Neutron Stars and Pulsars: Challenges and Opportunities after 80 years
Proceedings IAU Symposium No. 291, 2012
J. van Leeuwen, ed.

doi:10.1017/S1743921312024672

Electric current diagnostics in the magnetosphere of neutron stars

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Abstract. Active neutron stars – SGRs, reveal the high-quality QPOs at the 'pulsating tail' phase. We suggest diagnostics of the trapped fireball plasma, the source of high-frequency pulsations, using coronal seismology. The trapped fireball is represented as a set of current-carrying loops - equivalent of electric circuits. Our approach gives the following magnetosphere parameters in SGRs: an electric current of $(2-8)\times10^{19}$ A, magnetic field of $(0.6-2.7)\times10^{13}$ G, and electrons density of $(1.3-6.0)\times10^{16}$ cm^{-3}. We show high-frequency QPOs can be self-excited for a smaller electric current than the maximum current and/or due to the parametric resonance.

Keywords. Stars: neutron, SGRs, flares, oscillations, current-carrying loops, diagnostics.

1. Introduction

Three giant flares of neutron stars (Table 1) with the energy release of $10^{44}-5\times10^{46}$ ergs, were accompanied by high-frequency (from tens to thousands Hz) quasi-periodic X-ray pulsations (Barat *et al.* 1983; Strohmayer & Watts 2006). A model of the high-frequency pulsations should explain not only their periods and excitation mechanism, but also their very high quality factor $Q\geqslant10^4-10^5$. Currently, the most widespread models for high-frequency QPOs are based on global seismic oscillations of the magnetar. Some models of neutron stars implied the excitation of torsion oscillations of the crust with a shear during starquakes. However, Levin (2007) pointed out that the torsion modes of the crust decay very rapidly. Beloborodov & Thompson (2007) proposed that non-linear oscillations of the production of electron-positron pairs emerge during the formation of a magnetar corona consisting of a set of magnetic loops. Nevertheless, current models are unable to explain the total set of the observed properties of QPOs with frequencies of 20 to 2400 Hz and their very high Q-factor. We represent the source of the pulsations - a trapped fireball - as a set of current-carrying loops and use the analogy with the loop as an equivalent RLC-circuit (Alfven *et al.* 1967; Stepanov *et al.* 2012). The efficiency of our model is illustrated by the diagnostics of the magnetospheres of SGRs.

2. The suggested approach: an equivalent electric (RLC) circuits

The 'trapped fireball' can be represented as a set of current-carrying magnetic loops with various sizes whose eigen-frequencies, quality factors, and inductance are given by

$$\nu=(2\pi\sqrt{LC})^{-1},Q=\frac{1}{R}\sqrt{\frac{L}{C}},L=2l\left(\ln\frac{4l}{\pi r}-\frac{7}{4}\right). \quad (1)$$

Here, R and C are the resistance and capacitance of the coronal loop, and the inductance L specified by the length l and radius r of a loop. Given the energy $E=LI^2/2$ that

Table 1. Pulsating tail properties in giant flares of SGRs and magnetosphere parameters.

	SGR 0526-66 March 5, 1979	SGR 1900+14 August 27, 1998	SGR 1806-20 December 27, 2004
Duration, s	~ 200	~ 400	~ 380
Energy, ergs	3.6×10^{44}	1.2×10^{44}	1.3×10^{44}
Main pulse period, s	8.1	5.15	7.56
QPO frequencies, Hz	43	28,54,84,155	18,26,30,93,150,625, 720,976,1840,2384
Q-factor	$\sim 10^4$	$\sim 10^4 - 10^5$	$\sim 10^4 - 5 \times 10^5$
		Calculated parameters	
Electric current, A	8×10^{19}	3×10^{19}	2×10^{19}
Magnetic field, G	2.7×10^{13}	10^{13}	6×10^{12}
Electron density, cm^{-3}	6×10^{16}	$2{\times}10^{16}$	1.3×10^{16}

has been released in the flare tail, one may determine the current I in the coronal loop and hence the coronal plasma density and the φ-component of the magnetic field. From the observed energy release power $W = RI^2$ one can find the resistance R of a loop, while from the oscillations frequency the loop's capacity C may be estimated, that is, the quality factor Q of the oscillations. Let us illustrate the efficiency of the proposed model.

