CORRESPONDENCE.

FORMULA FOR THE FORCE OF MORTALITY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—May I ask the privilege of sufficient space in your *Journal* to observe, that the formula for computing force of mortality from numbers living,

$$\mu = \frac{8(l_{-1} - l_{+1}) - (l_{-2} - l_{+2})}{12l},$$

given by Mr. W. S. B. Woolhouse at p. 64 of the *Journal* for April last, is identical with one which had been published by me some

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years before (*Smithsonian Report* of 1873, p. 343). My notation was

$$\mu_3 = \frac{8(y_2 - y_4) - (y_1 - y_5)}{12 y_3},$$

and I remarked upon it as follows:—" Thus, if we have found by observation the numbers of persons annually attaining any five consecutive birthdays, or, what amounts to the same thing, the numbers living at those five ages out of a given number of persons born, the above formula will give the intensity of mortality at the middle birthday."

I am, Sir,
Your obedient Servant,

Watertown, Connecticut, U.S.A., E. L. DE FOREST.
1 *Oct.* 1878.

In connection with the above letter, it may be useful to remind our readers of the more exact formula for the force of mortality given by Mr. Sprague in vol. xiii, p. 210, and applied by him in vol. xvii, p. 268, namely

$$-\mu_x l_x = a_0 - \frac{c_0}{6} + \frac{e_0}{30} - \frac{g_0}{140},$$

where a_0, c_0, e_0, g_0, are the central differences of the first, third, fifth, and seventh orders, their values in terms of the l's being

$$a_0 = (l_{+1} - l_{-1}) \div 2$$
$$c_0 = \{l_{+2} - l_{-2} - 2(l_{+1} - l_{-1})\} \div 2$$
$$e_0 = \{l_{+3} - l_{-3} - 4(l_{+2} - l_{-2}) + 5(l_{+1} - l_{-1})\} \div 2$$
$$g_0 = \{l_{+4} - l_{-4} - 6(l_{+3} - l_{-3}) + 14(l_{+2} - l_{-2} - l_{+1} + l_{-1})\} \div 2.$$

Taking two terms in the value of μl, we get by substitution,

$$\mu = \{8(l_{-1} - l_{+1}) - (l_{-2} - l_{+2})\} \div 12l,$$

which is the formula given by Mr. Woolhouse. Taking three terms, we similarly get

$$\mu = [9\{5(l_{-1} - l_{+1}) - (l_{-2} - l_{+2})\} + l_{-3} - l_{+3}] \div 60l.$$

ED. *J.I.A.*
