$8r + 1$, $8r + 3$, $8r + 5$, $8r + 7$ where r is an integer. A little elementary algebra shows that on using the second equation and dividing by 2 or 4 or 8, these become $6r + 1$, $12r + 5$, $3r + 2$, $12r + 11$ respectively. Three of these are odd but $3r + 2$ is even when r is even and so, by the first equation, is then to be divided by 2 or a power of 2. Two of our four types are decreased by the process and two increased. It remains to show that on the average the decreases beat the increases so that in the long run there is a net decrease.

A few hours with a simple calculator show that all numbers less than 1000 sooner or later reduce to 1, so we may assume that r is greater than 100, so that in considering the approximate size of our numbers rather than their exact values we can ignore the $+1$, $+3$, $+5$ etc. above and say that after four steps, one of each of the above kinds, our number has changed approximately by a factor $\frac{6}{8} \times \frac{12}{8} \times \frac{3}{8} \times \frac{12}{8}$ that is 0·63 approximately. The actual factor will, on the average, be less than 0·63 because $3r + 2$ will sometimes be divided by 2 or a power of 2. Even using 0·63, however, the conclusion is that in the long run—and it might be a very long run—but in the long run any number is reduced and so eventually finishes at 1.

We have assumed here that the four types are equally likely or occur with equal frequencies, but there is good evidence that this is approximately true with perhaps the $8r + 3$ being slightly less frequent than the others. It is admitted that the argument here presented is incomplete. For example, it does not consider the possibility of the sequence getting into a loop."

My thanks go to all correspondents: it has been impossible to reply to them individually. These two 'stinkers' will be followed by a wide variety of puzzles of all difficulties.

Correspondence

Beating exponential growth

DEAR EDITOR,

The results in Dr Colin R. Fletcher's article (*Gazette* no. 430, December 1980) can be extended somewhat.

It is common practice in some financial institutions in the U.S.A. to calculate interest on a daily basis and to compound daily. In our part of the country such institutions include Savings and Loan Associations (the equivalent of U.K. Building Societies). Thus, for completeness, one should add an extra line to Fletcher's table. It appears to be the custom in many institutions to use a 360-day year for this purpose. We obtain the annual interest using the formula $A_n = P(1 + tr)^n$, where P = principal at the beginning of the year, r = interest rate per annum as a decimal, $t = 1/360$ is the compounding interval, the number of intervals $n = 360$ for one year, and A_n is the accumulated principal plus interest after the 360 intervals. We have calculated A_n for this case, and for the cases discussed by Fletcher, to six significant figures, to give the interest to the nearest cent on an investment of \$1000 for one year at 15% (the rate used by Fletcher to illustrate his article). The results are in our table.

Amount of \$1000 for one year

Simple interest	\$1150·00
Compounded yearly ($t = 1$, $n = 1$)	\$1150·00
Compounded monthly ($t = 1/12$, $n = 12$)	\$1160·75
Compounded daily ($t = 1/360$, $n = 360$)	\$1161·80
Exponential growth	\$1161·83
$t = 1/360$, $n = 365$	\$1164·22
$t = 1/360$, $n = 366$	\$1164·71

A further wrinkle has appeared. Some institutions are calculating their interest using $t = 1/360$ but putting $n = 365$ or, in leap year, $n = 366$. The results for these cases are listed in the table. We have beaten exponential growth! Well, not mathematically of course. But I did receive for 1980 interest calculated using $t = 360$, $n = 360$ from one institution and using $t = 1/360$, $n = 366$ from another.

Yours sincerely,

R. H. GARSTANG

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A pi-less proof

DEAR EDITOR,

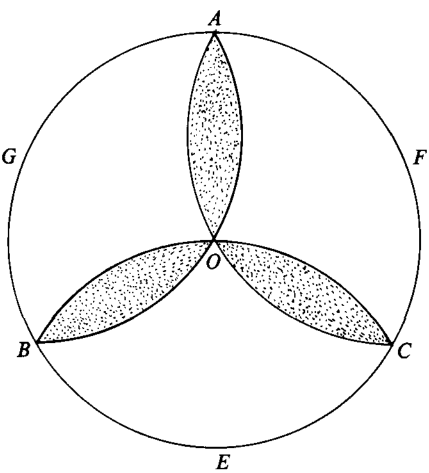
In J. V. Narlikar's note **65.3** (March 1981) the proof as well as the result can be pi-less.

If the area of the circle is S and that of the triangle ABC is T, the area bounded by the straight line AC and the arc AFC is $\frac{1}{3}(S - T)$. Six such areas cover the circle with overlaps as shaded. The addition of the unshaded areas would make up two complete circles, i.e.

$$2(S - T) + \text{unshaded areas} = 2S$$

and therefore

$$\text{unshaded areas} = 2T$$



Yours sincerely,

E. H. LOCKWOOD

18 West Hill, Charminster, Dorset

Editor's note: there will be more pi-less areas in next March's edition.

Odd odds

"Their chances of qualifying for the finals, to be played as the best of five, are not much more than mathematical." From the *Guardian*, 24th December 1980 (per Derek Middleton).

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