MODE IDENTIFICATION IN PULSATING STARS

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Abstract. During the past twenty years, different methods have been developed to identify the modes in non-radially pulsating stars. Before the introduction of high-resolution spectrographs with sensitive detectors, identifications were obtained from photometric observations. More recently, mode identification is obtained by means of spectroscopic methods. In this paper, we present an overview of the different mode-identification techniques currently used and we describe their accuracy to identify the modes present in different kinds of pulsating stars. By means of some applications of the moment method, we show that this method deserves far more attention than it has received until now.

1. Introduction

The displacement field generated in the photosphere of a star by an non-radial pulsation (hereafter called NRP; see e.g. Smeyers 1984, Unno *et al.* 1989) leads to periodic variations of observable physical quantities. By comparing the observed variations with those predicted by theory, one may hope to determine the most important parameters that appear in the expression for the displacement field. Specifically, mode-identification techniques try to assign values to the spherical wavenumbers (ℓ, m), the degree and the azimuthal number of the spherical harmonic Y_ℓ^m that describes the NRP mode. We will discuss various techniques in Section 2.

A caveat for many analyses is that the theoretical framework which is used only applies to the slow-rotation approximation, i.e. the case where the effects of the Coriolis force and the centrifugal forces may be neglected in deriving an expression for the components of the Lagrangian displacement. It is, however, not allowed to describe an oscillation mode for a rotating star in terms of a single spherical harmonic, and so to ascribe a single set of wavenumbers (ℓ, m) to a mode (see e.g. Lee & Saio 1990, Lee *et al.* 1992; Aerts & Waelkens 1993). The Coriolis force introduces a transverse velocity field that is of the same order of magnitude as the pulsation velocity for a non-rotating star if the ratio Ω/ω of the rotation frequency to the pulsation frequency approaches unity. It is clear that this last condition may be met in several stars discussed in the literature.

In this paper, we will summarise the different methods commonly used to identify modes in Section 2, where we also list their advantages and disadvantages. Some applications of the moment method will be discussed in Section 3. Finally, we present some concluding remarks in Section 4.

L. A. Balona et al. (eds.), Pulsation, Rotation and Mass Loss in Early-Type Stars, 75–86.

2. Mode-identification Techniques

2.1. Q-VALUES

By comparing the observed values of the period P and the pulsation constant $Q \equiv P\sqrt{\overline{\rho}/\overline{\rho}_\odot}$, where $\overline{\rho}$ is the average density, with the ones calculated from theoretical models, it should be possible to estimate the degree and the order of the pulsation mode. While this technique works well for radial pulsations, it is of low power for NRPs. Indeed, the errors on the physical parameters introduce uncertainties on Q that are often larger than the difference in theoretical Q-values associated with different pulsation parameters.

2.2. PHOTOMETRIC AMPLITUDES

This method is based on the photometric variations of a pulsating star and was introduced by Dziembowski (1977), refined by Stamford & Watson (1981), Watson (1988) and Heynderickx (1991), and also applied by Cugier & Boratyn (1992) and Cugier *et al.* (these Proceedings). Heynderickx (1991, 1992) has made a thorough study of a large group of β Cephei stars and slowly pulsating B stars (SPBs, see Waelkens 1991) for which he determined the degree of the pulsation modes. To achieve this, a theoretical expression for the photometric amplitude of a non-rotating, pulsating star is derived as a function of the wavelength λ and of the degree ℓ of the pulsation mode. By calculating this expression for different values of ℓ and for different values of λ, and by comparing the results with the observed photometric amplitudes at the same wavelengths, one finds the value of ℓ that best fits the observations. The azimuthal number m cannot be determined with this method because a non-rotating star is considered. The method appears to work well for β Cephei stars and SPBs (see Heynderickx 1991).

It would be interesting to extend the method of photometric amplitudes to a rotating pulsating star and apply it to photometric observations of the rapidly rotating Be stars. Such an analysis might help to decide whether or not the observed photometric and spectroscopic variations in these stars are due to NRP or to rotating spots.

The main difficulty with this method seems to be linked with the uncertainties on the stellar parameters needed to calculate the theoretical amplitudes. In some cases, inaccurate stellar parameters might lead to a misidentification of the mode.

