7. THE PROBLEM OF ULTRA-SHORT PERIOD BINARIES

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Recent spectroscopic and photometric studies have revealed the existence of three binary stars with orbital periods in the range 1^h to 2^h: the well-known old nova WZ Sge (1913, 1946) (**1**), EX Hya (**2**), and VV Pup (**3**). All are eclipsing binaries with remarkable photometric and spectroscopic properties.

WZ Sge is the binary of shortest known period, approximately 81·6 minutes. In the spectrum, one finds absorption lines of a DA white dwarf, with superposed doubled emission lines separated by about 1350 km/sec (**4**); the spectrum corresponds to a rather low effective temperature, probably about 13 000°K to 14 000°K (**1**). No radial velocity variation has been found for either the absorption lines or doubled emission, though a variation of the former would be extremely difficult to detect in any case. When the star is trailed continuously over a long slit, however, one finds (**5**) in addition an S-wave emission component with a mean velocity equal to the mean separation of the stationary emission components ($\gamma = -33$ km/sec) and a period of about 81 minutes. The amplitude of this wave is variable, ranging from perhaps 600 to 750 km/sec. Improvement in the period came with Krzeminski's (**6**) discovery that the object also eclipses; the light curve is roughly of the W UMa-type, but in the spectrum there is no trace whatever of the secondary component. Simultaneous photoelectric and spectroscopic observations (**1**) reveal also that the light and radial-velocity curves are 90° out of phase, in the sense that maximum velocity of recession occurs at principal minimum.†

Krzeminski and Kraft (**1**) have suggested a model in which the discrepancy in phase of the light and velocity curves is explained as due to a luminous stream ejected from the dark (unseen) secondary towards the primary through the inner Lagrangian point L_1; the dark star is pictured as filling its lobe of the inner zero-velocity surface. At principal eclipse, the dark star covers a portion of this stream and partially eclipses both the white dwarf primary and the ring around it. At secondary eclipse, the white dwarf itself eclipses most of the ejected stream. The velocity of ejection is quite high (~ 600 to 750 km/sec), and exceeds the relative motion of the two stars in the system. Because the double emission components do not move, limits can be placed on the mass ratio which must be unusually high. Krzeminski and Kraft use the properties of the DA spectrum of white dwarfs (following Weidemann (**7**)) to derive the mass of the white dwarf; the best present estimates, though subject to large uncertainties, of the physical parameters of the system, are: $\mathfrak{M}_1 = 0{\cdot}59\ \mathfrak{M}_\odot$, $\mathfrak{M}_2 = 0{\cdot}03\ \mathfrak{M}_\odot$, $\mathfrak{M}_2/\mathfrak{M}_1 = 0{\cdot}05$, $i = 82°$, $R_1 = 0{\cdot}87 \times 10^9$ cm, $R_2 \sim 7 \times 10^9$ cm, $a = 3{\cdot}7 \times 10^{10}$ cm, where the subscript 1 refers to the white dwarf primary. Estimates of the gas density in the ejected stream lead to a mass loss from the secondary of around 10^{21} g/yr; at least a portion of this ejected material forms a ring around the primary, but it is not known how much matter is collected by the primary and how much is lost from the system.

Quite similar to WZ Sge is the system of EX Hya; the period is about 99 minutes. The principal eclipses (**7, 8**) are sharp and deep and vary in depth (0·3-0·8 magnitude); there is no evidence for a secondary minimum. As in WZ Sge, the hydrogen lines show doubled emission components which are stationary but less sharp than in WZ Sge; there is no trace of a white dwarf spectrum, however. In addition, the emission lines of EX Hya are stronger relative to the continuum than is the case in WZ Sge, though the separation is about the same, $\sim$1400 km/sec.

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The writer is indebted to Dr. G. Haro for his generous hospitality.

† It should be mentioned that all p.e. observations have been obtained in the ultraviolet because of the presence of a red (purely optical) companion only 7″ away from WZ Sge. Nothing is known at present of the light curves in blue or visual light.

The degree of excitation of the spectrum is also somewhat higher in EX Hya, there being a faint trace of He II emission ($\lambda 4686$) accompanying He I ($\lambda 4471$) in some strength; He II never appears in WZ Sge and He I is present only occasionally.

