13

Heavy quark fragmentation and baryon production

13.1 Introduction

In this chapter we are going to consider a few further phenomena that should be included in a realistic model for hadron production.

We start by considering heavy flavor fragmentation. There should be no production of heavy flavors in the fragmentation process itself because of the very strong suppression from the tunnelling process. Heavy quark jets will nevertheless occur when the heavy flavor is produced in a process where there is a large energy concentration, e.g. in an e^+e^- annihilation process. Then the first-rank hadron in the jet contains the heavy flavor and such a hadron will, in general, have a larger mass than the ordinary hadrons, which are made up from the lighter quark flavors, u, d and s. We have seen (cf. Chapter 9) that for the usual Lund fragmentation function a larger-mass hadron will have a 'harder' z-spectrum, i.e. the typical value of the fragmentation variable z will be closer to unity.

We will consider a number of different models, both those that tend to give $1 - z \propto 1/M$ and those that give $1 - z \propto 1/M^2$ for the first-rank hadron with large mass M. We will also consider a rather different treatment which leads to the so-called Peterson formula, [99], for heavy quark fragmentation. The basic idea is to make use of the wave functions obtained in a lightcone-dynamical scenario.

After that we will continue with a discussion of baryon-antibaryon ($B\bar{B}$) production. A baryon, or at least a baryon resonance state, may well have a more complex structure than that which a $(1 + 1)$-dimensional dynamical scenario can provide. The number of parameters which occur for the description of $B\bar{B}$-production in the Lund model is rather large. The number of baryon states is eight if we count the ones in the $J^P = (1/2)^+$ octet (we use the usual notation with J the spin and P the parity of the states) (N, Λ, Σ, Ξ) and in the $J^P = (3/2)^+$ decuplet (Δ, Σ^*, Ξ^* and Ω^-).

The number of parameters to describe their yields is, however, basically seven! There are good reasons for complaints about the predictive powers of the model.

The two models we will discuss, the *diquark model* and the *popcorn model*, have, however, some endearing qualities. If we are only interested in describing the ordinarily observed baryon states, i.e. the nucleons N (proton and neutron) and the Λ-particle, then their rates are determined by a single number, the baryon-to-meson rate. This is so if we use an SU(6) (flavor-spin) symmetrisation of the wave functions.

Both models account for a strong increase with energy in the baryon-to-meson ratio (which is not only a kinematical effect) as well as an increased baryon fraction in gluon jets. This will be discussed after the introduction of the Lund gluon model (see Chapter 15). Also both models exhibit a string 'drag' effect in the sense that the B and $\bar{B}$ in a pair tend to go in different directions along the string. This is due to the correlation between flavor and color, i.e. a q has color and flavor and is therefore dragged by the string in the opposite direction to a $\overline{q}$, which has anticolor and antiflavor. This was experimentally observed early on by the TPC group at PEP.

The difference between the models is mostly related to the transverse momentum correlations. In the diquark model the B and $\bar{B}$ are neighbors in rank and therefore contain stronger transverse momentum correlations than $B\bar{B}$-production in the popcorn model, where there may be mesons produced in between. The experimental data tend to confirm the popcorn scenario.

We will end the chapter with a brief discussion of a different use of the Lund model fragmentation formulas in the way suggested by a group in UCLA, [38]. The basic idea is to make use of the Lund model area law to determine the *relative rates for different kinds of hadrons*. In this approach there is no use of the probabilities for producing different $q\bar{q}$-pairs. Instead it is the fact that the hadrons have different masses, and consequently will effectively use up different areas in the Lund model fragmentation formulas, that provides the relative probabilities. The model contains an intriguing picture, which, for some reason that is not understood, seems to mirror rather well the observed rates and spectra for different hadrons.

13.2 Heavy quark fragmentation

In Chapter 12 we learned that, at least within a tunnelling scenario with an available energy per unit length equal to the string tension $\kappa \simeq 1$ GeV/fm, there is no heavy quark production along the string.