Pulsating tail of SGR 1806-20 on 27 December, 2004. The energy released in this flare was of the order of 5×10^{46} erg, and the energy of the "ringing tail" was of the order of 10^{44} ergs. Taking into account the great variety of QPO frequencies (Table 1) we will suggest that the energy stored in an "average" loop in the course of "ringing tail" is roughly 10^{43} erg. Supposing $l = 3 \times 10^6$ cm and $r = 3 \times 10^5$ cm, we can use Eqs. (1) to find its inductance $L \approx 5 \times 10^{-3}$ Henry. Assuming that the stored energy of an "average" loop, $E \approx 10^{43} ergs = 10^{36}$ J, has been released we obtain the current $I = (2E/L)^{1/2} \approx 2 \times 10^{19}$ A, from which we estimate the φ-component of the magnetic field in the loop $B_\varphi \approx I/cr \approx 6 \times 10^{12}$ G. The electron-positron pairs density n in the source can be obtained from the electric current $I = encS$ and the cross section of the coronal loop S with the radius $r = 3 \times 10^5$ cm. For $I = 1.8 \times 10^{19}$ A, $n = 1.3 \times 10^{16}\mathrm{cm}^{-3}$, i.e., the Langmuir frequency $\nu_p \approx 1$ THz corresponds to the sub-mm wavelengths. The power of the energy release in the tail is of the order of 10^{41} erg/s, i.e., for an "average" loop $W = RI^2 \approx 10^{40}\ erg/s = 10^{33}\ W$. The resistance of a loop is $R = W/I^2 \approx 3 \times 10^{-6}\Omega$. One of the possible reasons for resistance may be the plasma wave instability driven by beams of high-energy electrons, accelerated in electric fields of the magnetar magnetosphere. The minimum ($\nu_1 = 18$ Hz) and maximum ($\nu_2 = 2384$ Hz) frequencies in QPOs allow the capacity of loops to be estimated from Eqs. (1): $C_1 \approx 1.3 \times 10^{-2}$ F, $C_2 \approx 7 \times 10^{-7}$ F. On the other hand, the capacity of a coronal loop may be presented as $C \approx \varepsilon_A S/l$, where $\varepsilon_A = c^2/V_A^2$ is the dielectric permeability of the medium for Alfven waves (Stepanov *et al.* 2012). It is known than in the magnetar corona $V_A \approx c$. Therefore, $\varepsilon_A \approx 1$ and we obtain $C \approx 10^5 cm = 10^{-7}$ F, which is several times lower than that C_2 calculated from the Eq. (1). It is easy to see that with increasing S as l decreases (thick loop), the coincidence of the capacitance with C_2 and C_1 can be achieved. Applying the second relation from Eqs.(1), we find corresponding Q-factors $Q_1 \approx 2 \times 10^5$ and $Q_2 \approx 10^7$, which exceed the observed quality factors of QPOs. Note, that the coronal loop in SGRs is a system with compact parameters and Eqs.(1) can be applied. Indeed, oscillations of electric current should be in-phase in all points of a loop. On the other hands, variations of the current propagate along the loop with the Alfven velocity ($\approx$ c for SGR). Therefore, the condition of phase coincidence $\nu \approx 20 - 2500$ Hz $< c/l \approx 10^4$ Hz is satisfied.

https://doi.org/10.1017/S1743921312024672 Published online by Cambridge University Press

3. Excitation of high-frequency QPOs of the current in coronal loops

For minor deviations of the electric current $|\tilde{I}| \ll I$, the equation for current oscillations in a loop can be presented as (Zaitsev *et al.* 2001):

$$L\frac{d^2\tilde{I}}{dt^2} + \alpha(I^2 - I^2_{max})\frac{d\tilde{I}}{dt} + \frac{\tilde{I}}{C} = 0 \qquad (2)$$

Eq. (2) indicates that oscillations will be excited for a smaller current than the maximum current in the giant pulse of the flare, $I < I_{max}$. Parametric resonance can be another way for the excitation of magnetic loop oscillations. The electric current oscillations due to perturbations in the crust with the pumping frequency ν trough a parametric interaction with a coronal loop can trigger oscillations in the loop at the frequency ν, at the subharmonics $\nu/2$, and at the first upper frequency of the parametric resonance $3\nu/2$. The variations in coronal loop parameters can be described by the equation

$$\frac{d^2y}{dt^2} + \nu_0^2(1 + q\cos\nu t)y = 0 \qquad (3)$$

where ν_0 is the frequency of the coronal loop eigen-mode. The parameter q defines the width of the zone near the parametric resonance frequency $\nu_n = n\nu/2, n = 1, 2, 3, \ldots$, namely $-\, q\nu_0/2 < \nu/2 - \nu_0 < q\nu_0/2$. The excitation occurs when the frequency of eigen-oscillations of the loop ν_0 falls on the first instability zone, i.e., it is close to $\nu/2$. Thus for a coronal loop to be excited parametrically, it must have suitable size, density, and magnetic field. In the flare in SGR 1901+14 27, the QPO were excited due to parametric resonance: $\nu = 54$ Hz, $\nu/2 = 27$ Hz (at the observed frequency 28 Hz), $3\nu/2 = 81$ Hz (at the observed frequency 84 Hz). Therewith the frequency width of the QPO peaks is about 1-5 Hz. Note that we obtain the observed frequencies ($\nu/2 = 28$ Hz and $3\nu/2 = 84$ Hz) with a high accuracy for ν equal to 56 Hz rather than 54 Hz.

We present the source of QPOs, a trapped fireball, as a set of current-carrying loops - an equivalent electric (RLC) circuits. Nevertheless this phenomenological approach is quite effective diagnostic tool for magnetospheres very active neutron stars - Soft Gamma Repeaters. With this approach we determined the electric current in magnetospheres of SGR 0526-66, SGR 1806-20, and SGR 1900+14, electron density, and magnetic field $B < B_q = m^2c^3/\hbar e = 4.4 \times 10^{13}$ G. It means that the physical processes in magnetar magnetospheres at the 'ringing tail' phase can be studied within non-quantum electrodynamics approach.

This work was partially supported by the RFBR grants No 12-02-00616a and 11-02-00103a, the Program of Leading Scientific Schools (1625.2012.2 and 4185.2012.2), the RAS Program 'Non-steady-state Universe', and the Federal Goal-Oriented Program (code 2012-1.2.1-12-000-1012-010).

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