EPHEMERIS OF MARS

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Newcomb's (1898) Tables of Mars with Ross's corrections have been used to compute its ephemeris in national ephemerides. Clemence (1949 and 1961) constructed a more precise theory of Mars and determined preliminary mean elements using a limited number of observations. Laubscher (1971) compared Clemence's theory with about 11000 observations by meridian circles from 1851 through 1969 and about 800 observations by radars and derived an improved set of mean elements. Duncombe and Kubo (1977) compared the ephemeris of Mars, in which Clemence's theory and Laubscher's elements were adopted, with the observations made by the meridian circle at Washington in 1972 and found large systematic residuals, which were larger than residuals by Newcomb's theory. Therefore, Duncombe and Kubo suspected that Laubscher's treatment of data was erroneous. On the other hand we had undertaken a reconstruction of Clemence's theory. In the meantime Kubo and Seidelman independently have found Laubscher's error in the determination of the inclination.

Clemence used the method of Hansen, that is the method modified by Hill in his theories of Jupiter and Saturn. We have followed Clemence's procedure as closely as possible to repeat his calculations and to obtain the same results as Clemence's calculations. One of the distinguished features of Hansen's method is that his method is applicable to orbits of higher eccentricity and inclination; numerical values for the mean elements other than mean anomalies are substituted in any stage of calculations. This feature inevitably introduces mixed secular terms in the expressions of perturbations, and, therefore, a solution by this method is only applicable to a limited time interval. In fact Clemence's goal was to attain a precision of $0''.01$ in the geocentric position of Mars for five centuries or more before and after 1850. This feature also introduces another problem in the calculation of the second-order perturbations; the term of the longitude having any argument $jl'+kl$ (l' and l are mean anomalies) are multiplied by $n^3/(jn'+kn)^3$, n and n' being the mean motions of l and l'. If the term with $jl'+kl$ is a long periodic one, this factor causes the loss of too many significant figures. In Hansen's method both long- and short-periodic perturbations are treated simultaneously, and the necessary places of decimals are set by the term of the

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longest period among the perturbations. Hence in order to determine both long- and short-periodic terms with the same accuracy, all the calculations have to be carried to the same number of decimals and require keeping a lot of terms in the intermediary calculations and costing a lot of time and memory of a computer.

All the calculations in our work are carried out by Poisson-Series-Processor developed by Nakai (1978). PSP manipulates operations of addition, multiplication, partial differentiation, integration, and so on for the following type of Poisson series:

$$PS=\sum_{ij} C_{ij}P_1^{j_1}P_2^{j_2}P_3^{j_3}P_4^{j_4}P_5^{j_5}P_6^{j_6}\times{}^{\cos}_{\sin}(i_1l_1+i_2l_2+i_3l_3+i_4l_4+i_5l_5+i_6l_6) \quad ,$$

where C_{ij} are numerical coefficients (rational or floating), j's are powers of P, and l's are angular variables. PSP adopts Hash coding method, which drastically reduces the computing time of multiplication of the two Poisson series, because this method does not require a large amount of sorting and moving the data in core and does not take a fair amount of storage space.

Let Δ be the mutual distance of two planets and α be a/a', where the primed quantity relates always to the outer planet. If the eccentric anomalies are taken as the independent variables, the series expansion of $(\Delta/a')^2$ can be written in a closed form. Let $(\Delta/a')^2=1+E$, where E becomes zero for $\alpha=0$, and denote a constant part of E by E_c and a periodic part by E_p:

$$(\Delta/a')^2=(1+E_c)D^2 \quad , \quad D^2=1+E_0 \quad , \quad \text{and} \quad E_0=E_p/(1+E_c) \quad .$$