There is nevertheless the possibility that a heavy quark pair $q_h, \overline{q}_h$ is

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produced initially in e.g. an e^+e^- annihilation event, if there is a large energy concentration available from the annihilating pair. Later, we will also discuss a particular 'hard' process in which a gluon may split up into a $q\bar{q}$-pair. For this we will once again need the possibility of fragmenting strings containing heavy quarks.

We will start with a few introductory remarks due to Bjorken. After that we will consider three different scenarios.

In the first we will use the Lund fragmentation function based upon the usual area suppression.

In the second we will use a prescription first proposed by Bowler, [32]. He noted that the area spanned by a heavy quark, moving until it meets its 'light' partner (from the $q\bar{q}$-pairs usually produced) to form a hadron, is smaller than that used in the Lund model formula. A heavy particle connected to a string moves along a hyperbola, while a massless quark moves along the lightcone which is the asymptote of the hyperbola. Therefore the area spanned is smaller for the heavy particle. We note that in the interpretation we used for the area law in Chapter 11, it was actually the area in space-time which should occur in the area law. We will show that a correction of the type Bowler advocates leads to a different version of the Lund model fragmentation formulas, in which there are different values for the fragmentation parameter a for the light and for the heavy flavor.

We will then pursue a very different approach to fragmentation and derive the Peterson formula, [99]. In this approach the basic idea is to build up a wave function for the final state from the lightcone dynamics that we sketched in Chapter 3. Such a wave function is based upon the off-shell nature of the state. This leads to a simple one-parameter formula for the distribution of the first-rank hadron in a heavy quark jet, which has been used rather successfully.

1 *Bjorken's remarks*

The following arguments for an average cascade correspond to the essence of Bjorken's ideas. Suppose that we consider an ordinary quark jet with, for simplicity, a single kind of hadron (mass m). The first-rank hadron will have rapidity y_l and the rest will have rapidities $y_l - \delta y, y_l - 2\delta y, \ldots$. This means that the total energy W is

$$m \exp y_l \sum_{j=0} \exp(-j\delta y) = \frac{m \exp y_l}{1 - \exp(-\delta y)} = W \tag{13.1}$$

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We conclude that the first-rank particle will take a fraction of the total energy

$$z_l = \frac{m \exp y_l}{W}, \quad z_l + \exp(-\delta y) = 1 \tag{13.2}$$

This is equivalent to the results in Eq. (9.11) in connection with the iterative cascade models.

Next we consider a heavy quark jet and assume that the first-rank hadron has (large) mass M at rapidity y_h. *All the remaining ones should, however, behave as before*, i.e. have the 'ordinary' average rapidities $y_h - \delta y, y_h - 2\delta y, \ldots$. This means that Eq. (13.1) is exchanged for

$$M \exp y_h \left[1 + \sum_{j=1} \frac{m \exp(-j\delta y)}{M}\right] = M \exp y_h \left[1 + \frac{m(1-z_l)}{M z_l}\right] = W. \tag{13.3}$$

Therefore the first-rank particle in a heavy quark jet should take a fraction of the total energy

$$z_h = \frac{M \exp y_h}{W} = \frac{1}{1 + m(1-z_l)/M z_l} \simeq 1 - \frac{m_0}{M} \tag{13.4}$$

The whole discussion is evidently an instructive demonstration of the difference between rapidity and energy-momentum. From a knowledge of ordinary quark jets we may guess that the characteristic mass m_0 should be of the order of 1 GeV.

2 Ordinary Lund area suppression versus a more literal interpretation

The Lund model fragmentation function for the production of a hadron with flavors α, β and with a mass m is given by

$$f_{\alpha\to\beta}(z)dz = \frac{Ndz}{z} z^{a_\alpha - a_\beta}(1-z)^{a_\beta} \exp\left(-\frac{bm^2}{z}\right) \tag{13.5}$$

It is mostly used with the same value for the parameters, $a_\alpha = a_\beta \equiv a$. For this case we found in Chapter 9 that for a commonly used value of $a = 0.5$ there is a maximum value of f when the fragmentation variable $z = 1 + bm^2 - \sqrt{1 + (bm^2)^2}$.

