The open mapping and closed graph theorems in topological vector spaces, by T. Husain. (Oxford math. monographs).

Of the seven chapters of this book, the first two are introductory: Chapter I reviews, without proofs, the elementary definitions and results of general topology and vector spaces (without topology). Chapter II is a brisk treatment of the classical results on topological vector spaces: Hahn-Banach theorem, barrelled and bornological spaces, Banach-Steinhaus theorem, duality; most of the proofs are given here.

With Chapter III begins the book's theme, the study of the open mapping and closed graph theorems, whose statements are as follows:
(A) A linear and continuous mapping f of E onto F is open.
(B) A linear mapping g of F into E with closed graph in $F \times E$ is continuous.

The problem is to find interesting pairs of topological vector spaces E, F for which these statements are both true: Banach's classical result is that one can take for E a complete metrizable space and for F a Baire space. The proof, together with some closely related generalizations, is given in Chapter III. Chapter IV goes beyond, with the introduction of the idea of B-complete spaces, due to V. Pták. A mapping $f: X \to Y$ of topological spaces is called almost open if, for any neighborhood U of any point $x \in X$, the closure $\overline{f(U)}$ is a neighborhood of $f(x)$; a B-complete locally convex space E is defined by the condition that any linear, continuous and almost open mapping of E onto a locally convex space F is open. Then, Pták and the Robertsons have proved that statements (A) and (B) are true if E is a B-complete locally convex space and F a barrelled space. Chapter IV proves these results as well as a fairly large number of characterizations and properties of B-complete spaces.

Chapters V and VI relate the notion of B-completeness and other notions refining "completeness" to the theory of duality. On the dual E' of a locally convex space, the ew^*-topology is defined as the finest which coincides with $\sigma(E', E)$ on each equicontinuous subset of E'. A complete space is characterized by the fact that each hyperplane in E' which is closed for $\sigma(E', E)$ is also ew^*-closed. If in this statement one replaces "hyperplane" by "vector subspace" one gets B-completeness; a still stronger property is the "hypercompleteness" of Kelley, which one obtains this time by replacing "hyperplane" by "convex circled set". These notions are studied in Chapter V; it is not known whether "B-completeness" and "hypercompleteness" are equivalent, but there are B-complete spaces which are not complete.

Another aspect of the theory of the ew^*-topology is the theory of S-spaces, created by the author and developed in Chapter VI: an S-space is a locally convex space E such that on E' the ew^*-topology coincides with the topology of uniform convergence in precompact sets of E. These spaces are not necessarily complete, but the completion of an S-space is hypercomplete and an S-space; the converse is unknown.

Chapter VI also takes up the properties of subspaces, quotients and duals of S-spaces.

Finally, Chapter VII is also an elaboration of original ideas of the author. Given a class $\mathcal{C}$ of locally convex spaces, one defines the class B($\mathcal{C}$) as consisting of those locally convex spaces E such that, for any space F in $\mathcal{C}$, statement (A) is true for almost open mappings; if $\mathcal{T}$ is the class of barrelled spaces, B($\mathcal{T}$)-spaces are thus the B-complete spaces defined earlier. The author chiefly investigates in that chapter what can be said of statement (B) for B($\mathcal{C}$)-spaces, for various classes $\mathcal{C}$.

A detailed historical note, a bibliography and two indexes end the book, which is concisely and clearly written.

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Geschichte und Theorie der Kegelschnitte und der Flächen zweiten Grades, by Kuno Fladt. Ernst Klett Verlag: Stuttgart, 1965. x + 374 pages. 185 figs. DM 48.

More than fifty years ago the author, then still a student, felt already the need for the type of book that he has now written. Of moderate size, it should contain the essential properties of the conic sections in a presentation which recapitulates the historical development of this theory. The result of the author's life-long occupation with the subject can be highly recommended to students and teachers of mathematics alike. Compared with J. L. Coolidge's "History of the conic sections and quadric surfaces" (Oxford, 1945, Dover Reprint 1947), Fladt's book puts emphasis on elementary methods in the sense of Felix Klein's "Elementary mathematics from an advanced standpoint".

Part I (140 pp.) traces the historical development of the theory of the conics and quadrics from antiquity to the present, summarizing the works of the more important mathematicians whose names are connected with it (Menaechmus, Euclid, Apollonius, Desargues, Pascal, Poncelet, Steiner, von Staudt, Plücker, Möbius, Salmon, Cayley, Klein and many more are discussed). The use of modern algebraic and geometrical formulas and language enable the author to do this in a concise and yet readable form. Part II (190 pp.) builds up the theory of the conics in a more systematic way. Guiding principle is Klein's classification of geometries according to invariants under groups of transformations. Euclidean, affine and projective treatment follow each other, with a special section on conics as projections of the circle. Part III (20 pp.) gives an outline of an elementary treatment of quadratic surfaces, while Part IV (16 pp.) contains a skeleton summary of modern methods (vectors, matrices, invariants, axiomatics, etc.), with reference to recent German literature. The importance of the conics for the foundation of geometry, which Cayley recognized first, is stressed, but a detailed treatment would have gone beyond the scope of this unique book.

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