2.3. LINE-PROFILE FITTING

The velocity field caused by the NRP(s) leads, through Doppler displacement, to periodic variations in the profiles of spectral lines. After having been first detected in the β Cephei stars, line-profile variations (hereafter called LPVs) have been observed in other kinds of variable OB-type stars as well (Petrie & Pearce 1962, Smith 1977, Baade 1982, Vogt & Penrod 1983, Baade & Ferlet 1984, Baade 1990, among others).

Since Osaki (1971) computed theoretical line profiles for various NRPs, the identification of modes from spectroscopic observations has become a popular technique. The identification of NRP modes from LPVs is achieved by line-profile-fitting on a trial-and-error basis. The idea is to compare the observed LPVs with those predicted by theoretical calculations. Until recently, this technique was used by most authors; examples are given by Baade (1984), Smith (1977,1983,1985,1989), Vogt & Penrod (1983), among others.

The main disadvantage of the method is the large number of free parameters that appear in the velocity expression due to the NRP. Indeed, the NRP model may be successful in reproducing the line profiles for different sets of input parameters, implying that the fitting technique does not necessarily lead to a unique solution. We also mention that the apparent quality of some fits is suspect in the sense that in modelling, one usually neglects temperature variations, which obviously must affect the profiles.

Some problems appear in applying the line-profile-fitting technique. It often appears necessary to assume that some modes temporarily disappear and reappear afterwards in order to obtain good fits over a large time scale. Also, the values found for the intrinsic profile sometimes have to be varied from one night to another in order to obtain reliable fits. Such assumptions are often introduced in an *ad hoc* fashion and cast doubt on the reliability of the model. In case of rapidly rotating stars, one usually assumes equator-on geometries and high-degree sectorial modes because these are the only ones that can produce the observed "moving bump phenomenon". On the other hand, high-degree modes are almost never seen in slowly rotating stars. Finally, it is always mentioned, but most often not taken into account, that one uses an expression for the pulsation velocity that is related to one spherical harmonic. This is, however, only valid in the case of a non-rotating star.

To overcome some of these problems, Kambe & Osaki (1988) have made a study of theoretically generated LPVs in rapidly rotating stars due to an NRP for moderate to high ℓ-values ($\ell = 2, \ldots, 8$). They also used, however, a pulsation velocity expression derived for a non-rotating star. On the other hand, they also considered purely toroidal modes in a non-rotating star and studied their associated LPVs. The authors find that moving bumps can be produced for modes with $\ell > 4$ in the following situations :

1. spheroidal sectorial modes with a large ratio of the horizontal to the vertical velocity amplitude at small inclination angles,
2. toroidal sectorial modes with intermediate inclination angles,
3. spheroidal tesseral modes with $\ell - 1 = |m|$ and an almost equator-on geometry.

It should be noted, however, that pure toroidal modes only become time-dependent in a rotating star. It is then not appropriate to use an expression for the velocity field associated with toroidal modes that relies on the solutions of order zero in the rotation frequency. Expressions for the toroidal displacement fields that are correct up to second order in Ω/ω have been computed by Smeyers *et al.* (1981).

2.4. DOPPLER IMAGING

In more recent studies, emphasis is especially laid on the spectacular LPVs of rapidly rotating OB stars, such as Be stars. Indeed, it has been recognised that the line profiles of rapid rotators allow a Doppler Imaging (DI) of the stellar surface (Vogt *et al.* 1987), so that a mapping of the pulsation velocity over the surface of variable stars should become possible (Baade 1987). Gies & Kullavanijaya (1988) presented an objective criterion based on DI to determine the periods and pulsation parameters of the modes in rapidly rotating stars. Fourier analysis of the LPVs at each wavelength point yields the periods of the variations by frequency peaks in the resulting periodogram. The true periods are distinguished from alias patterns with Robert's CLEAN algorithm (Roberts *et al.* 1987) or with the Akaike Information Criterion (Kambe *et al.* 1990). The azimuthal number m is obtained by considering the number of phase changes at each signal frequency versus the line position (see also Gies, these Proceedings).

It is usually assumed that equator-on-viewed sectorial modes appear. Moreover, although the method is developed for rapidly rotating stars, one uses an expression for the pulsation velocity based on the slow-rotation approximation. Although these assumptions seem to be rather restrictive, the DI technique is by now often applied, mostly to identify the modes claimed to be present in Be stars; examples are given by Baade (1988, μ Cen), Kambe *et al.* (1990, 1993, ζ Oph), Reid *et al.* (1993, ζ Oph), and Floquet *et al.* (1992, EW Lac).