Because of the considerable strength and width of the stationary emission components, the S-wave, while definitely present, is more difficult to discern in EX Hya than in WZ Sge. However a nearly simultaneous spectroscopic and photoelectric run indicates that, as before, the maximum velocity of recession occurs near principal minimum! It seems likely, therefore, that the system of EX Hya has physical properties similar to those of WZ Sge, including the ejection of matter at high velocity from the secondary into a ring surrounding the primary; all the evidence points again to a high mass-ratio although a direct mass estimate is not possible because no white dwarf spectrum is present.

It should also be mentioned that, contrary to the low-velocity character of WZ Sge, EX Hya has $\gamma = -124$ km/sec, if the mean of the stationary emission components can be taken as the radial velocity of the star in space.

VV Pup (**3**) differs in some important photometric and spectroscopic respects from both EX Hya and WZ Sge. To judge from Herbig's spectrograms, which have slightly less than half the dispersion of the spectrograms obtained for EX Hya and WZ Sge, there are no stationary emission components in a spectrum consisting of strong H, He II, and feeble He I superimposed on a hot continuum without absorption lines. The velocity range of the emission lines is very large ($2K = 880$ km/sec) and the period is 100 minutes. Contrary to the case of the other two stars, there is a rapid decrease in light from a maximum near $\varphi = 0{\cdot}75\ P$ to a minimum near $\varphi = 0{\cdot}00\ P$, where phase zero corresponds to conjunction with the emission-line star behind (as judged from the radial velocities). There is, however, a very slow recovery, the star remaining at or near minimum brightness until $\varphi = 0{\cdot}3\ P$ after which the light curve ascends more slowly than it declined to phase $0{\cdot}75\ P$. Thus the photometric and spectroscopic behavior of VV Pup is more nearly normal in that the light is near minimum when the radial velocity requires it to be so.

Herbig interprets the slow recovery from minimum as resulting from a 'hot-spot' on the primary (the secondary is assumed to be entirely dark) located on the following hemisphere approximately in a $45°$ position. Just before conjunction, the hot spot is exposed directly to view and one has maximum light. The slow recovery from minimum is a result of the presentation of the comparatively dark side of the primary to the observer. Indeed, the picture is such that the eclipse may be decidedly partial. Furthermore, the simultaneous drop in mean brightness and near cessation of light variations noted in VV Pup around 1948 are naturally accounted for by the hypothesis that the 'hot-spot' decreased considerably in brightness.

The writer has suggested elsewhere (**9**) that if the unseen companion ejects material into the lobe of the zero-velocity surface surrounding the primary, then from dynamical considerations one might predict an accumulation of this material near the following hemisphere of the primary (**10**). The 'hot-spot' might then be a result of continuum and line-emission from this accumulation region, possibly as a result of collisional excitation. One could then understand the secular decline in light as a result of a lower than average rate of ejection of material by the unseen secondary. If this picture were correct, however, it could require a slight re-interpretation of the radial velocities. The emission-line velocity actually measured then would represent a vector sum of the dynamical motion of the primary and the motion of the stream of gas through the accumulation region. If these velocities were, for example, approximately equal, then maximum recessional velocity would occur near maximum light, and zero (systemic) velocity just after true stellar conjunction. The spectroscopic and photometric data are not, at least, inconsistent with this view but a more nearly definitive picture must await spectroscopic observations with a larger telescope than that available to Herbig.

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If we adopt for the moment the picture advanced here for VV Pup, we can say then that VV Pup differs from the other two stars in three important respects: (*a*) the degree of excitation of the emission-line spectrum is higher; (*b*) the ring rotating around the primary does not exist or is not excited for reasons unknown at present; only that portion of the stream ejected from the secondary that passes near the following hemisphere of the primary is illuminated; (*c*) unless the mass of the unseen secondary is decidedly larger than the primary, which does not seem very reasonable, then the mass ratio of VV Pup is more nearly normal, i.e. nearer unity, in comparison with WZ Sge and EX Hya.