Therefore in this case the correspondence to z_h in Eq. (13.4) will depend upon the mass of the first-rank hadron according to

$$z_{1,oL} \simeq 1 - \frac{1}{bM^2 + \sqrt{1 + (bM^2)^2}} \tag{13.6}$$

(the notation oL stands for 'ordinary Lund'). For large values of M it behaves as $1 - (\mu_0/M)^2$ instead of linearly as in Bjorken's guess.

Phenomenologically the behaviour in Eq. (13.6) seems to be too stiff, i.e. to predict values of z that seem too large although they are not

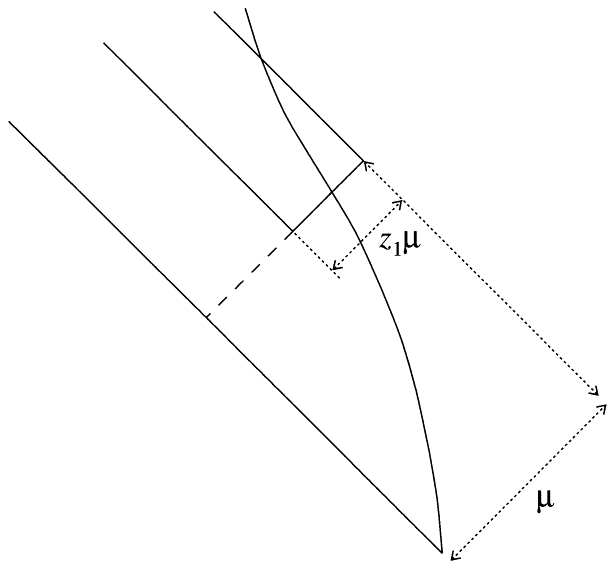

Fig. 13.1. The motion under the influence of the string tension of a heavy flavor parton in the coordinate system in which the parton was originally at rest.

actually excluded by the experimental data. We will now consider a modification introduced by Bowler (in the context of the Artru-Menessier-Bowler model; see Chapter 8).

We will go to a frame where the (original) q_h is initially at rest (see Fig. 13.1). According to the equations of motion for a heavy quark it will start to be dragged away by the string field along the hyperbola

$$(\kappa x_+ - \mu)(\kappa x_- + \mu) = -\mu^2 \tag{13.7}$$

(cf. Chapter 6 and note that in the rest frame of the q_h the total $p_+ = \mu$).

We will assume that a first-rank hadron with mass M is produced, from q_h together with a massless $\overline{q}_1$ stemming from the first vertex, with energy-momentum fraction z_1 as indicated in Fig. 13.1. We also find that the negative lightcone component of the first vertex x_{-1} is, from the mass requirement, given by

$$\kappa x_{-1} = \frac{M^2}{z_1 \mu} \tag{13.8}$$

Bowler then assumed that *only the area between the broken line and the orbit of the heavy quark* (Fig. 13.1) *should be counted in connection with* *an area suppression law*, i.e. he suggested the region

$$A_B = \kappa^2 \int_0^{x_{-1}} dx_- \int_0^{x_+(x_-)} dx_+ \tag{13.9}$$

where the quantity x_+ (x_-) is calculated from Eq. (13.7). The integrals are easily performed using Eqs. (13.7), (13.8) and we obtain for Bowler's area

$$A_B = \frac{M^2}{z} - \mu^2 \left[1 + \log\left(\frac{M^2}{\mu^2 z}\right)\right] \tag{13.10}$$

If we introduce Bowler's area into the fragmentation function in Eq. (13.5) (with a single value of a) then we obtain:

$$f_B = \frac{N' dz}{z} z^{a_h - a}(1-z)^a \exp\left(-\frac{bM^2}{z}\right) \tag{13.11}$$

where we have incorporated the difference between Bowler's area and the usual area as a change in the normalisation constant $N \to N'$ and as a new a-parameter

$$a_h = a - b\mu^2 \tag{13.12}$$

characteristic of heavy quark fragmentation.