Some attention should be paid to the fact that the DI technique is based on the usual slow-rotation approximation. Indeed, Lee & Saio (1990) and Aerts & Waelkens (1993) have shown that rotation can significantly affect the eigenfunction in a pulsating star. Aerts & Waelkens find that the effects of the toroidal correction terms due to the rotation become important when the ratio of the pulsation period to the rotation period becomes larger than 20 per cent. Therefore, one should be careful when applying DI to rapidly rotating stars for which this ratio is larger than 20 per cent.

An experiment in cooperation with A. Reid has been conducted to test the implications of the slow-rotation approximation when applying DI. Hereto, two sets of line profiles were generated for a non-radially pulsating star with the velocity expression taking into account the effects of the Coriolis force (see Aerts & Waelkens 1993). Application of DI in the usual slow-rotation approximation shows that a misidentification of both the period and the wavenumbers (ℓ, m) may appear in case of a low-degree non-sectorial input mode; for the test case of an $\ell = 2, m = -1$ input mode, the DI technique wrongly identified a high-degree sectorial mode ($\ell = -m = 5$ or 6). For a given $\ell = -m = 8$ mode, DI recovered the correct parameters (see Reid & Aerts 1993).

The findings of Reid & Aerts are based upon only two examples. It is clear that a more extensive range of NRP and stellar parameters has to be considered in order to obtain a final conclusion about the accuracy of the DI technique in combination with the CLEAN algorithm. In any case, the findings of Aerts & Waelkens and of Reid & Aerts could have some implications for mode identifications obtained thus far with the DI technique. Therefore, a re-evaluation of the mode identification based on DI seems necessary in case of rapidly rotating stars for which the effects of the rotation cannot be neglected.

2.5. THE MOMENT METHOD

To overcome the problems of line-profile fitting, Balona (1986a,b;1987;1990) proposed an alternative method of pulsation-mode identification from LPVs. The method is based on the time variations of the first few moments of a line profile. The periodograms of the moments can immediately be interpreted in terms of the periods that are present and in terms of the NRP parameters. The basic idea is to compare the observed variations of the moments with theoretically calculated expressions for these variations in case of various pulsation modes, and so to determine the mode that best fits the observations. This is achieved through the construction of a discriminant, which is based on the amplitudes of the moments.

At first sight, it may seem that by considering only a few numbers to characterise the profiles, one looses much of the richness of the observation. However, it turns out that much of the information necessary to determine the pulsation modes is contained in the first three moments. Moreover, the mode identification is based on the variation of the moments, and so is only marginally affected by uncertainties concerning temperature variations and the intrinsic profile.

The method as proposed by Balona (1986b, 1987) is developed in the case that the projected rotation velocity is much larger than the pulsation velocity. However, we suggest to combine Balona's approach with the velocity field as presented by e.g. Aerts & Waelkens (1993) since they explicitly take into account Coriolis correction terms, which might be important in identifying the correct modes. The combination of Balona's approach and the velocity field of Aerts & Waelkens may then provide a powerful tool to identify the modes in rapidly rotating stars.

The moment method in case of slowly rotating stars has been introduced by Balona (1990). His discriminant is a one-dimensional function based on the first two moments. It turns out that this discriminant has some limitations because no error on the velocity amplitude v_p is allowed. The moment method for slow rotators has been refined by Aerts *et al.* (1992), who have proposed a more accurate discriminant to obtain the mode identification. Indeed, their two-dimensional discriminant is based on the leading amplitudes of the first three moments and is less sensitive to errors on the estimates of both the velocity amplitude and the inclination angle. Together with the mode identification, this discriminant also gives estimates of the pulsation amplitude v_p and the inclination angle i of the star.

Aerts *et al.* (1992) also present a method to determine the projected rotation velocity v_Ω and the width of the Gaussian intrinsic profile σ from the other terms of the moments, once a mode identification has been obtained by the discriminant. Recently, the moment method for slow rotators has been generalised to a multiperiodic pulsation by Mathias *et al.* (1993), who use essentially the same discriminant as Aerts *et al.* for each of the present modes separately.