What the three systems, which we shall refer to hereinafter as WZ Sge-type stars, have in common appears to be: (i) ultra-short period; (ii) ejection of matter by the secondary into the lobe of the inner zero-velocity surface surrounding the primary; (iii) a primary that is a hot subdwarf or white dwarf, and is therefore an object in a late stage of stellar evolution; (iv) fairly large total masses. The last point must be justified a little further. The total mass of WZ Sge is $0{\cdot}62\ \mathfrak{M}_\odot$ and it seems unlikely that this can be in error by a factor of more than ± 25 per cent. The mass of EX Hya has not been measured, but it is probably similar to that of WZ Sge. For VV Pup, the mass function (3) leads to a minimum mass for the secondary of $0{\cdot}6\ \mathfrak{M}_\odot$; the primary is probably still more massive than this.

Now in every case the primary is a star in an advanced evolutionary stage. Thus it seems likely that the primaries must once have been more massive than they are at present, i.e. if $(\mathfrak{M}_1)_0$ is the primordial mass of the primary, then $(\mathfrak{M}_1)_0 \geqq 1{\cdot}0\ \mathfrak{M}_\odot$, if present ideas of stellar evolution are correct. In addition one can suppose, in view of the mass loss presently being observed for the secondaries, that their masses must also originally have been larger than they now are. Much of the original mass of the system must, therefore, have been lost. Now if we think of pairs of mass sequence progenitors for WZ Sge-type systems we must conclude the original periods were considerably longer than they are at present. To fix the situation, consider a W UMa-type system in which $\mathfrak{M}_1 = 1{\cdot}0\ \mathfrak{M}_\odot$ and $\mathfrak{M}_2 = 0{\cdot}5\ \mathfrak{M}_\odot$. Then the contact period would be $0^d\!.28 = 6.7$; even if we take the highly favorable case of a secondary with negligible mass ($\mathfrak{M}_1 = 1{\cdot}0\ \mathfrak{M}_\odot$ still), then the contact period is 3 hours, about twice as long as presently observed. Systems near the main sequence with large mass ratios have never been observed, and therefore we ask the following question: *Can we regard objects such as WZ Sge, EX Hya, and VV Pup as descendants of relatively normal binaries such as W UMa stars?* To answer this question we would have to show that considerable mass loss from a W UMa system can result in a severe shortening of the period.

Elsewhere, I have tried to show (11) that U Gem variables might well be the descendants of W UMa systems. The U Gem binaries are characterized by a mass ratio of approximately unity, a blue star that is a hot subdwarf or white dwarf, and a red star that overflows its lobe of the inner zero-velocity surface; the mean period is about 6 hours—the material flowing from the red star forms a ring around the blue. This type of system is qualitatively so much like the WZ Sge-type binaries that one may ask: *If the red star in a U-Gem system continues to lose mass into the ring (and also into space or on to the surface of the blue star), could such a system evolve into a WZ Sge-type binary?*

The problem of the period changes induced by mass exchange, mass loss, and ring formation in close binaries has been studied extensively by Huang (12, 13). Unfortunately, the problem is very complicated and only certain idealized cases can be studied; we will consider some of these. For definiteness, we take the dynamical parameters that are fairly likely to apply to a typical U Gem system: $\mathfrak{M}_{\rm bl} = 1{\cdot}0\ \mathfrak{M}_\odot$, $\mathfrak{M}_{\rm red} = 1{\cdot}0\ \mathfrak{M}_\odot$, $\mathfrak{M}_{\rm bl}/\mathfrak{M}_{\rm red} = 1$, $P = 6^{\rm h}\,40^{\rm m}$, $a = 1{\cdot}6 \times 10^{11}$ cm, $e = 0$. These are suggested by recent results on Z Cam (14). Estimates of the rate of mass loss $\sigma_{\rm U\ Gem}$ of the red components of U Gem systems are not available, and are difficult to obtain from spectroscopic observation. For WZ Sge, one observes directly the

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ejected stream through L_1, and the value $\sigma \sim 10^{21}$ g/yr has been obtained (**1**). In AE Aqr (**15**) $\sigma \sim 10^{25}$ g/yr. This latter value was obtained on the assumption that the rate of overflow of the inner Lagrangian surface is given by the rate of expansion through a fixed radius of the Sandage-Schwarzschild model of equal mass. However, the absolute magnitude of the red component was taken too high (**16**), and the rate of expansion should be reduced to about 10^{24} g/yr. The red component of our typical U Gem star is two magnitudes fainter than the red component of AE Aqr (**16, 17**), and if a similar evolutionary argument is applicable, we estimate $\sigma_{\text{U Gem}} \sim 10^{23}$ g/yr.

