must address the question: Do we want discrete mathematics in its modern form as a major component of 16–19 mathematics?

References

1. N. L. Bibby, A-level mathematics: where next?, *Math. Gaz.* **73**, 87–93 (1989).
2. S. French, The common core for mathematics: do we need one?, *Teaching Math. and its Appl.*, Vol. **8**, 180–183 (1989).
3. HMI, *A-level mathematics: proceedings of the invitation conference* (1989).
4. Math. Assoc. of America, *Calculus for a new century*, MAA Notes No. 8 (1988).
5. SEAC, Consultation on the draft principles for GCE, AS and advanced examinations (1990).

Yours sincerely,

JOHN MONAGHAN

Centre for Studies in Science and Mathematical Education, The University, Leeds LS2 9JT

Skills, knowledge and understanding

DEAR EDITOR,

The SEAC document *Examinations Post-16: Developments for the 1990's*, reporting on the response to SEAC's Consultation exercise, refers (p 20) to "a body of knowledge, understanding and skills" when defining what it means by a "syllabus", thereby repeating a by-now familiar phrase which may not be perfect, but which does have the virtue of combining reference to subject content, conceptual insight, and practised technique. However the same document also refers (p 4) to "core skills", and indeed the term "skills" is referred to far more often than either understanding or knowledge.

The query naturally arises: why should the concept of a core of study be associated with *skills* rather than with a core of *knowledge*, or a core of *understanding*, or all three? The obvious explanation is suggested by the document itself, which associates (ibid) "core skills" with "the vocational dimension in the education of the 16–19 age group". This suggests in turn the long standing association between "skill" and the "back to basics" movement; and the further association of "skills" and "basics" which is implicit in Kenneth Baker's insistence that it is skills which come first, and which are then applied to solve problems.

Unfortunately for those who seek simple-minded solutions to the problem of improving 16–19 provision, this association is potentially dangerous and misleading. I trust that no one of any consequence, in or out of the Mathematical Association, believes any longer that "Back to Basics" is an effective prescription for progress; or anything other than a facile slogan which ignores the richness and complexity of children's learning. Nor does Kenneth Baker's plausible analysis stand up. While skills may indeed sometimes be learnt first and then applied to problems, it is just as true to say that it is by tackling problems that individuals develop and hone the skills that they will then apply in the future. This is as true of the small child engaged in matching rows of counters, as one activity of many which over a period of time will lead to the skill of counting, as it is of the industrial mathematician who simultaneously uses skills at one level while solving a problem whose very solution will increase his repertoire of ideas and—yes—skills, for use in the future.

The entire tenor of developments in recent years has been towards an emphasis on an alliance between insightful understanding and technical accomplishment. Without such an alliance, at every level, and in every context, neither the pure mathematician nor the applied, neither the learning pupil nor the experienced adult, neither the strongest pupils nor the weakest, can produce their best work.

There is another possible interpretation of core skills which deserves a mention, though it is hardly consistent with SEAC's document: "skills" could be used to refer to those processes of doing mathematics which are associated with problem solving and the students' activities as young mathematicians. However, this interpretation stands up to scrutiny no better than the other. The ability to prove theorems, to generalise, to suggest hypotheses, to take control of one's own mathematical activity, are by no means merely skills to be practised and honed to perfection. Far from it—while practice will improve, it will not make such abilities perfect; and there is at least as large an element of penny-dropping insight and illumination in all such activities as there is of skilful technique.

This train of thought was prompted by an incident at a recent symposium on Mathematics 16–19 at the West Sussex Institute of Higher Education. Clive Hart, of SEAC, while giving an account of the SEAC position on 16–19 reform, described (to an ironical burst of laughter because unknown to him a related point had been made previously at the same conference), how he had decided that "core skills" was an unfortunate phrase, had looked up both halves in Roget's Thesaurus and concluded that "essential accomplishments" was the best alternative.

I hope that Clive Hart will not object if I commend the virtues of that phrase, and suggest that, should it seem too much of a mouthful then "core accomplishments" would be a most euphonious compromise. By all means let us acknowledge, and seek to extend by all means at our disposal the varied accomplishments, in skill, knowledge, and understanding, of all our pupils; without falling into the trap of singling out one member of the trinity as dominant, thereby unwittingly demoting the other two, and giving hostages to those who do not have our pupils' best interests at heart.

Yours sincerely,

DAVID WELLS

19 Menelik Road, London NW2 3RJ

Sixth form mathematics

DEAR EDITOR,

Sixth form mathematics in the grammar and independent schools had a golden age in the 20–30 years before comprehensive schools came in, caused partly by the influx of good teachers between the wars. But it was, of course, taken for granted, and is still not appreciated. Able sixth formers, mostly boys in those days, reached a better understanding of mathematical ideas, a better standard of problem-solving and had better technical skills than many graduate mathematics students in the world of today. They were able to move on to careers in science, engineering and so on at an early age if they wished, because they were already *mathematically* literate. They didn't have to battle with mathematical language later in life, when it can become more difficult.

It is interesting to note that the Kodaly primary music schools in Hungary, with unselective intake, produce *ten-year-olds* who are *musically* literate. They can hear and sing what they see, and see and write the music they hear. Indeed Americans with masters degrees in Music Education have been heard despairing about their own limitations compared to those children. Mathematics shares with music this ability of the young to make rapid progress under suitable circumstances. In the case of the sixth forms of the golden age the boys had often been taught in lower forms by people who themselves really understood and enjoyed mathematics, and who had good problem-solving abilities which they could pass on by a kind of apprenticeship process. This is a situation which is rare indeed in U.S.A. and Australia, and is probably becoming increasingly rare in England and Wales nowadays.

The situation in U.S.A. is instructive. My article in *Math. Gaz*. May 1963 explained the underlying differences between the education there and in England at the time, and described the differences in mathematics teaching. For those who do not have a copy, it highlighted the short-term nature of the learning; the focus on grades rather than the subject; the one-year course (unit?) as an entity in itself rather than as a stage of development; the importance of the packaged textbook, used by teachers as a prescription rather than an aid; and the supremacy of testing in its most arid form. It was written while I was still there and before I became aware of some of the more pernicious features of American schools. Nor did