The accuracy of the moment method is fully described by De Pauw *et al.* (1993), who studied the shape of the moments, the ability of the discriminant to obtain a correct mode identification, and the accuracy of the estimates of the velocity parameters for a large set of theoretically generated profiles with all kinds of NRP parameters. They conclude that a quantitative determination of the pulsation mode is possible for $\ell < 4$; for higher-degree modes the discriminant is not very convenient, but these modes are distinguished from those with a lower degree. In their paper, De Pauw *et al.* also studied the influence of the noise of observed spectra and of the number of available line profiles on the mode identification; their paper is a useful guide for users of the moment method in the formulation given by Aerts *et al.* and Mathias *et al.*

3. Applications of the Moment Method

The moment method in the slow-rotation approximation has recently been applied to β Cephei stars (see Aerts 1993). We have considered these stars to be ideal test cases for the moment method since they are regular pulsators that usually have a long record of photometric observations. We can then rely on photometric studies for period determinations and sometimes also for estimates of the degree ℓ of the mode (see e.g. Heynderickx 1991, Cugier *et al.* these Proceedings).

A first star that has been analysed with the moment method is δ Ceti, a well-known radially pulsating star. The analysis of this star (Aerts *et al.* 1992) has shown that the moment method works well for a star exhibiting such a simple pulsation. By means of illustration, we show in Fig. 1 some discriminants for δ Ceti, representing the four best solutions in the parameter space (ℓ, m, v_p, i). The best solution for (ℓ, m) is the one for which the

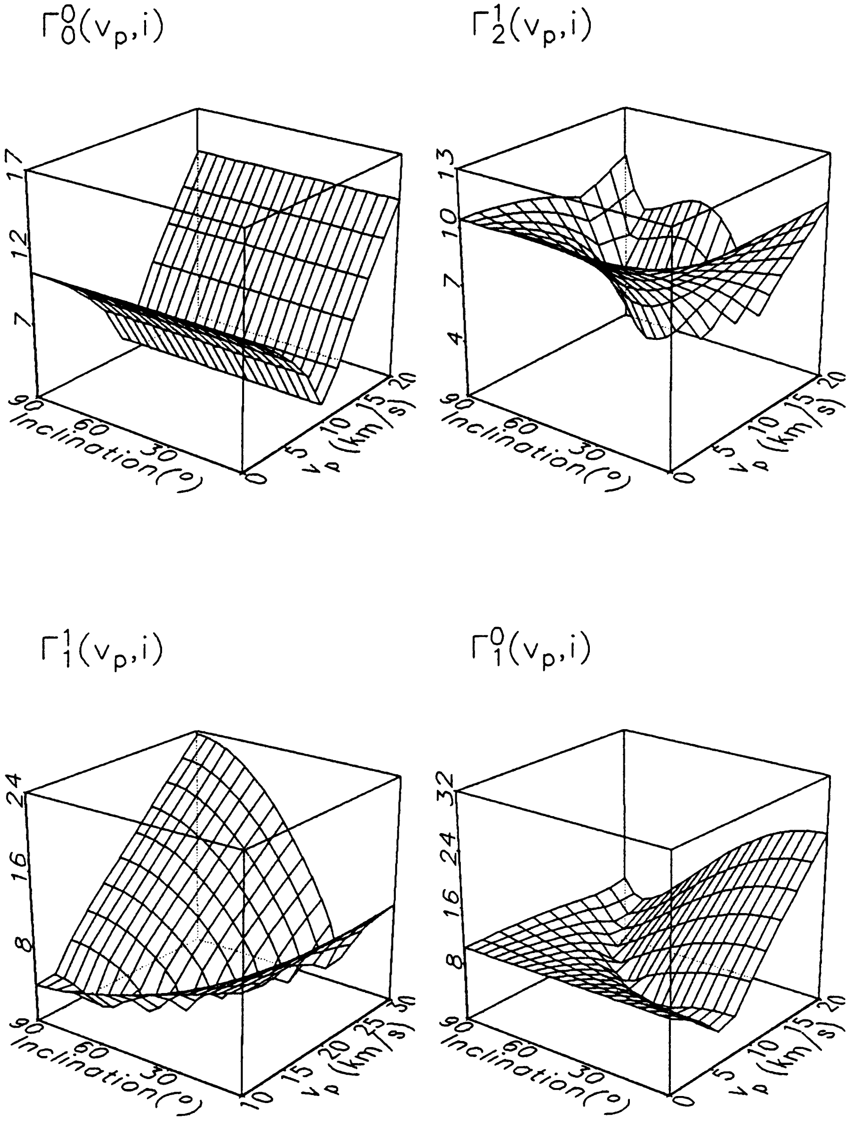

Fig. 1. Four discriminants for the β Cephei star δ Ceti representing the best solutions in the parameterspace (ℓ, m, v_p, i)