In order to obtain a series expansion of 1/D with respect to the eccentric anomalies, we use the following two kinds of iteration formula:

$$\text{a)}\ \frac{1}{\delta_1} = 1-\frac{1}{2}E_0+\frac{3}{8}E_0^2 \quad , \qquad \text{b)}\ \frac{1}{\delta_1} = 1-\frac{1}{2}E_0+\frac{3}{8}E_0^2 \quad ,$$

$$E_n = \frac{5}{8}E^3_{n-1}-\frac{15}{64}E^4_{n-1}+\frac{9}{64}E^5_{n-1} \quad , \qquad \varepsilon_n = (\frac{1}{\delta_n})^2D^2-1 \quad ,$$

$$\frac{1}{\delta_{n+1}} = \frac{1}{\delta_n}(1-\frac{1}{2}E_n+\frac{3}{8}E_n^2) \quad , \qquad \frac{1}{\delta_{n+1}} = \frac{1}{\delta_n}(1-\frac{1}{2}\varepsilon_n+\frac{3}{8}\varepsilon_n^2) \quad .$$

The order of magnitude of both E_n and ε_n is $E_0{}^{3n}$. In case (a), as number n increases, the amount of calculations usually decreases. However, rounding-off errors accumulate. On the other hand, in case (b), as number increases, the amount of calculations does not decrease, but rounding-off errors do not accumulate. In the actual calculations, we obtain an approximate series expansion by (a), then improve it by (b). Taking into account higher powers of E_n or ε_n, we can make more rapidly convergent formula than (a) or (b). However, our experience shows that such iteration formulae are usually time consuming because of increase of multiplications of Poisson series.

In developing a'/Δ in terms of the eccentric anomalies by (a) and (b) and transforming it into a series in terms of the mean anomalies by means of Besselian functions, we use ten places of decimals, which are larger by two places than Clemence's value. a'/Δ for the Earth includes 2173 terms. If this series is truncated at eight places of decimals, the number of terms reduces to 1377, which is less by twenty than in Clemence's theory. The largest difference in the numerical coefficients of the trigonometric series reaches up to 75×10^{-8} for the term with $35l_M-29l_E$, where l_M and l_E are mean anomalies of Mars and the Earth, respectively. Our results are in good agreement to the ninth digit with the results obtained by double Fourier analysis. Also we check the accuracy of the series expansion by substituting numerical values of the mean anomalies into the series expansion and a'/Δ for the Earth. The maximum difference is about 4×10^{-7} for the series truncated at 10^{-8}.

Using series expansions for a'/Δ and adopting the same formulae as Clemence did, we obtain the first-order perturbations of $n\sqrt{z}$, ν, and $u/\cos i$, which are perturbations in the mean anomaly, the radius vector, and the latitude, respectively. In any calculation, we keep at least one more digit than Clemence did. The following table shows the terms, which have the largest difference between Clemence's results and ours (K. and N.).

	$\times10^{-4}$	Clemence	K. and N.	Diff.	$n_M/(in+jn')$	
	$n\delta z$	missing	- 19	19	-38.3	$\cos(17l_E-32l_M)$
Earth	$u/\cos i$	-295	-296	1	1.3	$\cos(2l_E-3l_M)$
	ν	73	75	2	- 1.1	$\cos(8l_E-16l_M)$
	$n\delta z$	-471	-453	18	20.6	$\cos(l_M-6l_J)$
Jupiter	$u/\cos i$	0	- 35	35	0.5	$\sin 2l_M$
	ν	- 75	- 72	3	- 1.0	$\sin(-2l_M+6l_J)$

The large difference in the periodic perturbation of $n\delta z$ might have been caused by small divisors due to its long periodic. The discrepancy in $u/\cos i$ by Jupiter seems to be a typographical error, because this term is listed in the table in which Clemence compared his results with Newcomb's theory. The difference in mixed secular terms factored by nt does not exceed 7×10^{-6} of the second of arc, which appears in $n\delta z$ by Saturn.

Clemence compared his theory, which includes second- and third-order perturbations, with the results obtained by the numerical integration by Herget. The largest difference in the orbital longitude was $0''.042$, which cannot be explained by the above discrepancies in the first-order perturbations between Clemence's results and ours. The calculations of second-order perturbations are being undertaken.

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DISCUSSION

Henrard: Would you comment on the advantage of Fourier analysis over iteration procedure for the computation of the distance between two planets?

Kinoshita: Yes. A double Fourier analysis is the most efficient way to obtain a trigonometric expansion of the disturbing function in terms of the mean anomalies. However, for the next stage of calculation, we definitely need a formula manipulation system.