This result is interesting for several different reasons. Firstly it is evident that the spectrum will become softer than the one obtained in Eq. (13.6) because in Eq. (13.11) there is a negative z-power. Due to the very strong suppression in the exponent there is, of course, no trouble from this term in the normalisation of f_B.

An observational problem is that some of the b-flavored particles will decay into c-flavored ones. Therefore the observed c-flavored mesons have a different spectrum from that obtained using any of the formulas discussed above. Also, ordinary Lund model fragmentation looks very similar to the observed data, e.g. for D^*-mesons, because of this decay contribution. Only when the experimentalists have been able to disentangle the corresponding B-meson signal will it be possible to know whether any of these alternatives is correct.

A second reason is related to the findings in Regge theory. We have in an earlier discussion (Chapter 10) related the parameter a to the Regge intercepts and we see from the result in Eq. (13.12) that there should be a smaller intercept if there are larger-mass constituents involved. This is in accordance with the early phenomenological findings (cf. [46]).

3 A different approach based upon a wave function from lightcone dynamics

The Feynman propagator, in the contexts in which have met it, contains the notion of the *virtuality of a state*. In particular the free propagator describes the way in which a quantum fluctuation may develop in space-time according to Heisenberg's indeterminacy relation. We will find a similar description of a decaying state, i.e. a resonance, in connection with the Breit-Wigner form factor, in Chapter 14. In that case the correspondence to the Feynman limiting $\epsilon \to 0$ in Eq. (3.81) has a size corresponding to the inverse lifetime of the state.

A bound state in quantum theory is often described in terms of an exponentially falling wave function in space. This corresponds in energy-momentum space to an inverse power in the momentum. Our experience of form factors, which corresponds to the description of extended distributions in space, cf. the example of the elastic proton form factors in Chapter 5, confirms this general inverse-power behaviour in energy-momentum space.

In [37] there is a treatment of the possible bound-state wave functions using the lightcone formalism for the Feynman rules that we considered in Chapter 3. A possible wave function for a $c\bar{q}_l$-state (index l for light so that the state corresponds to a D-meson) is deduced from the distribution

$$|\psi|^2 \sim N \frac{\delta(1 - x_1 - x_2)}{(M_D^2 - m_c^2/x_1 - m_l^2/x_2)^2} = N' \frac{\delta(1 - x_1 - x_2)}{(1 - 1/x_1 - \epsilon/x_2)^2} \tag{13.13}$$

for the energy-momentum fractions x_j of the two constituents. Here N is a normalisation constant and the masses M_D, m_c, m_l correspond to the D, the c-quark and the light quark, respectively. In the second line it is assumed that the mass $M_D \simeq m_c$ is divided out and that $\epsilon = m_l^2/m_c^2$.

In [99] the authors take a step further and assume that the gross features of the amplitude for a heavy quark Q to fragment into a hadron $H = Q\bar{q}$ and a light quark q should be determined in a similar way. This means that it is the value of the energy transfer, ΔE, in the breakup process which will determine the distribution. Therefore the amplitude should behave basically as $(\Delta E)^{-1}$, ΔE being proportional to the denominator in Eq. (13.13).

They include a factor $1/z$ from the longitudinal phase space, perform the integral over the δ-distribution to get $x_1 = z$, $x_2 = 1 - z$, and obtain the shape

$$D_Q^H(z) = \frac{N}{z[1 - 1/z - \epsilon/(1-z)]^2} \tag{13.14}$$

as a possible fragmentation function for a heavy quark into a first-rank hadron. The formula evidently contains only one parameter and has been used extensively and successfully with reasonable values of ϵ.

This means that it is possible to use a very different approach to fragmentation than that usually advocated in this book. The lesson is that although only experimental data can distinguish between possible theoretical developments, it is actually quite difficult to observe such differences since very different functional shapes give closely similar predictions for the experimental data.

13.3 Baryon-antibaryon production

1 Preliminaries

We will make use of two general assumptions in $B\bar{B}$-production.

B1 We will only be concerned with the baryons in the octet, $J^P = (1/2)^+$, and decouplet, $J^P = (3/2)^+$, representations of SU(3) flavor and we will neglect all other production channels.

