds-Wills) were treated separately in each redshift bin and the properly weighted averages were used in the Hubble diagram (for the optical case, the limits of the

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three samples are so similar that this averaging was unimportant). We show in Table 1 the numerical corrections and their variances that were calculated for use with the present sample. We also calculated corrections for a specific case treated by Bahcall et al. (N=15, log F_{2500}(minimum) = 22.4) with Monte-Carlo computer calculations on a numerical luminosity function derived by Schmidt (1970) for a sample of optically-selected quasars. Our results are in good agreement with the previously derived corrections, indicating that the computed values of Δm and $\sigma^2(\Delta m)$ are not sensitive to details of the luminosity function. Strictly speaking, we should have used a bi-variate luminosity function (or both optical and radio luminosities, L_O and L_R) to calculate the corrections. However, our approximation is exact if the bi-variate luminosity function can be factored as a product of independent functions of L_O and L_R or (for the optical corrections) as a product of a function of L_O times a function of L_O/L_R. We intend to carry out corrections using bi-variate luminosity functions, but we expect that the improvements introduced by this more complicated procedure will be smaller than the variances $\sigma^2(\log L_B, Z)$.

TABLE I. CALCULATED CORRECTIONS

Z median (bin)	Δm OPT	$\sigma(\Delta m)$ OPT	$\Delta \log f$ radio	$(\log f)$ radio	Brightest in Optical	Brightest in Radio
0.309	0.0	0.51	0.00	0.30	3C 273	3CR 48
0.531	0.0	0.51	0.32	0.25	4C 23.24	3CR 147
0.704	0.0	0.51	0.43	0.21	4C 16.30	3CR 380
0.872	0.05	0.50	0.53	0.19	3CR 454.3	3CR 196
1.031	0.15	0.45	0.62	0.17	3CR 208	3CR 208
1.220	0.25	0.42	0.70	0.17	4C 06.41	3CR 181
1.561	0.43	0.40	0.81	0.15	3CR 298	3CR 298
2.060	0.68	0.34	0.93	0.12	4C 29.1	3CR 9

We note a simple prediction that follows from the joint assumptions of no luminosity evolution and a cut-off (or monotonically-decreasing) luminosity function at large luminosities (cf. equation 3). These ideas imply that the standard deviation of the luminosity of the brightest objects in equally-populated redshift bins should decrease monotonically with increasing redshift if the sample is flux limited (neglecting intrinsic variability). This prediction may be testable since the effect is rather large (see Table I, columns 3 and 5).

Our final results are shown in Figures 3c and 2d. The indicated errors bars are, in all cases, the theoretical standard deviations calculated as described above. Both the optical and the radio Hubble diagrams are good fits (in the χ^2 sense) to the predicted cosmological curve obtained by assuming no luminosity evolution and the calculated (Table 1) corrections and variances. This is satisfying but not surprising since the uncorrected Hubble diagrams (cf. Fig. 2a) already show that the basic assumptions of cosmological redshifts and no

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luminosity evolution are good approximations. We note in closing that the radio and optical Hubble diagrams refer to different objects; in six of the eight redshift bins the brightest optical quasar is not the brightest radio quasar.*

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DISCUSSION

Arp: What effect do the violently variable quasars have on your analysis?

Bahcall: They are very rare and do not influence importantly the statistical averages. The brightest quasars at the epoch studied are shown in Fig. 1. If quasar variability either in frequency or amplitude were strongly correlated with redshift, then this should show up in the Hubble diagram.

Petrosian: The fact that the m-log(z) slope is five after application of correction is an indication of the correctness of calculation but not an independent proof of the cosmological hypothesis. Is the difference between the expected slope of -2 for the $\log(S_{radio}) - \log(z)$ relation and the calculated one due to uncertainty in the calculated luminosity function or some other cause?

Bahcall: The strong correlation observed in the raw data between the luminosities of the brightest quasars and their redshifts supports the cosmological hypothesis. The corrected Hubble diagrams are consistent, to present accuracy, with the joint assumptions of cosmological redshifts and no appreciable pure luminosity evolution. The difference between the expected and observed slopes for the radio Hubble diagram is not significant.

Setti: I am not clear about your limit on the luminosity evolution, because suppose that the luminosity function steepens with increasing redshift, then the expected luminosity of the most luminous objects in the various redshift bins may not change while the average luminosity per bin would increase with z.

Bahcall: If by "steepen" you mean that the characteristic luminosity L_* increases with cosmic time, then the expected luminosity of the brightest objects also increases in the same way; it just scales with $L_*(t)$. This is the case we have treated in the text. On the other hand, one can imagine special cases in which the shape of the luminosity function varies in just such a way that the average luminosity in a bin changes with time but $L_{Brightest}$ remains constant. These special cases are probably best studied using a maximum-likelihood estimate that utilizes all the members of a bin.

Setti: Recently Zamazani and I have derived the shape of the optical luminosity function for radio selected quasars. This has been done following a step by step procedure in the redshift-apparent magnitude diagram of all known radio emitting quasars, taking into account possible strong selection effects. We find that the shape of the optical luminosity function can be best represented by two power laws with a rather sharp break at $F_{2500} \simeq 23.4$ (H_o = 100 km s^{-1} Mpc^{-1}, q_o = 1), in very good agreement with the findings reported by Dr. Bahcall.

McCrea: Does the constant in the m-z relation give any new determination of the Hubble constant? What is the descriptive meaning of the particular number-evolution that you have used - does it mean that there were more quasars in the past?

Bahcall: First question: no.
Second question: yes. It is the same as used by Schmidt and others.

Jauncey: It will be interesting to see where Malcolm Smith's Tololo quasars fit on your m-z diagram.