discriminant $\Gamma_\ell^m(v_p, i)$ reaches the lowest minimum. This minimum then also gives the best values for v_p and i for this given set (ℓ, m). In case of δ Ceti, the discriminants point towards a radial mode with $v_p = 10.6\,\mathrm{km\ s^{-1}}$; the second to fourth best solutions have minima that are at least a factor two higher than the minimum of the radial solution (see Fig. 1 and Aerts *et al.* 1992). It should be stressed that the theoretical profiles constructed for δ Ceti with the set of parameters found by the discriminant do not lead to perfect fits, because the temperature effects become important in studying the shape of the line profiles. We recall that the moment method is less sensitive to temperature variations, because it is based on the time *variations* of the moments of the line profiles.

Two more β Cephei stars, that were thought to be monoperiodic, were analysed with the moment method: β Cephei and KK Velorum. The analyses have shown, however, that both stars pulsate in more than one mode (Aerts 1993). In β Cephei, the two newly discovered modes seem to be a sectorial $\ell = 2$ one and an axisymmetric $\ell = 2$ one, both having an amplitude that is much smaller than the one of the well-known radial mode. The two additional modes lead to beat-periods of 7.4, 2.2, and 3.0 days (Aerts *et al.*, in preparation). The mode identification of these two newly discovered modes is uncertain because of the small amplitudes. KK Velorum exhibits a second radial mode, besides the already reported $\ell = 4$ g-mode (Heynderickx 1991, Waelkens & Aerts 1991). We note that the theoretically calculated instability region for β Cephei stars obtained with the new opacities is not only confined to the p-mode region, but extends to the g-modes as well (Dziembowski & Pamyatnykh 1993; Dziembowski, these Proceedings). KK Velorum may thus be an interesting object for further testing the pulsation mechanism for β Cephei stars.

Several multiperiodic stars were also analysed with the moment method (Aerts 1993). The analysis of α Lupi (Mathias *et al.* 1993) has shown that the moments of different spectral lines can be combined in applying the moment method. A problem for the analysis of LPVs of multiperiodic stars are the long beat periods that appear due to the interaction of the different modes, making it very difficult to obtain a full cover of the total interaction period. For such cases, it is very useful to combine photometric and spectroscopic analyses.

In principle, the method should also apply to less regular and/or long-period variables, such as e.g. the SPBs. These multiperiodic stars are known to pulsate in g-modes (see Waelkens 1991) but are very slow rotators so that the formalism as presented by Mathias *et al.* (1993), which neglects the effects of the Coriolis force, still applies for them.

4. Conclusions

It is clear that the introduction of accurate detectors into spectroscopy has been a very important issue for the mode identification in pulsating stars. Indeed, the photometric methods are less informative on the identification of the modes, since information on the total energy of the star is used, i.e. we have to work with information that is an integration of a physical quantity over the stellar hemisphere. However, photometric observations are very useful to find accurate periods and also allow an estimation of temperature effects. Moreover, in case of long beat periods, it is very useful to combine photometric and spectroscopic results since a full cover of the beat period may be very difficult to obtain spectroscopically.

Contrary to photometric measurements, spectroscopic observations offer a detailed picture of the visible stellar surface and allow determination of all the parameters that appear in the velocity expression. Due to the richness of the NRP model, it has become clear that quantitative identification techniques are preferred to the direct line-profile-fitting technique.

For slow rotators, the moment method is by far the best one available thus far. Rapid rotators can be analysed with the two different quantitative techniques presented in the literature, although we urge caution in using them both in their actual formulation. The moment method should be generalised by combining Balona's (1987) approach with a velocity field that takes into account the effects of the rotation on the velocity expression due to an NRP; the DI technique should be tested more thoroughly on synthetically generated LPVs for all kinds of NRP parameters as is being done at the moment by e.g. Reid (PhD Thesis, in preparation).