B2 The octet and decouplet can be made into a 56-representation (with completely symmetrical wave functions) of the group SU(6) in combined flavor-spin assignment. We will assume that all wave functions of the constituents are determined in accordance with the SU(6) requirements and that only the states in the 56-representation are actually produced.

The first assumption is one of economy. We know that the higher baryon resonances occur in only very tiny fractions in those exclusive channels that have been studied.

The second assumption is very basic in the Lund model. The production mechanism in the model is determined not only by the fragmentation probability (e.g. the area suppression law) but also by the available number of states (e.g. the phase-space factors).

Although it is known that SU(6) symmetry is rather badly broken (the different states do not have the same masses and here also we will break the symmetry in a similar way) we will insist on a projection to completely symmetric flavor-spin states for the baryons. If we did not use this requirement then the probability of picking up three u-quarks at random (making a Δ^{++}-baryon) would be one chance in 27 (assuming only u, d and s are produced, each with, for simplicity, the same probability). But the probability of picking up a u-quark, a d-quark and an s-quark at random, producing any of the three states Λ, Σ^0 or Σ^{*0}, would be six

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times larger, by pure combinatorics. Such predictions would be very much in conflict with the experimental findings.

The requirement, of a totally symmetric baryon flavor-spin wave function, means that if we start by producing e.g. an (effective) diquark $(ud)_1$ (corresponding to a state with spin = isospin = 1) and add an s-quark we have a probability of only 1/2 of obtaining a symmetric state; of this 1/3 will correspond to a decouplet (Σ^{*0}) and 1/6 to an octet (Σ^0). The remaining 1/2 corresponds to the production of a state outside the 56-representation.

2 The diquark production model

We will next outline a possible production mechanism for effective diquarks. We do not believe that diquarks, although we may equip them with different internal quantum numbers like color, spin and isospin, are basic quanta of the QCD force field. A diquark is a qq-state, which from a color point of view behaves as a $\overline{3}$, i.e. effectively as a $\overline{q}$-charge.

It is, however, reasonable that even an extended charge may have an effective coupling to the color force field in the same way as a heavy $\overline{q}$. The density of virtual q-particles may be sufficiently large to make the probability of finding a partner in a color-$\overline{3}$ diquark state approximately unity. Then the diquark may, together with an antidiquark (a $\overline{qq}$-pair) tunnel out like a heavy quark pair. This is the basic production mechanism in the diquark model.

In this way we obtain the following tunnelling properties.

- All hadrons have the same transverse momentum production mechanism.
- A $B\bar{B}$-pair, sharing a $(qq, \bar{q}\bar{q})$-pair, are neighbors in rank and are therefore sufficiently close in rapidity for correlation studies.
- As the tunnelling probability predicts a fast falloff for (effective) color charge masses above $\sqrt{\kappa/\pi} \simeq 0.25$ GeV a small difference in mass means a large change in probability.

The last property implies that the strange diquarks (and antidiquarks) are strongly suppressed compared to the non-strange ones. Λ-particles contain an s-quark and a $(ud)_0$-diquark (spin = isospin = 0), and also (small) $(d + us)$-components and so on. Due to the suppression of strange diquarks a Λ will not be produced very often with a $\overline{\Lambda}$ (which may occur for the latter diquark combination) but instead with an $\overline{N}$ (antiproton or antineutron, which contains only non-strange diquarks) and a K or a K^* to compensate the strangeness. However, if we do find a $\Lambda\overline{\Lambda}$-pair in an event then they are almost always rank neighbors in the diquark production model.

In order to make calculations in the model it is necessary to determine the 'masses' of the diquark pairs. The lightest diquark masses, obtained by consideration of the masses of, and mass differences between, the baryons may well be around 0.4 GeV. This would set the overall ratio of the production of diquark pairs to quark pairs at around 0.1. Such a production ratio of $B\bar{B}$ to mesons seems from the experiments to be in the right range (see the discussion after the popcorn model has been presented).