A second estimate that provides an upper limit to $\sigma_{\text{U Gem}}$ is based on the assumption that all the material lost by the red star is accreted by the blue. Then $\sigma_{\text{U Gem}} = 4\pi r^2 \rho v$, where ρ is the density and v the velocity of infall at some typical distance within the ring around the star. Typical rotational velocities in the ring run around 600 km/sec; thus $r = \dfrac{G\mathfrak{M}}{v^2} = 3{\cdot}7 \times 10^{10}$ cm $= 0{\cdot}23\, a$. With $N_e \sim 10^{11}$ cm^{-3}, corresponding to $\rho \sim 10^{-13}$ g cm^{-3}, and taking the half-half width ($\sim$200 km/sec) of a typical emission component as an upper limit to the infall velocity, we obtain $\sigma_{\text{U Gem}} \lesssim 10^{24}$ g/yr.

A third upper limit may be found by assuming that all the luminosity of the blue component is due to accretion heating. Then $\sigma_{\text{U Gem}} = \dfrac{R_{\text{bl}} L_{\text{bl}}}{G\mathfrak{M}_{\text{bl}}}$ (**15**), and we obtain the value $\sigma_{\text{U Gem}} = 5 \times 10^{23}$ g/yr. The $U - B$ vs $B - V$ colors for U Gem systems showing only the blue component seem compatible with a flux resulting from Balmer lines and Balmer, Paschen, and free-free continua (**18**). Thus the last estimate is probably a considerable upper limit.

If we can take the estimate for WZ Sge as a lower bound, then it is probably safe to suppose that $10^{21} \leqq \sigma_{\text{U Gem}} \leqq 5 \times 10^{23}$ g/yr; we take a value of 5×10^{22} g/yr in what follows. We consider first some of the idealized cases of Huang (**12**), neglecting the coupling being orbital and rotational angular momentum.

Case I. Slow Mode with Ring Formation

In this case we assume that no mass is lost from the system, so that $\delta(\mathfrak{M}_{\text{bl}} + \mathfrak{M}_{\text{red}}) = 0$ and $\delta\,\mathfrak{M}_{\text{red}} < 0$. Then from Huang's equations we have

$$\begin{cases} \dfrac{\delta P}{P} = \alpha\, \dfrac{\delta \mathfrak{M}_{\text{red}}}{\mathfrak{M}_{\text{red}}} & \text{(1a)} \\[2ex] \alpha = 3\left[\dfrac{(\mathfrak{M}_{\text{bl}} + \mathfrak{M}_{\text{red}})}{\mathfrak{M}_{\text{bl}}}\dfrac{r}{a}\right]^{1/2} + 3\left(\dfrac{\mathfrak{M}_{\text{red}}}{\mathfrak{M}_{\text{bl}}} - 1\right) & \text{(1b)} \end{cases}$$

where r is the radius of the ring. In the present case $r/a = 0{\cdot}23$, and $\alpha = 2{\cdot}04$, or $\dfrac{1}{P}\dfrac{\delta P}{\delta t} = -\,2{\cdot}04 \dfrac{1}{\mathfrak{M}_{\text{red}}}\left|\dfrac{\delta \mathfrak{M}_{\text{red}}}{\delta t}\right|$, whence $\delta P/\delta t = -\,4{\cdot}1 \times 10^{-14}$. At this rate, the period can be reduced to 94 minutes (the mean value for the WZ Sge-type stars) in about 10^{10} years, and the amount of matter lost by the secondary is certainly of the right order of magnitude if the rate remains 5×10^{22} g/yr.