Finally, after having obtained the most likely set of all the velocity parameters, theoretical LPVs should be generated in order to check if their evolution during a pulsation cycle is consistent with the observed ones. We stress that perfect fits should not be expected since the used pulsation models neglect many physical processes, such as temperature effects. While these effects may have a minor influence in studying the relative variation of the line profiles, they cannot be neglected in studying the absolute shape of the observed line profiles.

Acknowledgements

I am indebted to Christoffel Waelkens for his helpful comments on the manuscript and I also thank Andy Reid for explaining many aspects of the Doppler Imaging technique.

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Discussion

Smith: I agree with some of the shortcomings of the line-profile-fitting technique, but perhaps not all of them! Certainly, when confronted with multiple modes in line profiles, one should try to obtain the most reliable ephemeris by an "objective" period-finding technique. I prefer a Doppler Imaging technique (e.g. with CLEAN) to the moment method because it is easier to understand in terms of the derived NRP parameters and it does not assume a Gaussian intrinsic profile (Gaussians tend to mask the appearance of bumps for $m \leq 4$ modes).

Once one has derived periods from whatever technique, one wants to put some physics back into the LPVs because they contain so much information. The treatment of profiles by a short series of mathematical functions will be unable to explain e. g. the asymmetric wings to the continuum, the ratio of narrow/broad line phase widths, the profile footprints, the appearance/disappearance of profile bumps and even signatures of NRP nonlinearities. So perhaps what is needed is to combine the strengths of both objective and profile-fitting methods, in addition to light and color curves.

Aerts: I disagree with your first comment since the variations of the moments are immediately interpreted in terms of the periods and the NRP parameters, contrary to the CLEANed periodograms that are integrated quantities without a physical meaning necessary to obtain the most likely periods when using the CLEAN algorithm. The variations of the moments are hardly affected by the assumption of a Gaussian intrinsic profile, since we consider the time variations of the normalised moments, the normalisation factor being the equivalent width of the line profiles.

As I concluded, we should indeed generate theoretical LPVs with the most likely set of parameters found by an objective method. This should be done however *after* the mode identification and not as a method to obtain the most likely parameters.

Baade: Have you tested how easily the moment method gets confused by the presence of shock effects ?

There would be some interest in applying the moment method to Be stars because a comparison of their Hα emission line profiles with the models of Poeckert & Marlborough often permits one to make a coarse estimate of the inclination angle. Therefore, one may pre-select targets seen at a low inclination which should show much stronger evidence of tesseral modes, if any. From the same arguments it follows that the Doppler Imaging method does not always require the assumption of equator-on views.

Aerts: So far, we only applied the moment method to LPVs of β Cephei stars. In these stars, only very weak shock waves are present, if any (ν Eri, α Lupi). Our analyses have shown that such small shocks do not affect the accuracy of the method. We plan to apply the moment method to 12 Lac and BW Vul in the near future. This should give an idea about the effect of stronger shocks on the mode identification with the moment method.

It would indeed be interesting to apply the moment method to the rapidly rotating stars, such as the Be stars. However, the method should then first be generalised by taking into account the toroidal correction terms as well as temperature variations. The accuracy of the estimate of inclination angle using the Doppler Imaging technique needs to be tested by numerical simulations.

Henrichs: May I strongly suggest that you change or annotate your terminology of "slow" and "rapid" rotators. Traditionally, rapid rotators are stars with high $v \sin i$, and those stars might be slow or rapid rotators in your description. I suggest that you discriminate between slow/rapid rotators in the gravitational sense and slow/rapid rotators in the pulsational sense.

Aerts: OK. To avoid confusion: in this talk I have always used the term rapid rotator in the pulsational sense, i.e. for stars that have $\Omega/\omega > 20$ per cent, no matter how their rotation velocity relates to their break-up velocity.

Jerzykiewicz: You mentioned that the pulsation constant for KK Velorum is 0.050 d. This is quite unusual for a β Cephei star. How did you get your value ?

Aerts: We got the Q-value from photometry as well as from spectroscopy. We mention that the Q-value for the main mode is indeed exceptionally large and points towards a g-mode, but the Q-value of the second mode is 0.025 d which is a medium value in case of a p-mode.