The remaining parameters in this model, $(ud)_1/3(ud)_0 \simeq 0.05$, the suppression ratio of strange diquarks $(us)(d)/(ud)(s) \simeq 0.4$ and the suppression ratio of double-strange diquarks $(ss)_1/(ud)_0 \simeq 10^{-3}$ are at best fitted parameters. The decouplet-to-octet ratio has been kept at unity but could of course be changed if future experiments should require it.

It is the diquark-to-quark production ratio which is the main parameter for the relative ratio of Λ's, protons and neutrons. If one were to decrease e.g. the ratio $(ud)_1/3(ud)_0$ then the increase in directly produced Λ's would compensate the decrease in Λ-production through the channels $\Sigma \to \Lambda + X$, $\Sigma^* \to \Lambda + X$, etc. (X is any other decay product).

Such a change would, however, produce a different amount of e.g. Δ's in a jet. At present a thorough search is being conducted for the resonance content in e^+e^- annihilation events. It will be interesting to see whether one can understand the spectra from these simple considerations (although we necessarily have five parameters for the detailed content already!).

Let us consider some of the consequences of the present scheme. For 30 GeV e^+e^- annihilation events, about 30% of these will contain one $B\bar{B}$-pair and about a further 6% two pairs. About half of the $B\bar{B}$-pairs will decay into $p\bar{p}$-states. Therefore in about 25% of the observed $p\bar{p}$-combinations the p stems from one original pair and the $\bar{p}$ stems from another. This evidently waters down the correlations stemming from the fact that the $B\bar{B}$-pairs are produced as rank neighbors.

For a directly produced $p\bar{p}$-pair in a quark jet, the $\bar{p}$ is about half a unit in rapidity behind the p (in a q-jet it is evident that we will first produce the baryon and only afterwards, i.e. at lower rank, the antibaryon). Regarding the analysis of an event, however, the quark and the antiquark directions are not defined a priori.

If we use one of the customary axes for an event, the thrust axis, cf. Chapter 15, to study the rapidity difference $|y_p - y_{\bar{p}}|$ we end up with an average distance around 1.3, which is evidently far away from the primary result. The transverse momentum correlations are also watered down and we find that

$$\frac{\langle \mathbf{p}_{\perp p} \cdot \mathbf{p}_{\perp \bar{p}} \rangle}{\langle \mathbf{p}_{\perp}^2 \rangle} \simeq 0.3 \tag{13.15}$$

There are some particular features of $B\bar{B}$-production found in connection with events where there is gluon radiation. These features are discussed in Chapter 15.

3 *The popcorn mechanism*

The gluon radiation influences the transverse momentum correlations. When gluons are included the correlation parameter in Eq. (13.15) becomes only half as large as in the diquark production model. But, according to the experiments, this is still a bit too big.

The tunnelling out of non-fundamental quanta such as the diquark-antidiquark pairs is also a less pleasing aspect of the model. Therefore we have produced another model within the Lund scheme, the *popcorn model*. This is based upon an idea first advocated in [40], i.e. that the baryons are produced in a stepwise manner, 'popping out'.

Unfortunately the popcorn model necessarily increases the number of parameters for the detailed content of the baryon species to seven, although it is still the same for the proton, neutron and Λ-particle.

A general fact about the number of parameters in a model is that even if they are not in practice used (an example is the probability of producing the decouplet baryons more or less often than the octet baryons) they are still parameters! This general definition of the notion 'parameter' means 'a possible degree of freedom not fixed by the present dynamics'.

The popcorn mechanism diminishes the correlations between the B and $\bar{B}$ in a pair (they will no longer necessarily be neighbors in rank) but it also provides a scenario in which massive but loosely bound 'diquarks' are produced, without invoking the tunnelling scenario for diquarks *per se*.

In order to understand this consider Fig. 13.2. We will assume that there are space-time regions, such as A, B and C in the figure, where there are fluctuations with the 'wrong' colors in the field. In simple language the original $q_0\bar{q}_0$-pair may start out as a color $\mathsf{r}\bar{\mathsf{r}}$-pair (although we remember that the state is actually a coherent color state). The field is then a $\mathsf{r}\bar{\mathsf{r}}$-colored one; 'wrong' means that there may be fluctuations inside regions of the field which are $\mathsf{g}\bar{\mathsf{g}}$ or $\mathsf{b}\bar{\mathsf{b}}$).