However, as the mass of the red component declines, the second term in equation (1b) becomes negative. To take WZ Sge as a limiting case, we then obtain $\alpha = -1{\cdot}9$ (note that $r/a \sim 0{\cdot}1$), and the period increases! We conclude that after some time, there is a compensation between the two terms of equation (1b), and the period stops decreasing. Thus the time it takes to reduce the period to 94 minutes is longer, by a significant amount, than 10^{10} years, the age of the galactic disk (Sandage, private communication), a rather uncomfortable conclusion.

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Case II. 'Intermediate Mode'; Mass Ejected from the System

In this case the ejection velocities are high enough that a significant amount of matter is ejected from the system. Strictly speaking, there is no observational evidence that any U Gem variable actually ejects matter secularly: certainly there is no evidence for a ring around the whole system as in the case of β Lyr. For this reason, we cannot estimate the angular momentum actually carried away by the ejected material, if any. For our case

$$\frac{\delta P}{P} = -\frac{\delta \mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}} + 3\,\frac{\delta h_0}{h_0},$$

where h_0 is the angular momentum per unit mass of the material. Now, according to Huang, if the escaping matter carries away more than its average share of angular momentum, δh_0 will be negative, and since $\delta\mathfrak{M}_{\mathrm{red}} < 0$, the direction of the period change will depend on the relative sizes of $-\dfrac{\delta \mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}}$ and $3\,\dfrac{\delta h_0}{h_0}$. On the other hand, if the matter carries away less than its average share of angular momentum, or if $\delta h_0 = 0$, the period will increase. Since this case is not supported by observational evidence in any event, we will not consider it further.

Case III. Case I with Coupling

Earlier we supposed that there was no coupling between the orbital and rotational angular momenta; in the present case we go to the other extreme and suppose the rotation and revolution to be synchronized. According to Huang (**13**) the only difference between this and the uncoupled case (equation 1a) is that the left-hand side is multiplied by $(1 - 3A_{\mathrm{bl}} - 3A_{\mathrm{red}})$ and the right hand side by

$$-3\left(A_{\mathrm{bl}}\,\frac{\delta\mathfrak{M}_{\mathrm{bl}}}{\mathfrak{M}_{\mathrm{bl}}} + A_{\mathrm{red}}\,\frac{\delta\mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}} + 2A_{\mathrm{bl}}\,\frac{\delta k_{\mathrm{bl}}}{k_{\mathrm{bl}}} + 2A_{\mathrm{red}}\,\frac{\delta k_{\mathrm{red}}}{k_{\mathrm{red}}}\right)$$

where k_{bl} and k_{red} are the radii of gyration of the two stars and the constants A are given by

$$\begin{cases} A_{\mathrm{bl}} = \dfrac{\mathfrak{M}_{\mathrm{bl}} + \mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}}\left(\dfrac{k_{\mathrm{bl}}}{a}\right)^2 \\[2ex] A_{\mathrm{red}} = \dfrac{\mathfrak{M}_{\mathrm{bl}} + \mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{bl}}}\left(\dfrac{k_{\mathrm{red}}}{a}\right)^2 \end{cases} \tag{2}$$

Now $A_{\mathrm{bl}} << A_{\mathrm{red}}$, and we neglect it, since the blue star is very small compared with a. Thus we have $1 - 3A_{\mathrm{bl}}$ on the left-hand side, and

$$-3A_{\mathrm{red}}\left(\frac{\delta\mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}} + 2\,\frac{\delta k_{\mathrm{red}}}{k_{\mathrm{red}}}\right)$$

on the right-hand side. Following Huang (**13**) we obtain $A_{\mathrm{bl}} \sim 0{\cdot}025$, so that the left-hand side becomes $0{\cdot}925$, and the right-hand side

$$-0{\cdot}075\left(\frac{\delta\mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}} + 2\,\frac{\delta k_{\mathrm{red}}}{k_{\mathrm{red}}}\right).$$

These terms will scarcely affect equation (1a) for the starting value of α and become important only when α becomes small, i.e. when the two terms on the right-hand side of equation (1b) become equal. The conclusion that the period cannot be changed to 94 minutes in a time $< 10^{10}$ years does not seem to be greatly affected by the introduction of coupling.

