Einführung in die Zahlentheorie von Arnold Scholz, überarbeitet und herausgegeben von Dr. B. Schoeneberg. Third ed. Sammlung Göschen No. 1131. Walter de Gruyter and Co., Berlin 1961. Price DM. 3.60.

An introduction to Number Theory that in its four chapters covers roughly the same ground as Dirichlet-Dedekind's classic in its first four chapters. Taking into account the smallness of the volume (126 pages, 4 x 6 in^2) it contains a considerable amount of information, of course at the price of a terse style, carefully avoiding repetition. It will be appreciated by intelligent students and by every instructor planning a course on these lines.

Contents: I. Divisibility, including arithmetical functions. II. Congruences, Residue Classes, including primitive roots, power residues, representation by sums of squares: Lagrange's theorem for four squares and the classical condition for two squares, proved by means of a strikingly simple theorem of Thue, which is based on the pigeon hole principle. III. Quadratic residues, with two proofs of the reciprocity law and a few remarks on biquadratic residues. IV. Quadratic forms, including finiteness of the class number proved for definite and for indefinite forms, ending with a brief discussion of Pell's equation.

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Mathematical Logic and the Foundations of Mathematics, by G.T. Kneebone. Van Nostrand. 400 pages. 1963.

This book is subtitled "An Introductory Survey", and it is exactly that. It is introductory in that no previous knowledge is assumed, and it is a survey in that its range is very wide and finer details are omitted except for references. The author himself compares it with Baedeker.

It succeeds admirably in its purpose. Something is said about a remarkably large number of well selected topics; what is said is well enough said and sufficiently detailed to awake interest, and excellent references are provided for detailed study. It should be irresistibly inviting to a student who works in the presence of a good library (without this latter condition, however, other books would be preferable). For those who appreciate such things, it may be added that it is written in excellent English, rather than in the mathematically corrupted form

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which is well adapted for certain purposes, but so often applied far beyond them.

One caution might be given, namely that, when supplemented as it should be by reading of the sources it recommends, it is a very substantial introduction indeed, and might well be studied over a period of two academic years.

A sample will be discussed, to give some idea of Professor Kneebone's treatment.

The sample is section 4 "Completeness of the restricted calculus of predicates" of Chapter 3, "The Restricted Calculus of Predicates". This section begins by mentioning the notion and fact of completeness of propositional calculus. The notion of a formula of predicate calculus being identical for a domain D is then explained, and similarly for satisfiability. It is taken as obvious that F is identical in D if and only if $\bar{F}$ is not satisfiable in D, and that only the cardinal number of D is relevant. $F(y) \rightarrow (x) F(x)$ is given for illustration, and also a simple example requiring a domain of cardinality γ_0 for satisfiability.

There follows a brief discussion of normal forms, the distinction being made between deductive equivalence and convertibility (reference to Hilbert and Bernays: Grundlagen). $F(y)$ and $(x) F(x)$ are given as a specimen pair. Reduction to prenex normal form is briefly explained, some details being left as an exercise. The Skolem normal form is then discussed, the relaxation of convertibility to deductive equivalence being clearly stated, and a readable proof is given of the main result.

Given the Skolem normal form of the given formula, the sequence of disjunctions of values

$$A(x_0, \ldots, x_0, x_0; x_1, x_2, \ldots, x_e)$$

$$A(x_0, \ldots, x_0, x_1; x_{e+1}, x_{e+2}, \ldots, x_{2e})$$

$$\vdots$$

of the matrix is presented (without motivation) and the reduction to formulae of propositional calculus is given. Reference is then made to Hilbert and Ackermann, Gödel, and Henkin, for completion of the proof, and alternative methods. Finally, Löwenheim's theorem is stated and proven as a corollary.

The book concludes with an Appendix on developments since 1939.

This amounts to a basic bibliography with running comments, and should be very useful.

The Chapter headings are:

Part I

Traditional Logic. Symbolic Logic I--The Propositional Calculus. Symbolic Logic II--The Restricted Calculus of Predicates. Further Development of Symbolic Logic.

Part II

The Critical Movement in Mathematics in the Nineteenth Century. The Logistic Identification of Mathematics with Logic. Formalized Mathematics and Metamathematics. Gődel's Theorems on the Inherent Limitations of Formal Systems, Intuitionism, Recursive Arithmetic, The Axiomatic Theory of Sets.

Part III

The Epistomological Status of Mathematics, The Application of Mathematics to the Natural World, Logic and the Activity of Thinking.

Appendix. Developments since 1939 in Foundations.

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Representations of Groups with special consideration for the needs of Modern Physics, by H. Boerner. J. Wiley and Sons Publ. Inc., New York, 1963. xii + 325 pages.

This book is a translation, with only minor changes from the German edition [Grundlehren der Math. Wiss. Bd 74 Berlin-Gottingen-Heidelberg, Springer-Verlag, 1955]. It is a self-contained treatment of representations of the classical groups written primarily for physicists. There are preliminary chapters on vector spaces and matrices, groups and the general theory of group representations. The remaining chapter headings are: Representations of the symmetric group; Representations of the full linear, unimodular and unitary groups; Characters of the linear and permutation groups; Characters and single-valued representations of the rotation group; Spin representations, infinitesimal ring, ordinary rotation group; The Lorentz group.

The original German edition was reviewed in Math. Reviews 17(1956), 710 and in Amer. Math. Soc. Bulletin 63 (1957), 204.

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