Then in such a region we would see the color combination $\mathsf{rg} = \bar{\mathsf{b}}$ in one direction and the combination $\bar{\mathsf{g}}\bar{\mathsf{r}} = \mathsf{b}$ in the other (if the colors happen to combine in this particular way). Under these circumstances we find that the existence of the color fluctuation does not change the energy

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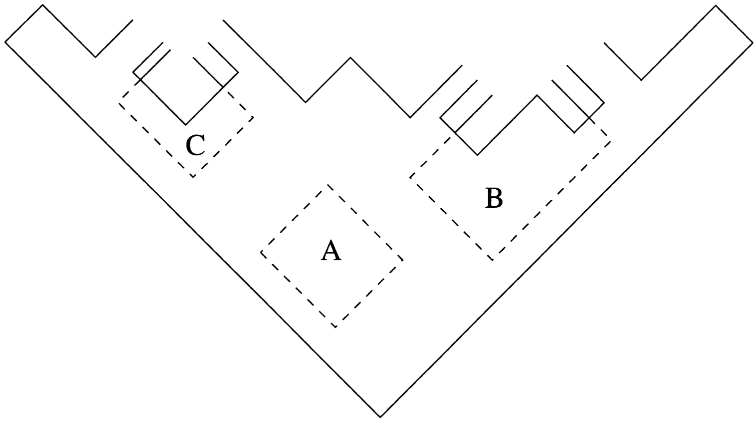

Fig. 13.2. A space-time description of a string field with color fluctuations having disallowed colors. For the regions B and C this leads to $B\bar{B}$ pair production situations, in B with a meson in between.

density in the string field; it is still κ-characteristic for a $3\bar{3}$ color field in the region with the wrong colors. The force direction changes, however, to the opposite one. This implies that:

- the wrongly colored charges are pulled with the same magnitude of force in the two directions, i.e. there is no net force on them;
- one or more $b\bar{b}$-colored $q\bar{q}$-pairs may tunnel out within the region. If that happens then we evidently obtain a qqq-state at one end and a $\overline{qqq}$-state at the other, i.e. a $B\bar{B}$-pair is produced. There may also be one or more mesons produced in the $b\bar{b}$-part of the field, i.e. in between the B and $\bar{B}$.

The fact that the two 'wrongly' colored particles, q_1 and $\overline{q}_1$ (each assumed to have transverse mass μ_1) just float around without any force upon them means that we may estimate the probability for them to be at a (spacelike) distance x_1 apart by the same method as in Chapter 11, i.e. using the Heisenberg indeterminacy relation we obtain

$$|\Delta_F(x_1,\mu_1)|^2 \simeq \exp(-2\mu_1|x_1|) \qquad (13.16)$$

If there is a pair $q_2\overline{q}_2$ (with transverse masses μ_2) produced in between q_1 and $\overline{q}_1$ then the probability for the particles in the pair to tunnel the distance x_2 is suppressed by the tunnelling factor

$$\exp\left(-\frac{\pi\mu_2|x_2|}{2}\right) \qquad (13.17)$$

If no other pair is produced in the region the resulting pair B, $\bar{B}$ will be neighbors in rank. The minimum distance to which the quarks must separate to come onto the mass shell is related to the effective transverse mass, μ, of the 'diquark' q_1q_2 and the antidiquark $\overline{q}_1\overline{q}_2$ by

$$|x_1| = |x_2| = \frac{2(\mu_1 + \mu_2)}{\kappa} = \frac{2\mu}{\kappa}. \tag{13.18}$$

Then we obtain for the total probability for such a situation to occur, from Eqs. (13.16), (13.17),

$$\exp\left(-\frac{4\mu_1\mu + \pi\mu_2\mu}{\kappa}\right) \sim \exp\left(-\frac{\pi\mu^2}{\kappa}\right) \tag{13.19}$$

Therefore if we stretch the value of π to 4 we obtain back the ordinary tunnelling formula for the diquark-antidiquark! Thus in the popcorn model we obtain a similar suppression to that in the diquark production model for the heavier 'diquarks'.