Case IV. Explosive Ejection by the Red Star

Krzeminski (**19**) has presented evidence that in the eclipsing binary U Gem it is the red component that is responsible for the outbursts characterizing the U Gem phenomenon. Once again, while direct spectroscopic evidence is lacking, we may nevertheless consider the effect on the period of ejection of material during an outburst. The amount of matter ejected is, however, completely uncertain. In ordinary novae, where the ejected shells can be studied spectroscopically, there is evidently approximate equipartition between the excess luminosity integrated over the time of the outburst and the kinetic energy of the ejected shell (**20**). If this were true during the outburst of a U Gem star, we would have

$$E = 10^{39}\ \mathrm{erg} = \tfrac{1}{2} mv^2 \tag{3}$$

The velocity of escape from the surface of the red star is certainly less than 600 km/sec, the static value corresponding to a non-rotating frame of reference. If we take $v = 600$ km/sec, then the mass ejected is bounded by

$$\delta\mathfrak{M}_{\mathrm{red}} \geqq 5 \times 10^{23}\ \mathrm{g/outburst}.$$

The average U Gem star has about six outbursts per year, so that the rate of mass loss would not be less than 3×10^{24} g/yr. If this matter is ejected in a statistically spherical way, then Jean's mode of mass ejection (**13**) applies and we have $\delta P/P = -2\delta\mathfrak{M}_{\mathrm{red}}/(\mathfrak{M}_{\mathrm{bl}} + \mathfrak{M}_{\mathrm{red}})$, and since $\delta\mathfrak{M}_{\mathrm{red}} < 0$, this mode leads to a minimum *increase* in the period of $\delta P/\delta t \geqq 2 \times 10^{-13}$. If it makes sense to speak of thermodynamic equilibrium in the ejected spherical shell, then its temperature would have to exceed 100 000°K in order to avoid producing noticeable hydrogen absorption (we assume a shell, optically thin in the continuum, of radius $\sim 10^{11}$ cm, and mass $\sim 5 \times 10^{23}$ g). We emphasize once again that there is no spectroscopic evidence for the ejection of material during an outburst (**21**).

Another possibility is ejection from the red star in the form of a jet. In the presence of a magnetic field, the jet material may carry away more than its average share of angular momentum; the red star would undergo a braking action (**22**). For a solar-type star with sunspots similar to those of the Sun in strength and number, and under the assumption that 10^{23} g is ejected with a velocity of 10^8 cm/sec over a cross-sectional area of 3×10^{18} cm^2, we obtain from Schatzman's equations

$$\left(\frac{\omega}{\omega_0}\right)^{2/5} \cong 1 - 0.5\left|\frac{\delta\mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}}\right|$$

where ω_0 is the original angular velocity, and synchronous revolution and rotation have been assumed. Thus, per year, the change in ω/ω_0 is of the order of 10^{-8}, or a loss of angular momentum of $\sim 10^{-8}$ per year. Now if there were coupling between the orbital and rotational angular momentum, the period of the orbit might be decreased by an amount $\sim 10^{-10}$ per year in order to maintain synchronism, since the radius of gyration of the red star is ~ 0.1 separation of the centers of mass. Substituting this value of $\dfrac{\delta h_0}{h_0}$ into the equation $\dfrac{\delta P}{P} = -\dfrac{\delta\mathfrak{M}_{\mathrm{red}}}{\mathfrak{M}_{\mathrm{red}}} + 3\dfrac{\delta h_0}{h_0}$, we find the two terms on the right-hand side agree to within one order of magnitude. Thus it is not possible to state, within the errors, whether the period will increase or decrease. It should be mentioned also in connection with the supposed ejection of a jet, that there is no spectroscopic or photometric evidence in favor of such an hypothesis. There is nothing about the spectroscopic description of a flare in, for example, UV Ceti (**23**) that suggests a similarity with U Gem stars during outburst. Naturally we could decrease the period more rapidly through Schatzman's equation by changing the density or velocity of the ejected material, or increasing the magnetic field, but there exist no observational guidelines.

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Case V. Gravitational Waves

We have see that, in the idealized cases considered above, no definite conclusion can be reached regarding the direction or rate of change of the period largely because we do not know the mechanism by which matter is removed from the system. It seems to the writer likely that more matter is removed explosively (either in shells or jets) than secularly since, in the latter event, the temperature and density in the stream ejected from the system would have to be something like those in the ring and it seems very unlikely that such matter could go undetected spectroscopically. In the former case, we would have to imagine that the violently ejected gas is too highly ionized to produce a spectroscopic effect.

However, regardless of these considerations, the relativity theory predicts that binary systems emit gravitational waves which act always to decrease the period. Let us consider the effect of the emission of gravitational waves on a typical U Gem system, even though, among physicists, the reality of the radiation remains questionable.

According to Kraft, Mathews, and Greenstein (**24**), the change in period is given by

$$\frac{dP}{dt} = -\frac{192\pi}{5}\frac{G^{5/3}}{c^5}\frac{\mathfrak{M}_{bl}\,\mathfrak{M}_{red}(\mathfrak{M}_{bl}+\mathfrak{M}_{red})^{1/3}}{a^{5/3}}.$$

For our typical system we have

$$\frac{dP}{dt} = -1{\cdot}5 \times 10^{-13}$$

number at least as large as any of the idealized estimates given earlier. Thus in 4×10^9 yrs, the period would be reduced to the mean value characteristic of WZ Sge-type systems. This is a rather, but not impossibly, long time and would tend to indicate that WZ Sge-type binaries are to be found among the oldest stars. It is therefore not surprising to find a high-velocity object, viz., EX Hya, among them.

Actually, however, the problem is not so simple as is indicated above. For if the red star fills its lobe of the inner Lagrangian surface, then any decrease in the period owing to the emission of gravitational waves will be accompanied by a loss in mass for the red star as the surface decreases in size. We can estimate the amount of this mass loss from

$$\frac{d\mathfrak{M}_{red}}{dt} = -4\pi R_{red}^2\,\bar{\rho}\,\frac{dR_{red}}{dt}$$

where $\bar{\rho}$ is the mean density in the atmosphere of the red star.

Now

$$\frac{dR_{red}}{dt} \sim \frac{1}{2}\frac{da}{dt}, \text{ and } \frac{da}{dt}$$

is given by

$$\frac{da}{dt} = \frac{-64}{5}\frac{G^3}{c^5}\frac{\mathfrak{M}_{bl}\,\mathfrak{M}_{red}(\mathfrak{M}_{bl}+\mathfrak{M}_{red})}{a^3}$$

Thus we find

$$\frac{d\mathfrak{M}_{red}}{dt} = -\bar{\rho}\frac{dV}{dt} = -\bar{\rho} \times 7 \times 10^{15}.$$

If the atmosphere of the red star were similar to that of the stream ejected through L_1, then

$$\bar{\rho} \sim 10^{-12}, \text{ and } \frac{d\mathfrak{M}_{red}}{dt} \sim -10^4 \text{ g/sec} = -10^{11} \text{ g/yr}.$$

On the other hand if the atmospheric density were as high as that of the Sun, we would have

$$\bar{\rho} \sim 10^{-7} \text{ and } \frac{d\mathfrak{M}_{red}}{dt} \sim -10^{16} \text{ g/yr}.$$

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The larger of these is still $\sim 10^{-7}$ times smaller than the mass loss estimates given above that lead to a period change of the same order as that given by gravitational radiation. We conclude, therefore, that the emission of matter brought on by the shrinkage of the surface resulting from the emission of gravitational waves can be neglected, at least in the beginning. Later, however, as the surface cuts more and more deeply into regions corresponding to the interior of the original star, we must encounter a large pressure gradient which will cause matter to flow out very rapidly. It may be, then, that the emission of gravitational waves provides a triggering mechanism whereby large amounts of matter can be forced to leave one component of a U Gem system. This brings us back full circle to the question of mass loss and transfer in the system. It is finally a question of whether the mass lost by the red component induces a period decrease that reinforces, or an increase that competes with, the decrease in period resulting from the emission of gravitational waves.

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DISCUSSION

Hoffmeister: According to the model, the two components of a close binary move within a gaseous envelope. Is not this motion in a resisting medium sufficient to explain the decrease of periods? There is a long enough time available, about 10^9 years.

Kraft: It is not sufficient.

Greenstein: What about tidal effects?

Kraft: Tidal effects have not been considered, but the blue star is too small for this to be important for the blue component.