For a meson to be produced, we need an even larger color fluctuation between q_1 and $\overline{q}_1$. We need an extra piece of the order M/κ to produce a meson with transverse mass M, which leads to an extra suppression factor

$$\exp\left(-\frac{2\mu_1 M}{\kappa}\right) \tag{13.20}$$

The mass of a mesonic system increases quickly with the multiplicity. It is not difficult to convince oneself that, with an average rapidity distance $\delta y \simeq 1$ between subsequent mesons, we obtain for the mass M_n of an n-particle system

$$M_n \simeq M \exp\left[\frac{(n-1)\delta y}{2}\right] \tag{13.21}$$

Therefore the probability of producing more than a single meson in between the $B\bar{B}$-pair will be small. The heavier vector mesons, in particular, may be even more suppressed than the pseudoscalars.

From Eqs. (13.20), (13.21) we may evidently estimate the average number of mesons in between as well as the multiplicity distribution. We obtain that in the mean about 0.5 mesons should be included.

We will therefore introduce the probability factor $(BMB) = 0.5$ of producing a single meson in between the $B\bar{B}$-pair and will neglect the larger multiplicities. Such larger multiplicities are nevertheless indirectly included by allowing the production of both pseudoscalar mesons and vector mesons (which afterwards decay into two or more pseudoscalars) in the same way as in any other part of the string field.

13.4 A different use of the Lund model formulas, the UCLA model

A group at UCLA has proposed, [38], the use of the Lund model formulas in a way different from the one we have presented up to now. They have been quite successful in interpreting the area law as a density for the final-state particle ratios also.

The prescription is the following. They make use of the ordinary projection coefficients, the Clebsch-Gordon coefficients, between a given $q\bar{q}$-state and a hadron state. These are used also in the normal version of the Lund model but then there is (as we have described in Chapter 12) extra suppression of the s-flavor and no production of c- and b-flavors in the fragmentation.

All are allowed in the UCLA version. It is, however, noted that if one produces e.g. an $s\bar{s}$-flavored pair then there must be two strange particles produced, with correspondingly larger masses than mesons that are composed of the u- and d-flavors. Suppressions of this kind can be determined by a few iterations of the basic method.

They then use the results in Chapter 10, [19], according to which every hadron with mass m is given stochastically a value of z according to the fragmentation function

$$f(z, m^2) = N\frac{(1-z)^a}{z}\left(1 - \frac{m^2}{zs}\right)^a \exp\left(-\frac{bm^2}{z}\right) \tag{13.22}$$

The authors use as a finite-energy correction the term we derived in Eq. (10.10). The basic assumption is that N, a and b are the same for all particles. The relative normalisation for different species is given by the integral of f. There are a set of extras, however, and we will mention a few.

They include a transverse momentum generation mechanism, which is an approximation of the mechanism we presented in Chapter 12. There is a claim that the approximation is good, [38].

Further, the method contains the Bowler implementation of heavy quark fragmentation in accordance with the description we gave in subsection 2. They formulate it in such a way that they can keep their general normalisation constant N by writing

$$z_{eff} = \frac{z}{1 - (\mu^2 z/m^2)[1 + \log(m^2/\mu^2 z)]} \tag{13.23}$$

Finally they use the popcorn mechanism for $B\bar{B}$ production, allowing any number of mesons in between. They provide each meson produced in this way with a factor $\exp(-\eta m)$, i.e. an exponential suppression proportional to the mass.

This means that in total they use the distribution

$$f(z_{eff}, m^2_{\perp,D})dzd^2k \times \text{Clebsch-Gordon coefficients} \times \text{popcorn distribution} \quad (13.24)$$

in order to generate a cascade.

In such a fitting-scheme it is of course essential to describe the data well and this method seems to do so although, as of yet, no basic reason why it should work has been put forward.